# Last class

- Greedy best-first search
- A* search
- Recursive best-first search
- Heuristic generation
- Heuristic goodness

Informed (heuristic) search

*Uses problem-specific knowledge beyond the problem definition*

南京大学
人工智能学院
SCHOOL OF ARTIFICIAL INTELLIGENCE, NANJING UNIVERSITY

# Heuristic Search and Evolutionary Algorithms

## Lecture 4: Local Search and Evolutionary Algorithms

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# Classical search

A search problem can be defined formally by five components:

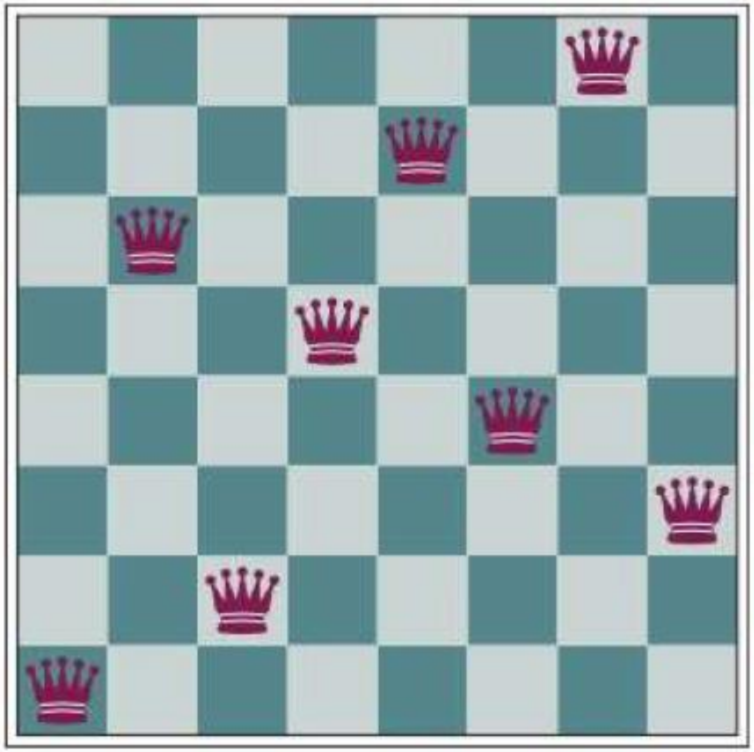
- Initial state
- Actions
- Transition model
- Goal test
- Path cost

Solution: a path (i.e., an action sequence) from the initial state to a goal state

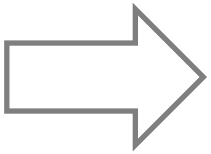
Optimal solution: a path with the lowest cost

# Search example: Path is irrelevant

8-queens problem: to place eight queens on a chessboard such that no queen attacks any other



Heuristic function $h$: number of pairs of queens that are attacking each other



What is a goal state, i.e., a state with $h = 0$?

The path to the goal state is irrelevant

# Search and optimization

General Search: to find a goal state, i.e., a state with $h = 0$

⇩

Optimization: to find an optimal solution

$$\arg\min_x h(x) \quad \text{or} \quad \arg\max_x f(x)$$

Note that: classical search can be transformed into this form by treating an action sequence as a solution and the cost as the objective to be minimized

# Hill-climbing search

Hill-climbing search: maintain only the current state

**function** HILL-CLIMBING(*problem*) **returns** a state that is a local maximum
    *current* ← *problem*.INITIAL
    **while** *true* **do**
        *neighbor* ← a highest-valued successor state of *current*
        **if** VALUE(*neighbor*) ≤ VALUE(*current*) **then return** *current*
        *current* ← *neighbor*

Select the best neighbor state

Stop until no neighbor has a higher objective value

Need to define a neighbor space

# Hill-climbing search – example

8-queens problem: to place eight queens on a chessboard such that no queen attacks any other

Heuristic function *h*: number of pairs of queens that are attacking each other


The current *h* value: 17

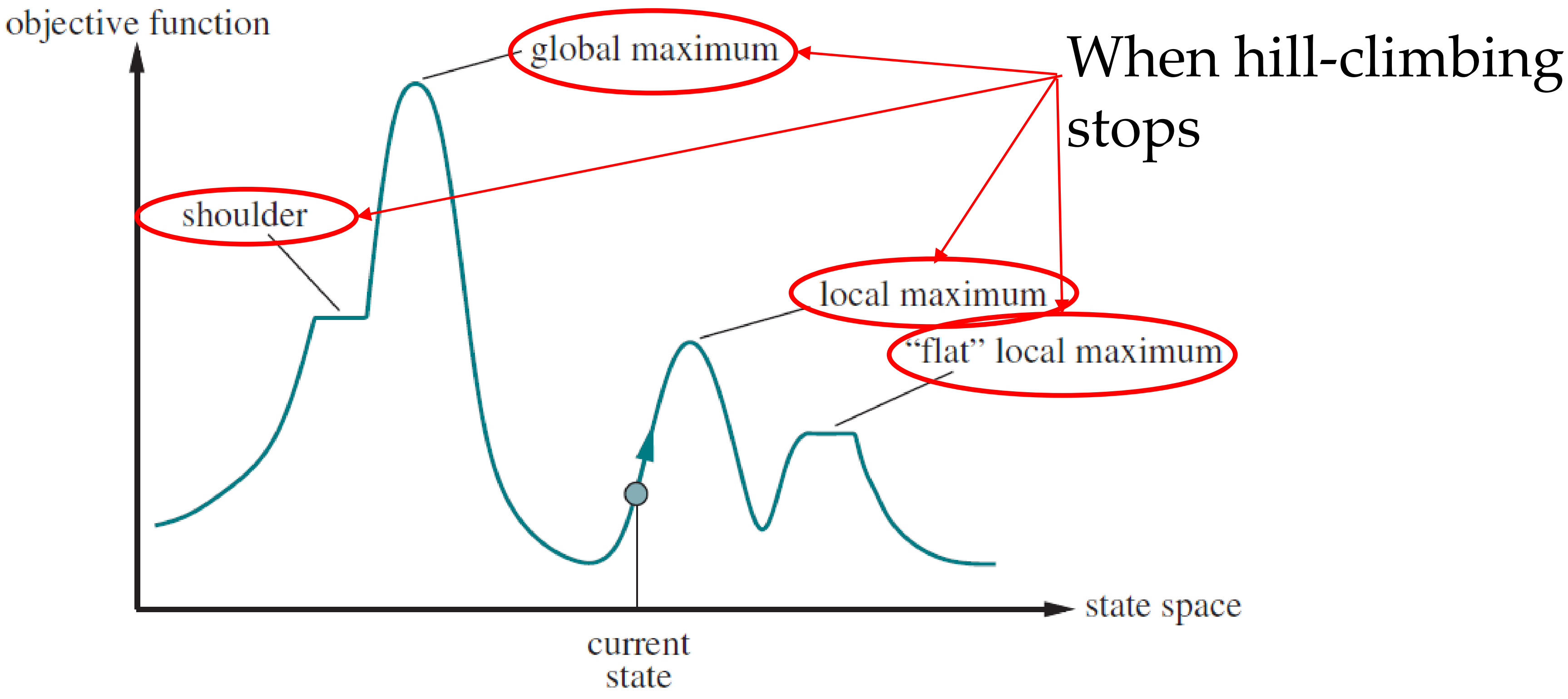
Neighbor space: states generated by moving a single queen to another square in the same column

The number of neighbors: 56

Move to the best neighbor with *h* value 12

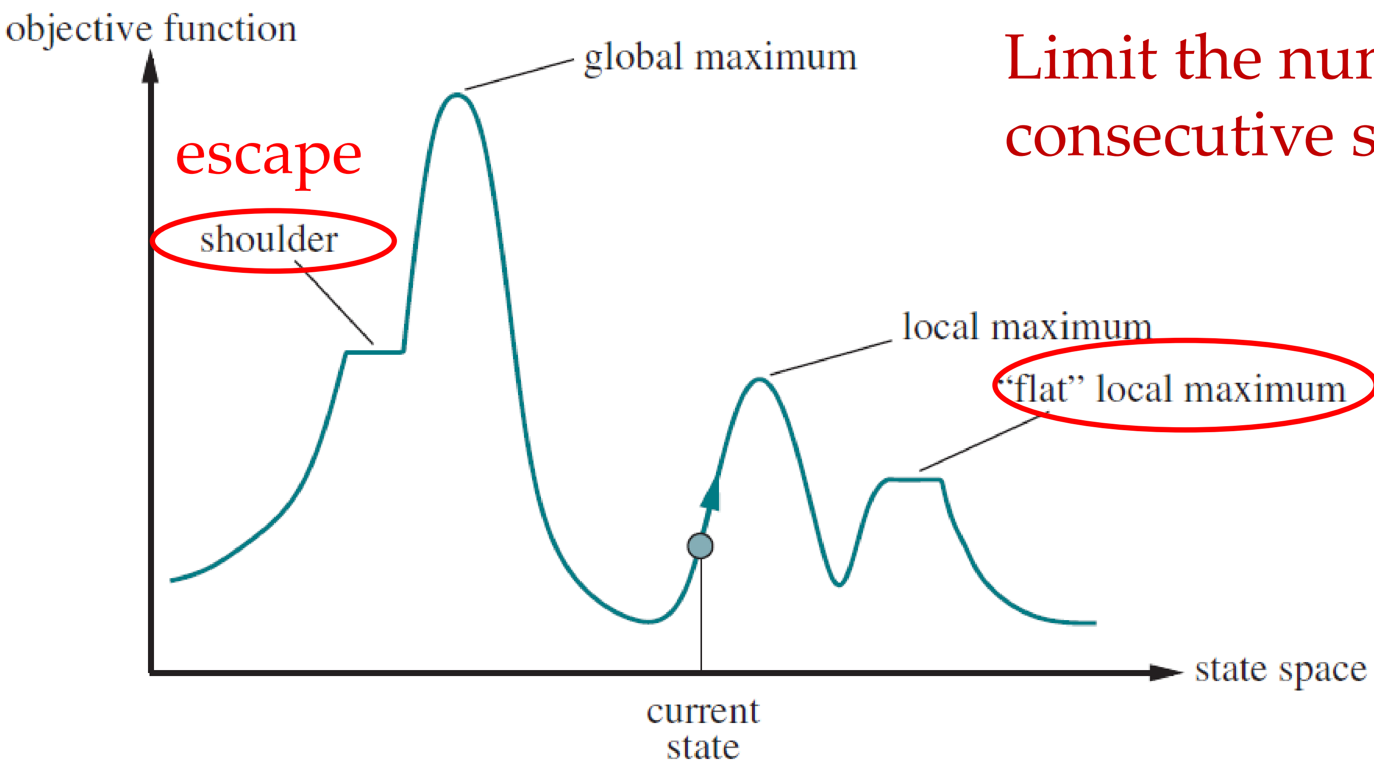
# Hill-climbing search

An example of one-dimensional state-space landscape



# Hill-climbing search

Hill-climbing search with sideways move: accept the best neighbor if it has the same value as the current state




Limit the number of consecutive sideways moves

infinite loop

# Hill-climbing search

8-queens problem: to place eight queens on a chessboard such that no queen attacks any other

Heuristic function *h*: number of pairs of queens that are attacking each other

Neighbor space: states generated by moving a single queen to another square in the same column

| Hill-climbing | Without sideways move | With sideways move |
| --- | --- | --- |
| Success rate | 14% | 94% |
| Average steps for a success | 4 steps | 21 steps |

# Random-restart hill-climbing search

Random-restart hill-climbing search: conduct a series of hill-climbing searches from randomly generated initial states

Given unlimited time, it will eventually find a goal state

The success probability of each hill-climbing search: $p$

⇩ geometric distribution with parameter $p$

The expected number of restarts: $1/p$

# Variants of hill-climbing search

hill-climbing: move to the best neighbor state

Stochastic hill-climbing: find all better neighbor states, and select one as the next state with probability related to its objective value

First-choice hill-climbing: repeatedly generate neighbor states randomly, and select the first better neighbor as the next state

Can be applied to continuous spaces

# Simulated annealing

Hill-climbing search: efficient, but may get trapped in local optima

Random search: find global optima, but inefficient

Simulated annealing

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
  current ← problem.INITIAL
  for t = 1 to ∞ do
      T ← schedule(t)
      if T = 0 then return current
      next ← a randomly selected successor of current
      ΔE ← Value(next) − Value(current)
      if ΔE > 0 then current ← next
      else current ← next only with probability e^(ΔE/T)
```

randomly generate a neighbor

if the neighbor is better, move to it

Otherwise, move to the worse state with some probability

# Simulated annealing

## Simulated annealing

**function** SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state
    *current* ← *problem*.INITIAL
    **for** $t = 1$ **to** $\infty$ **do**
        $T \leftarrow schedule(t)$
        **if** $T = 0$ **then return** *current*
        *next* ← a randomly selected successor of *current*
        $\Delta E \leftarrow Value(next) - Value(current)$
        **if** $\Delta E > 0$ **then** *current* ← *next*
        **else** *current* ← *next* only with probability $e^{\Delta E/T}$

randomly generate a neighbor

if the neighbor is better, move to it

Otherwise, move to the worse state with some probability

Can be applied to both discrete and continuous spaces

# Simulated annealing

## Simulated annealing

**function** SIMULATED-ANNEALING($problem$, $schedule$) **returns** a solution state
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randomly generate a neighbor

if the neighbor is better, move to it

Otherwise, move to the worse state with some probability

The probability $e^{\Delta E/T}$ of accepting the worse state

- Increase with $\Delta E$
- Increase with the temperature parameter $T$

# Simulated annealing

## Simulated annealing

The probability $e^{\Delta E/T}$ of accepting the worse state

- Increase with $\Delta E$
- Increase with the temperature parameter $T$

$T$ is initially set to a large value, and gradually decreased to 0

The probability of accepting worse states gradually decreases

Inspired from the annealing process in metallurgy

# Local beam search

Local beam search: maintain $k$ states

- The initial $k$ states are generated randomly
- In each iteration, generate all neighbors of the current $k$ states, and select the best $k$ ones

Different from hill-climbing search with $k$ random-restarts

Can be applied to discrete spaces

# Local search for continuous spaces

Gradient descent: for minimization

$$\boldsymbol{x} = \boldsymbol{x} - \alpha \cdot \nabla f(\boldsymbol{x})$$

Gradient ascent: for maximization

$$\boldsymbol{x} = \boldsymbol{x} + \alpha \cdot \nabla f(\boldsymbol{x})$$

Converge to $\nabla f(\boldsymbol{x}) = 0$: local optimum or saddle point

There are many variants of gradient descent/ascent, as well as methods using the Hessian matrix, e.g., Newton-Raphson

$$\boldsymbol{x} = \boldsymbol{x} + \mathbf{H}_f^{-1}(\boldsymbol{x}) \cdot \nabla f(\boldsymbol{x})$$

# The theory of evolution

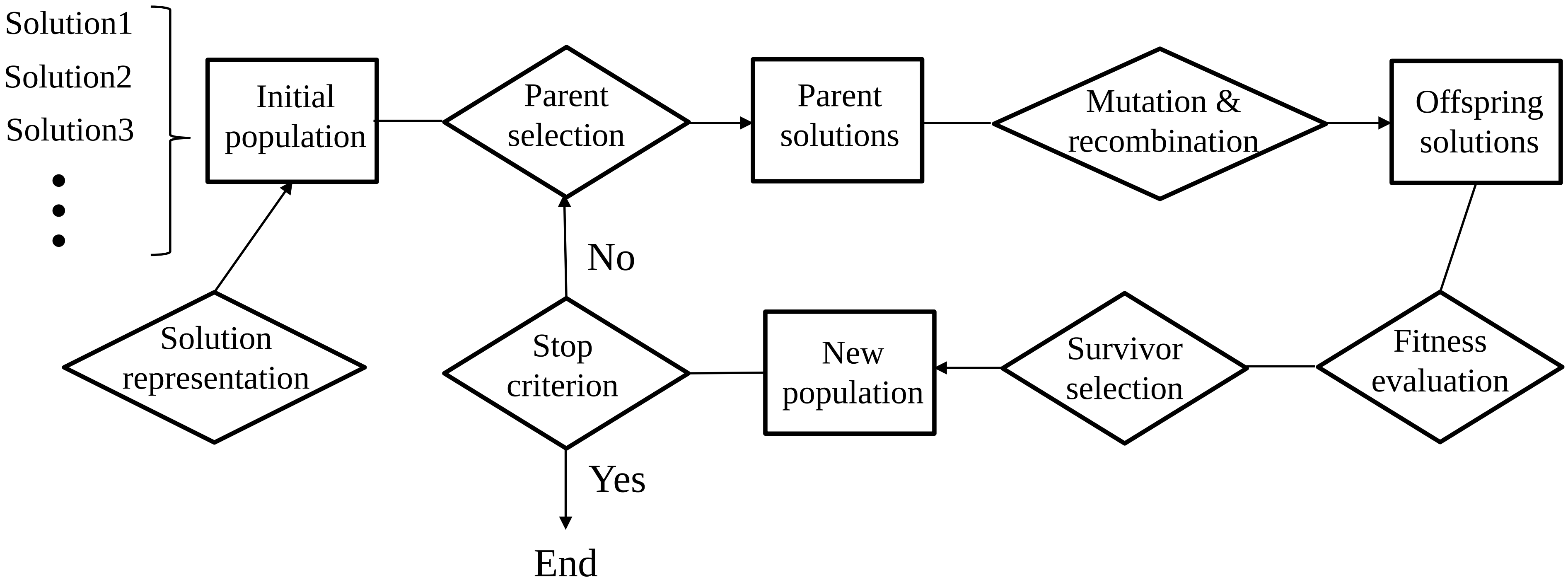
Central idea of Darwinism: reproduction with variation and natural selection based on the fitness

Core components of Darwinian evolutionary system:

- *One or more populations of individuals competing for limited resources*
- *The notion of dynamically changing populations due to the birth and death of individuals*
- *A concept of fitness which reflects the ability of an individual to survive and reproduce*
- *A concept of variational inheritance: offspring closely resemble their parents, but are not identical*

# Evolutionary algorithms

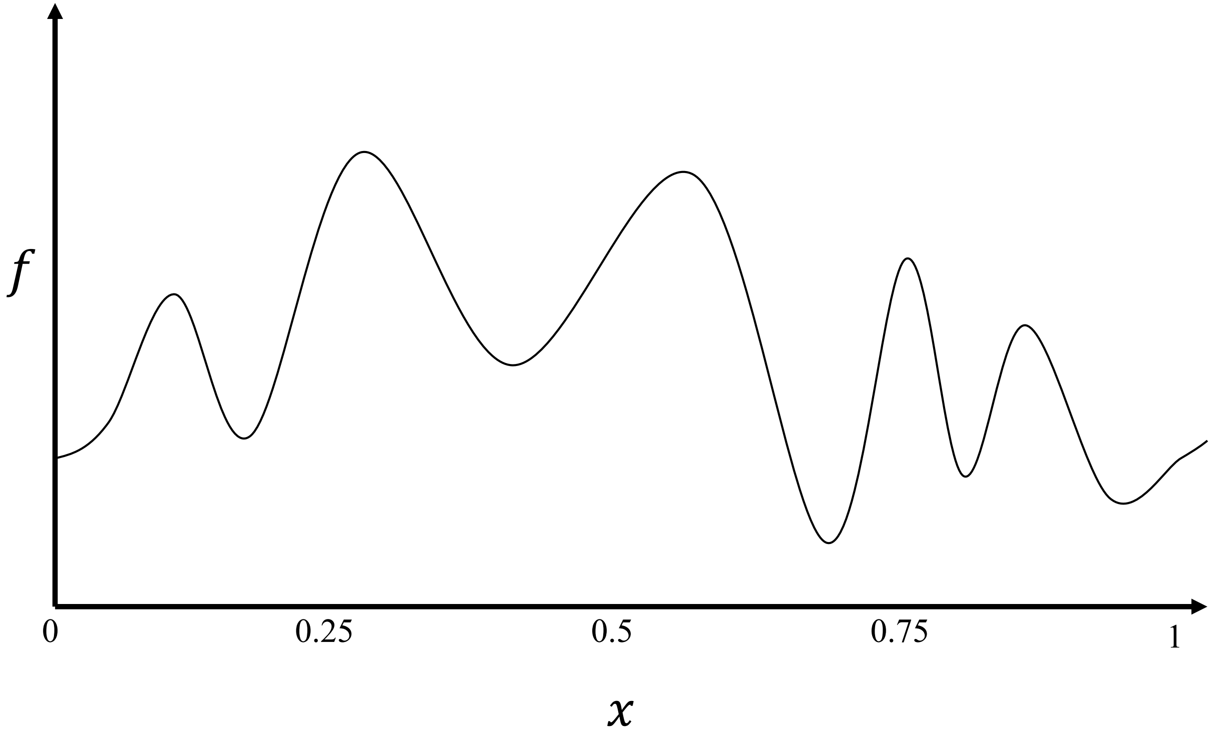
General structure of evolutionary algorithms for $\arg\max_x f(x)$




Can be applied to both discrete and continuous spaces

# An illustration of running

$\mathcal{X} = [0, 1]$



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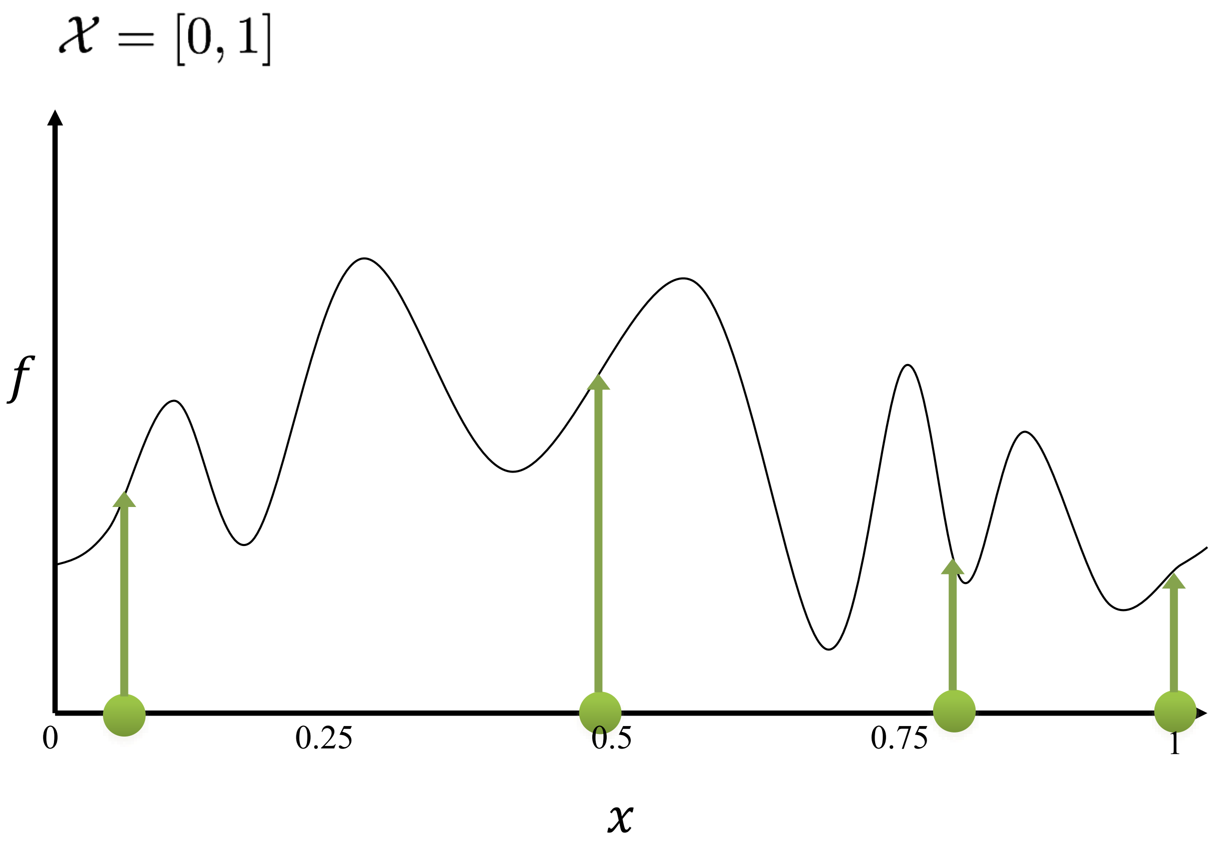
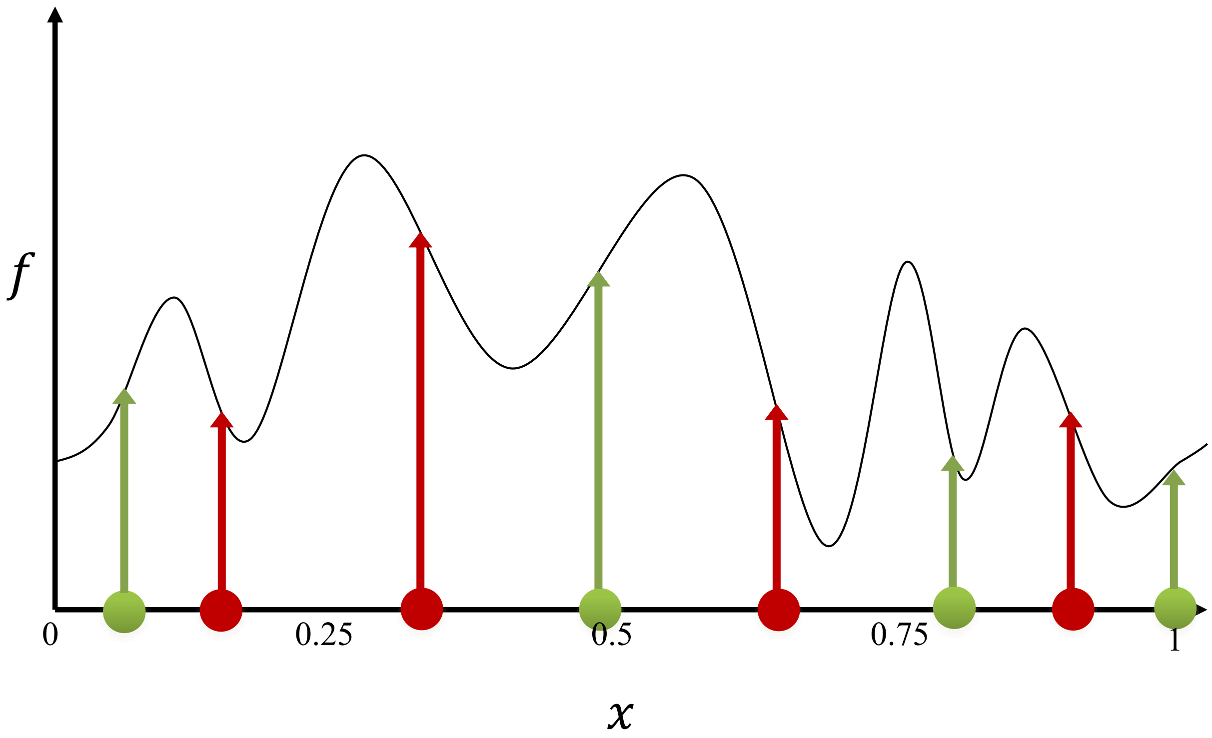



initialization
evaluation

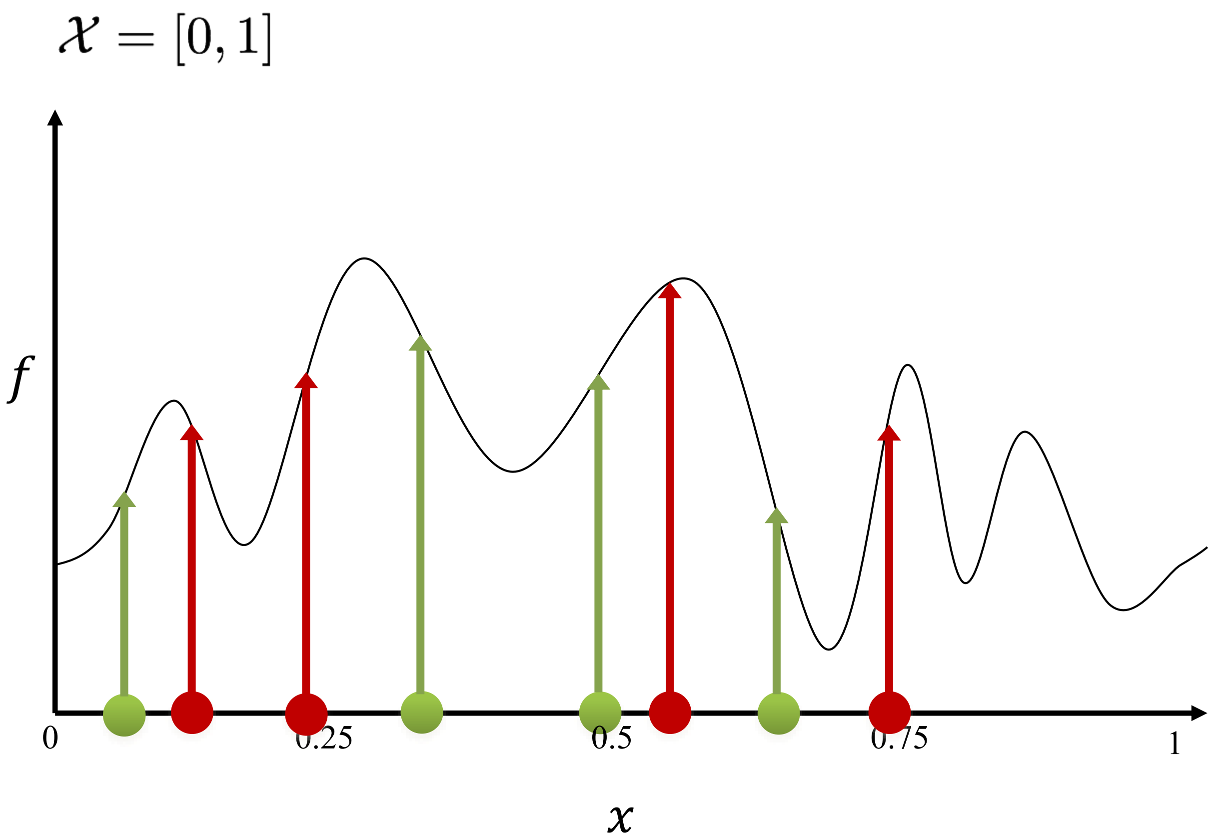
# An illustration of running

$\mathcal{X} = [0, 1]$



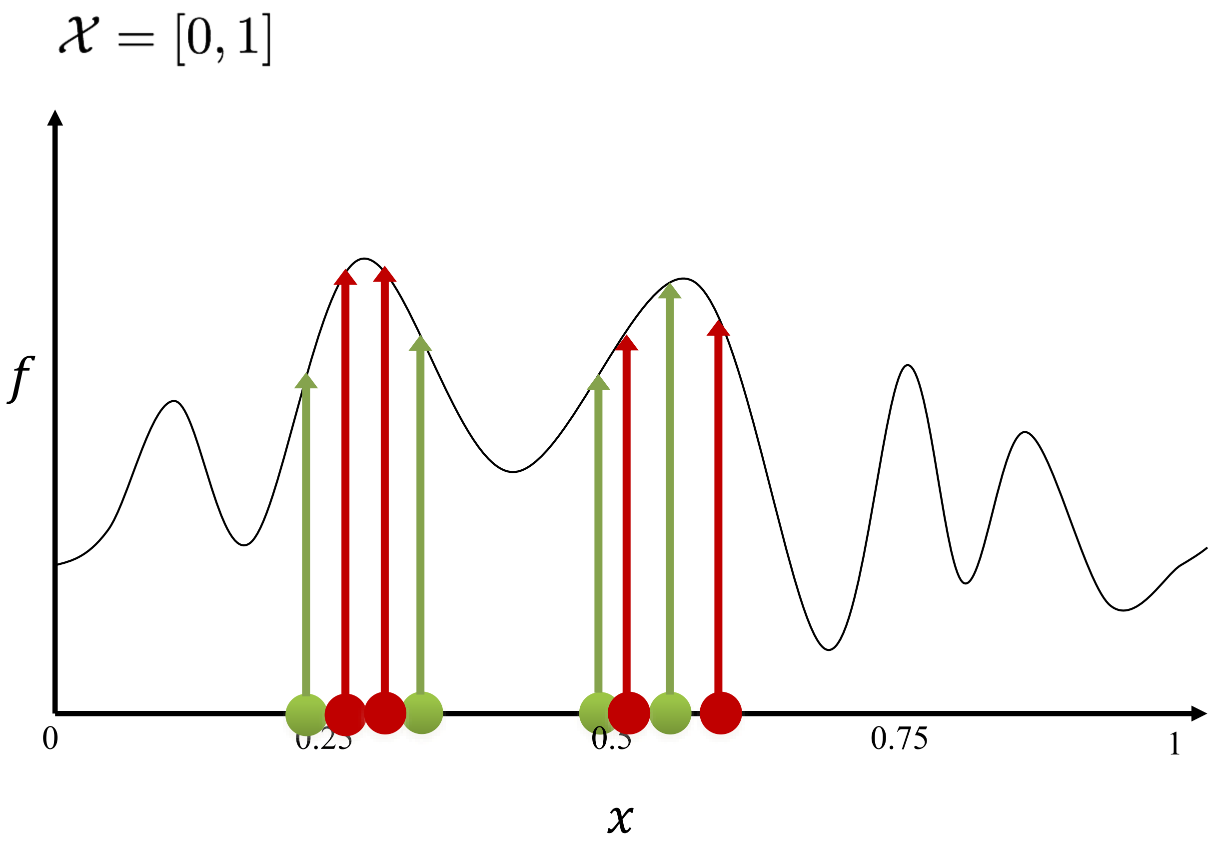

initialization
evaluation
reproduction
evaluation

# An illustration of running



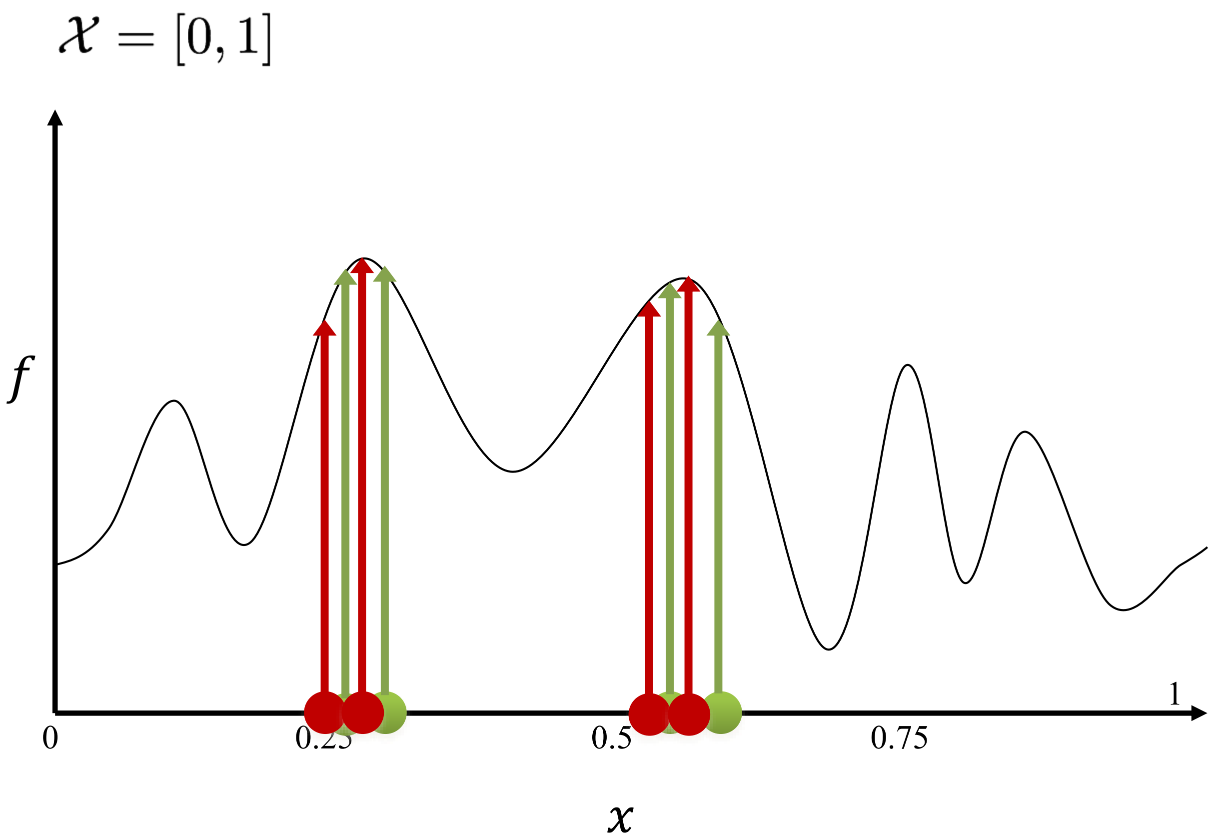

initialization
evaluation
reproduction
evaluation
selection
reproduction
evaluation

# An illustration of running




initialization
evaluation
reproduction
evaluation
selection
reproduction
evaluation
selection
reproduction
evaluation

# An illustration of running




initialization
evaluation
reproduction
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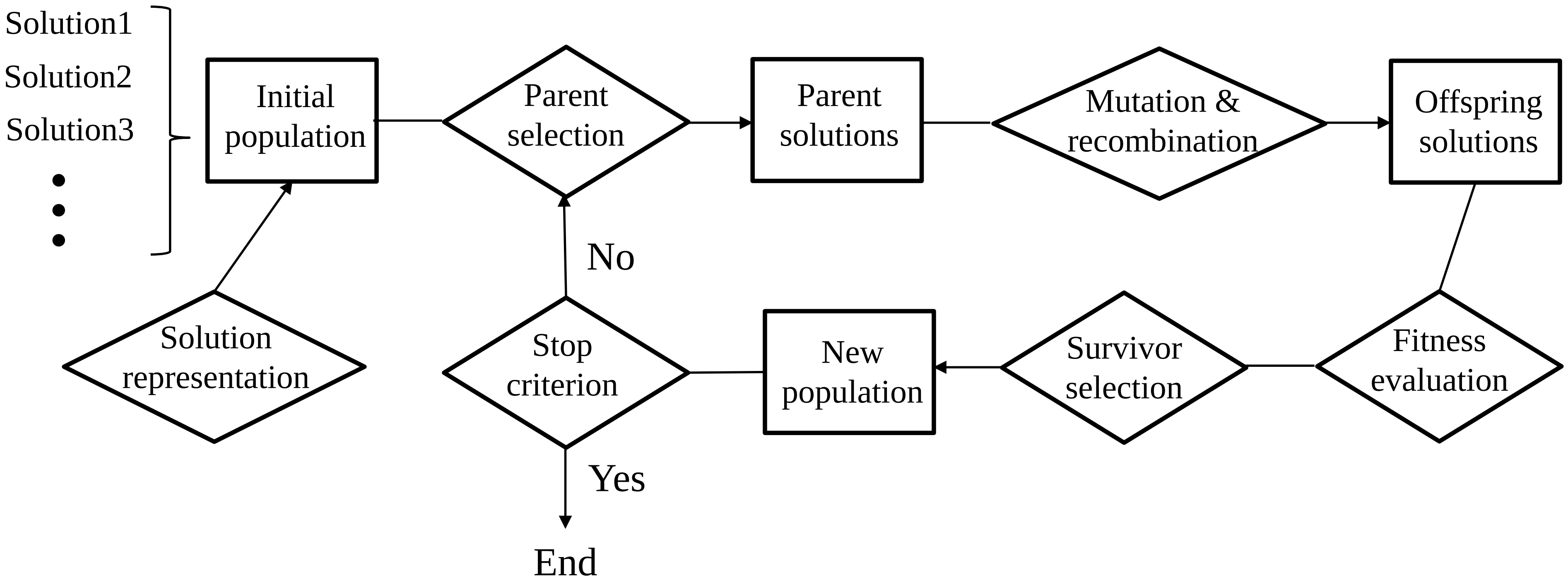
# Evolutionary algorithms

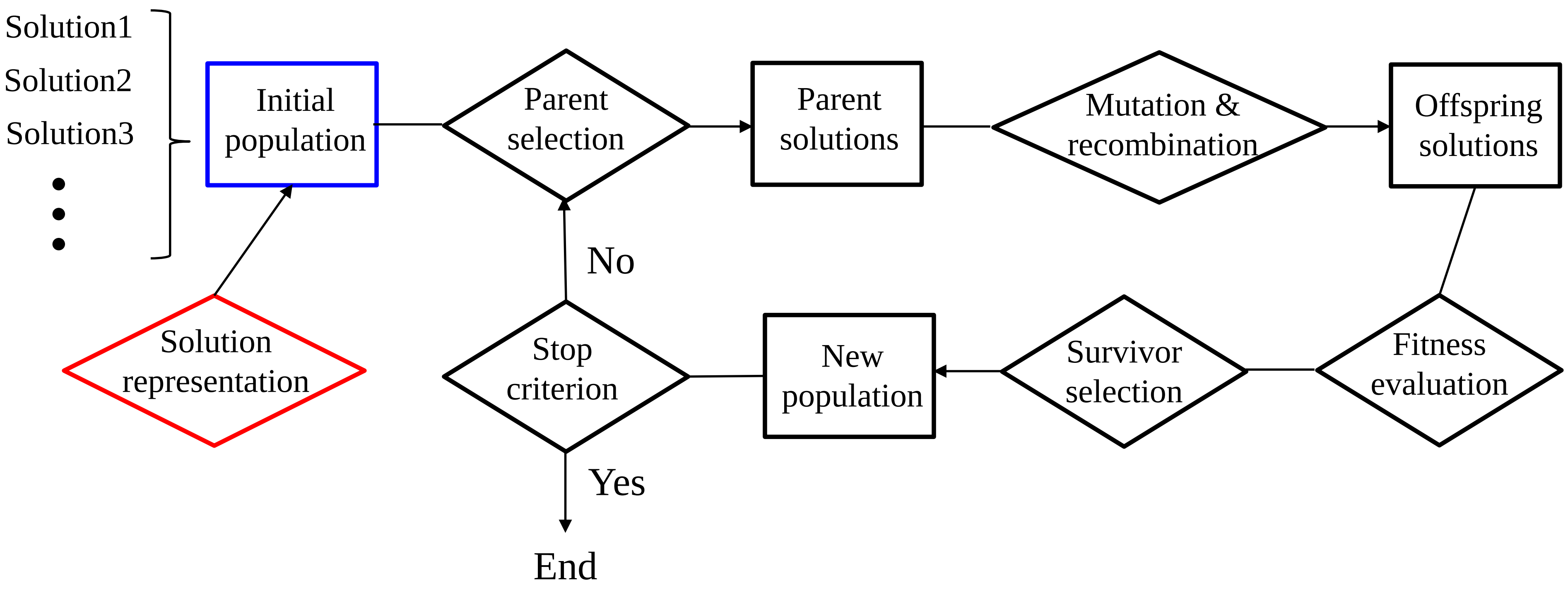
## General structure of evolutionary algorithms




Need to design each component of evolutionary algorithms

# Evolutionary algorithms

## General structure of evolutionary algorithms

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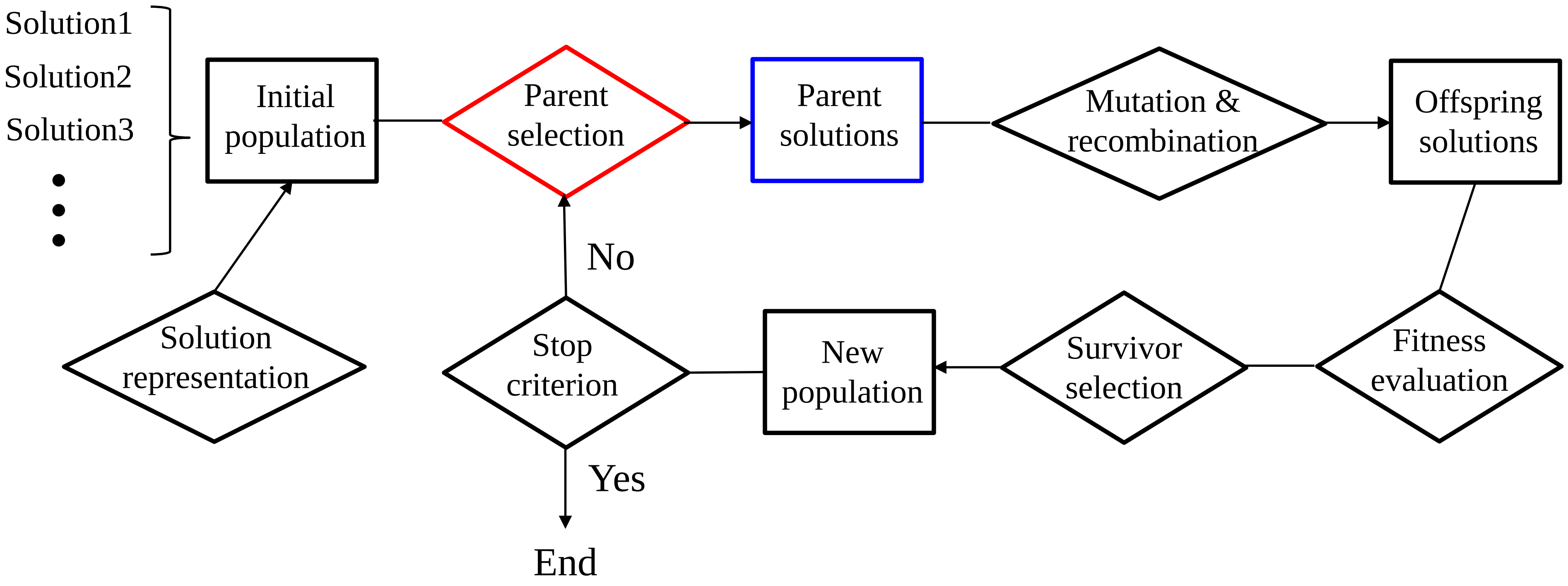
# Evolutionary algorithms

## General structure of evolutionary algorithms

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# Evolutionary algorithms

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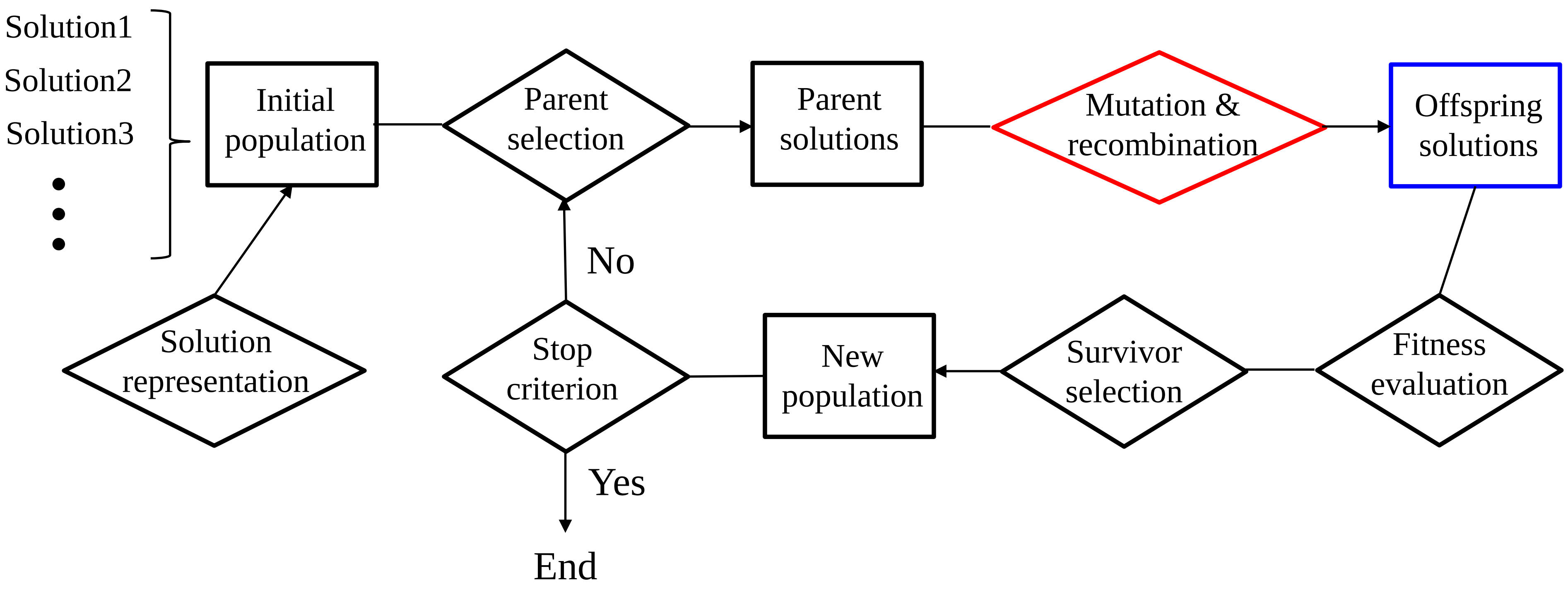


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# Evolutionary algorithms

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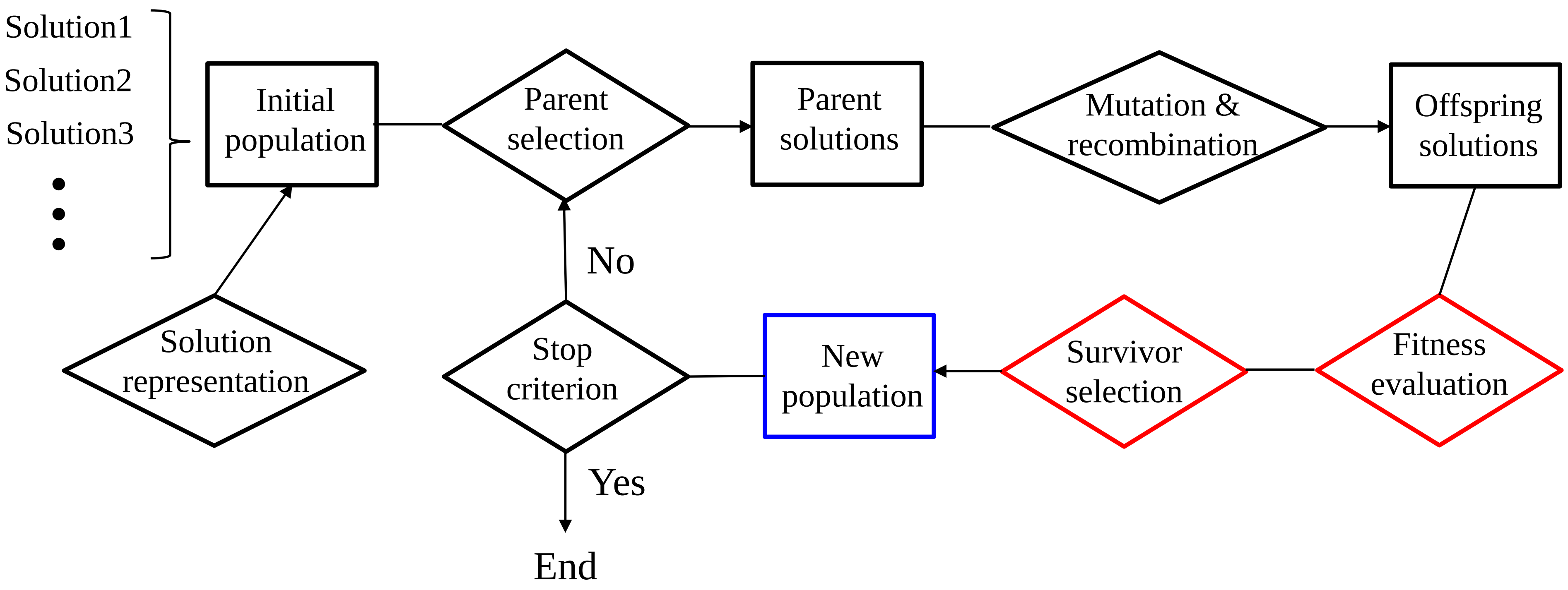


Need to design each component of evolutionary algorithms

# Evolutionary algorithms

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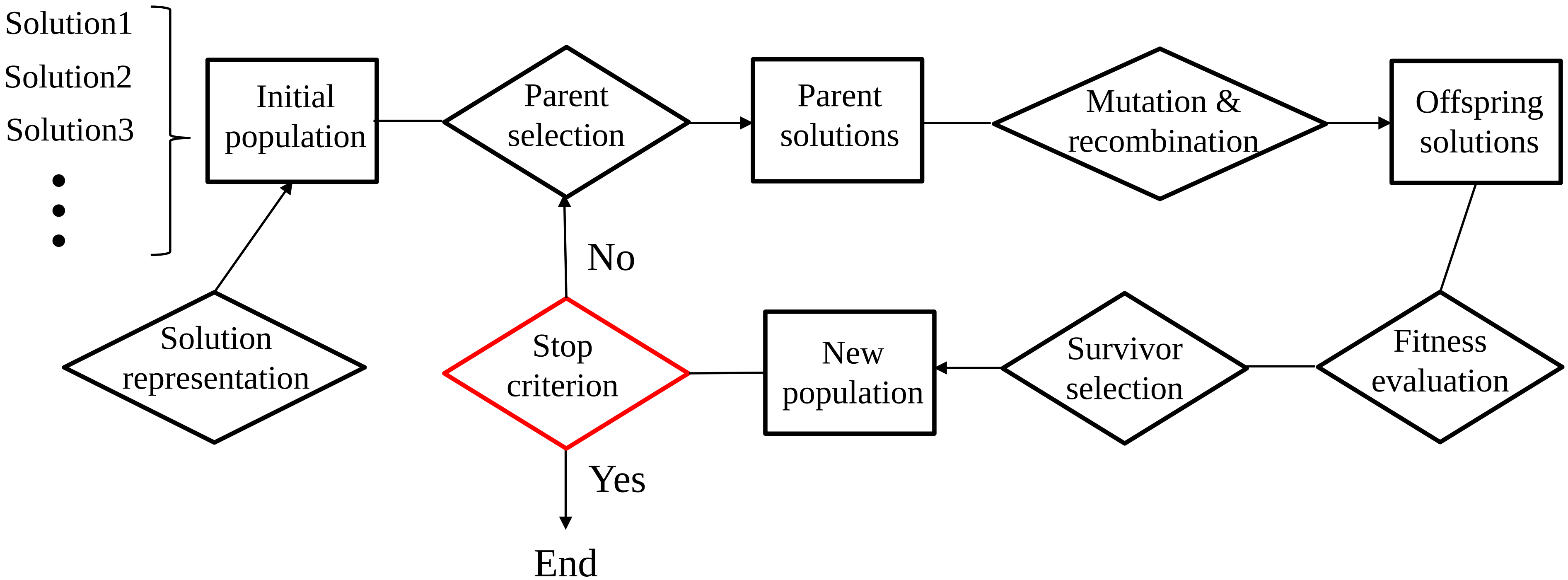
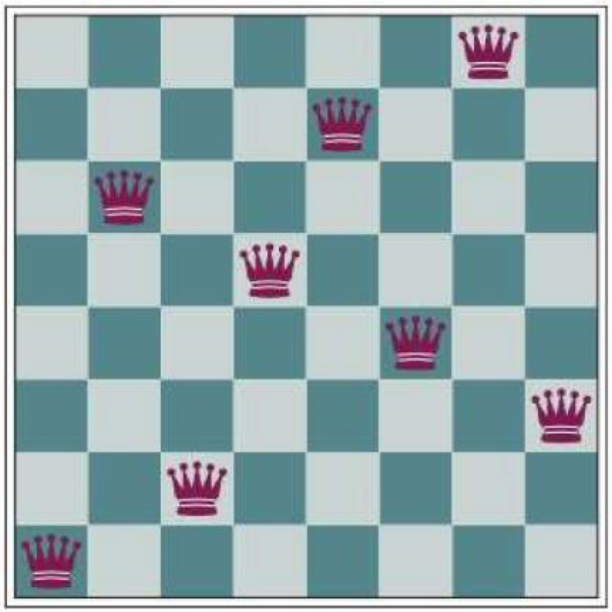



Need to design each component of evolutionary algorithms

# An application to 8-queens problem

8-queens problem: to place eight queens on a chessboard such that no queen attacks any other

Objective function $f$: number of nonattacking pairs of queens



Solution representation

Integer vector

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 6 | 2 | 5 | 7 | 4 | 8 | 3 |

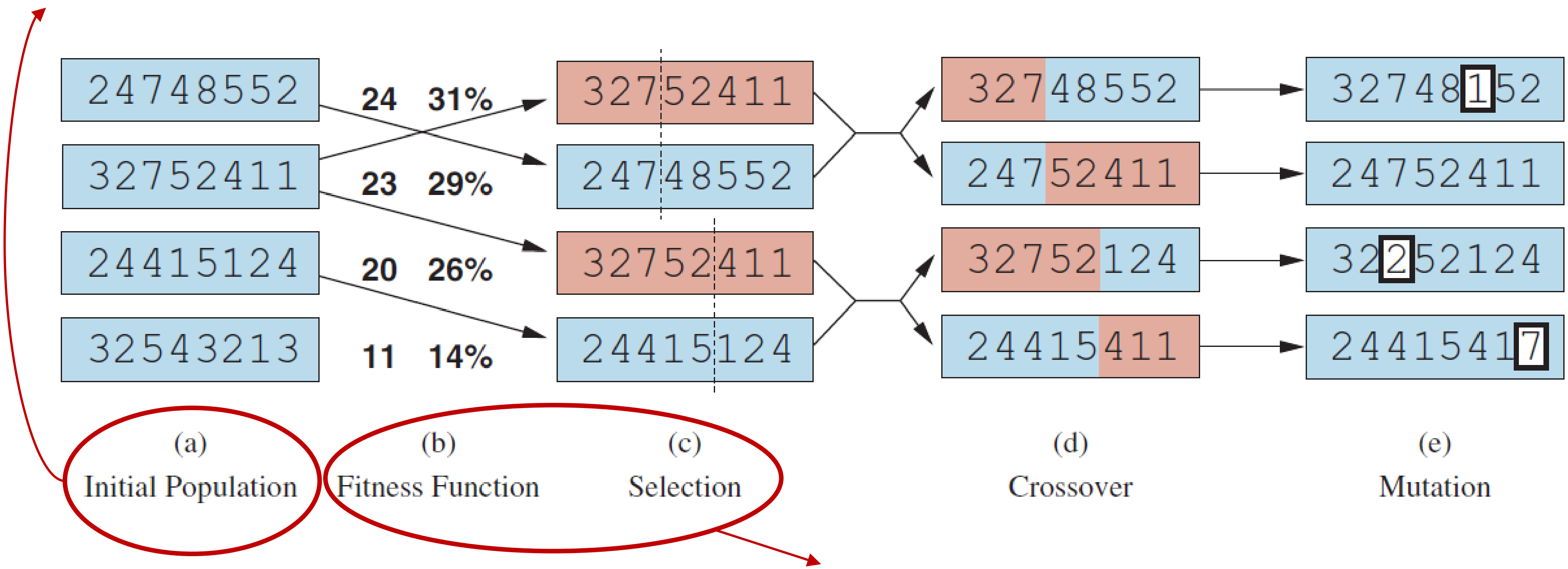
position of the queen on each column

Binary vector

000101001100110011111010

# An application to 8-queens problem

Initialization: four randomly generated solutions

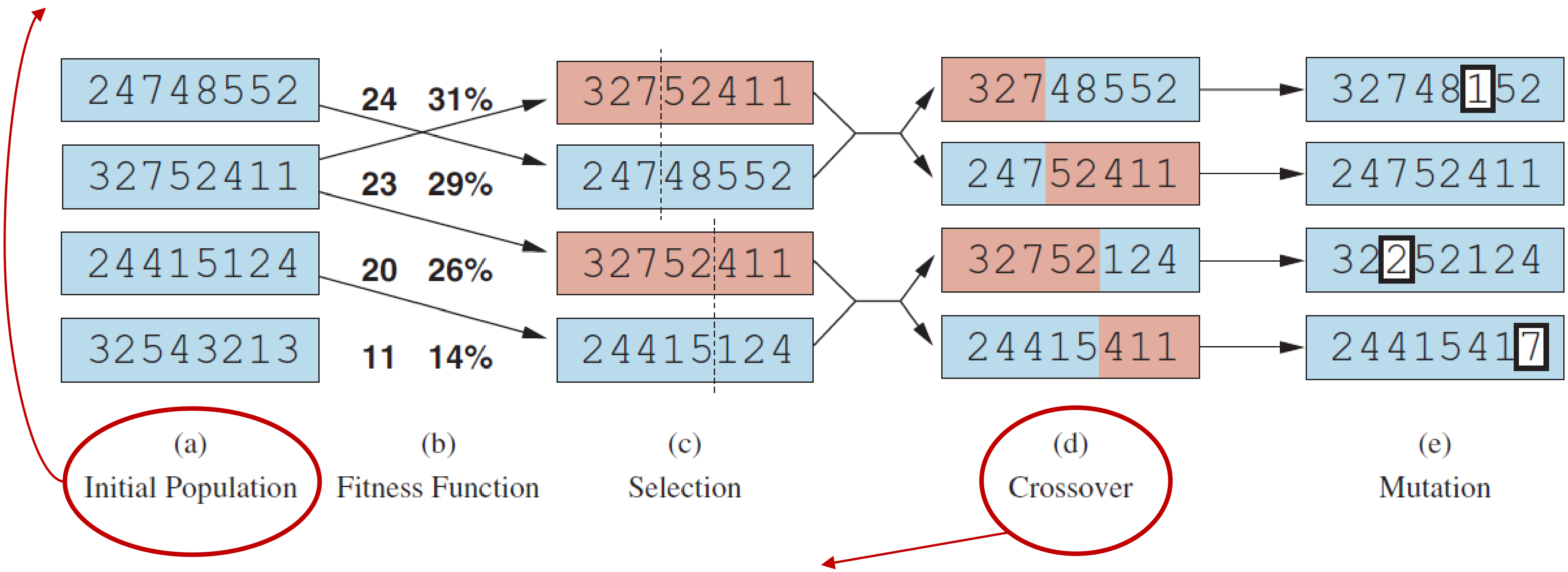



Parent selection: fitness proportional selection

Probability of selecting the $i$-th solution ⟵ $p_i = \frac{f_i}{\sum_{j=1}^{\mu} f_j}$ ⟶ Fitness (objective) value of the $i$-th solution

# An application to 8-queens problem

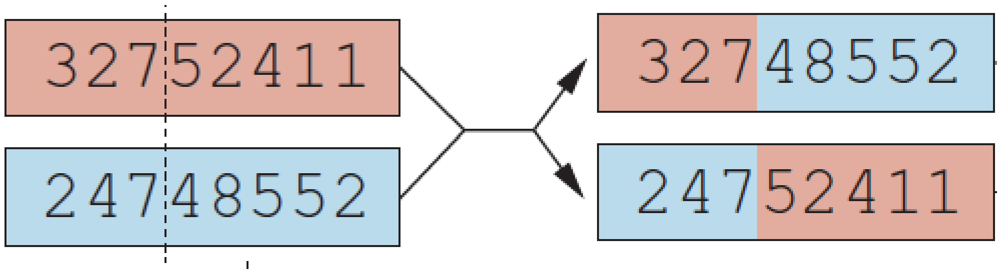
Initialization: four randomly generated solutions

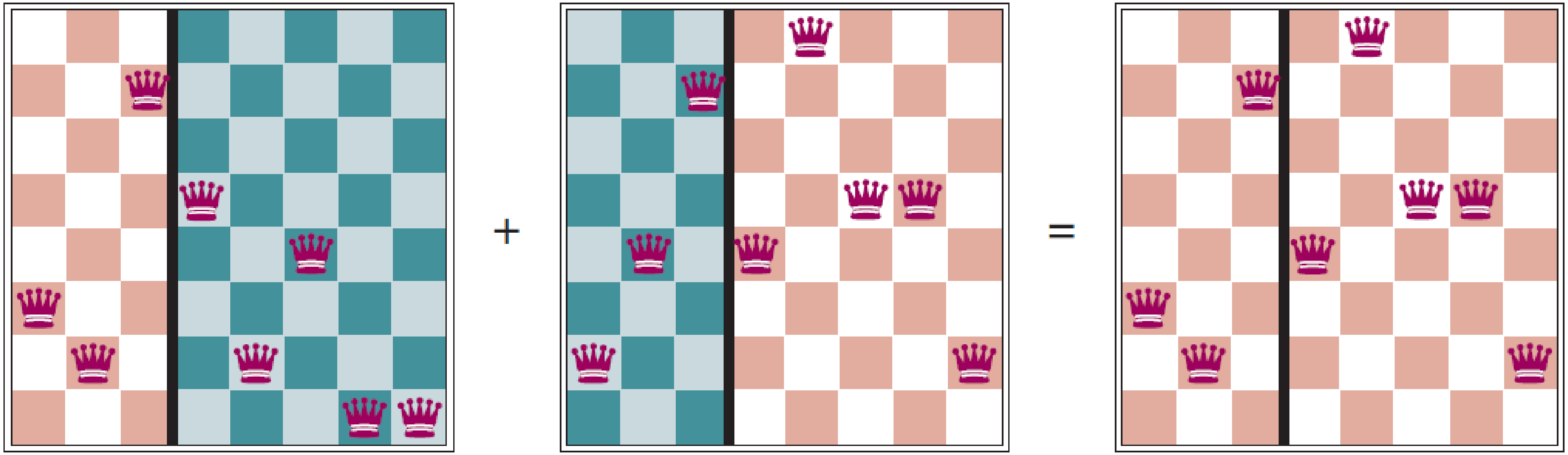



Recombination: one-point crossover

Select one crossover point randomly, and exchange the parts of the two solutions after the point

# An application to 8-queens problem



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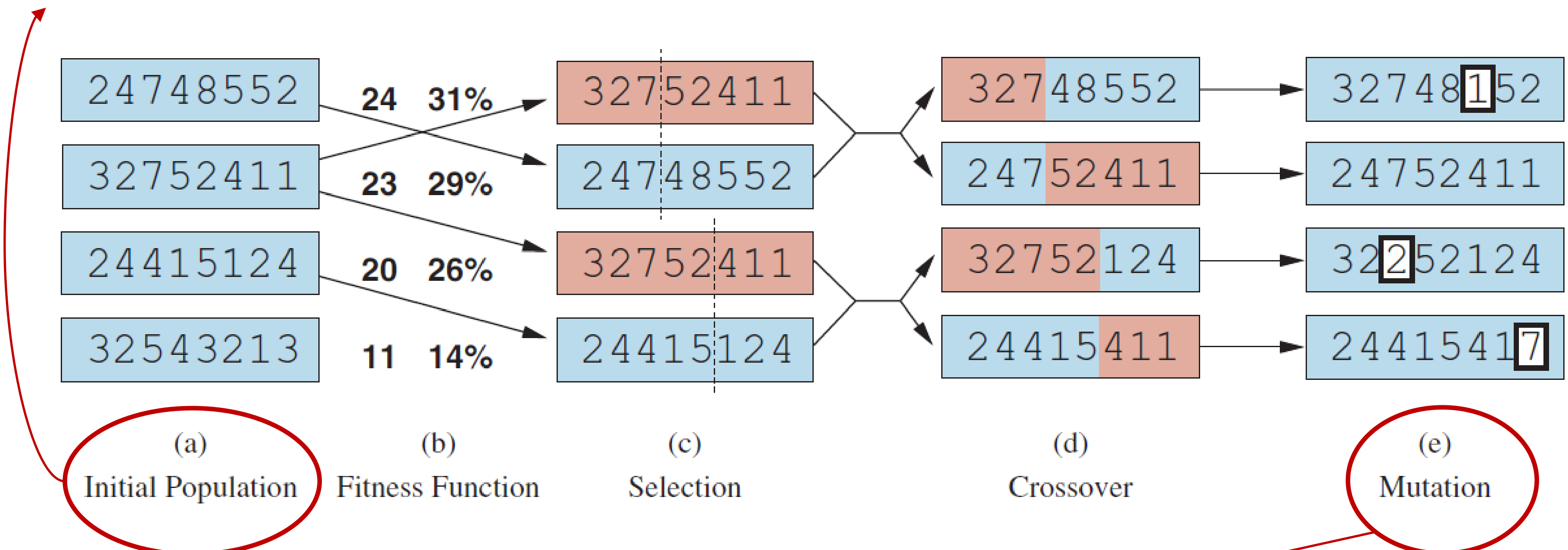
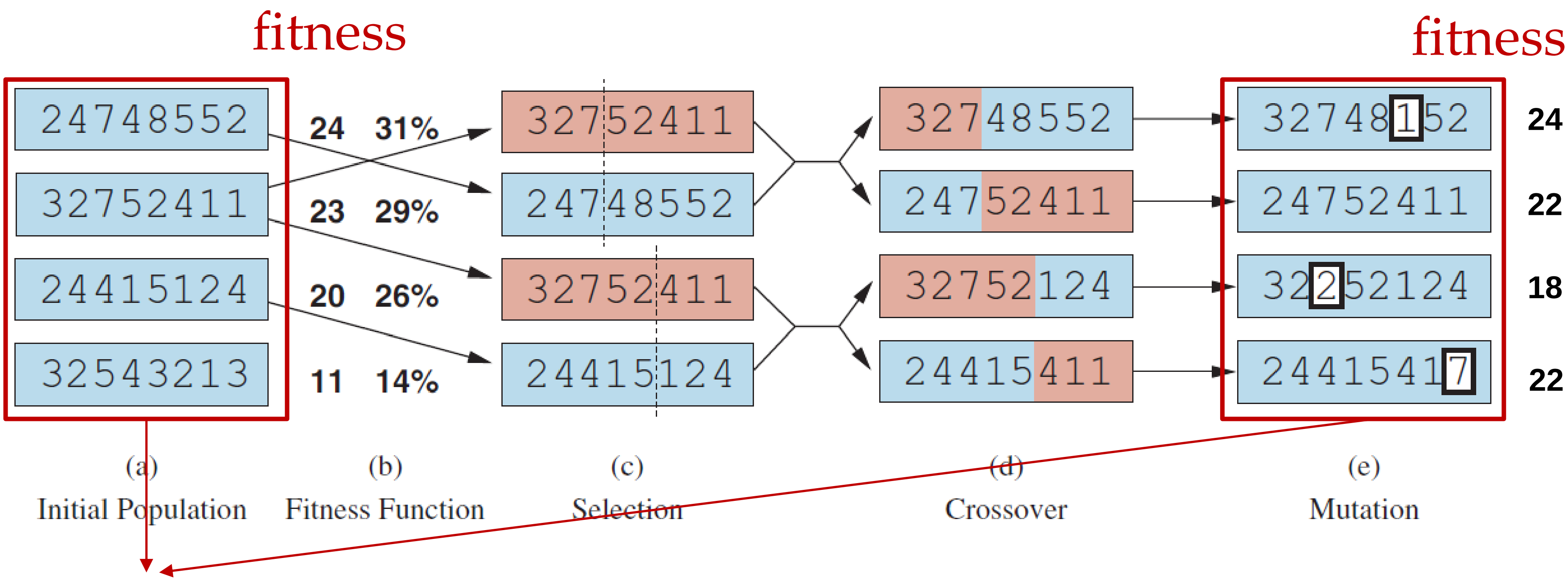
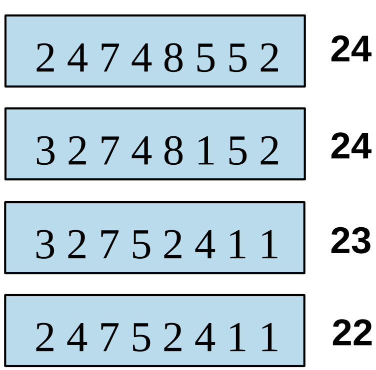



Mutation:

For each element of a solution, change it to a randomly chosen different value with probability 1/8

# An application to 8-queens problem




Survivor selection:

Select the best four solutions from the current population and offspring solutions to generate the next population

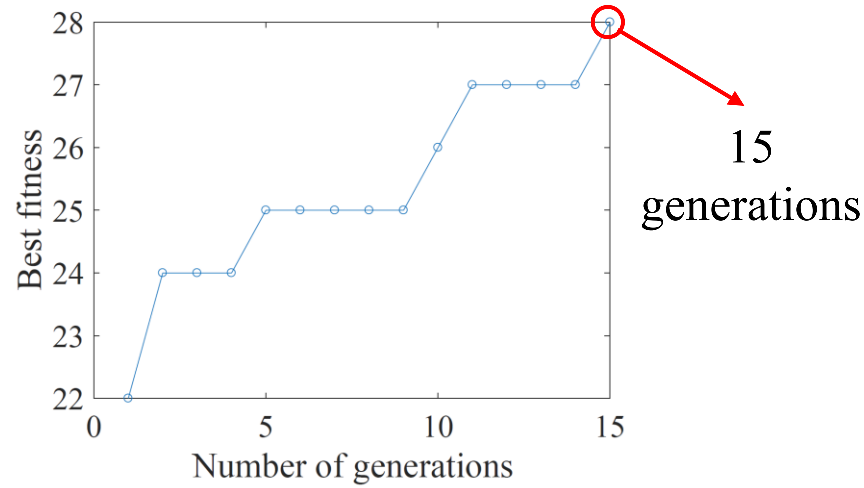
# An application to 8-queens problem

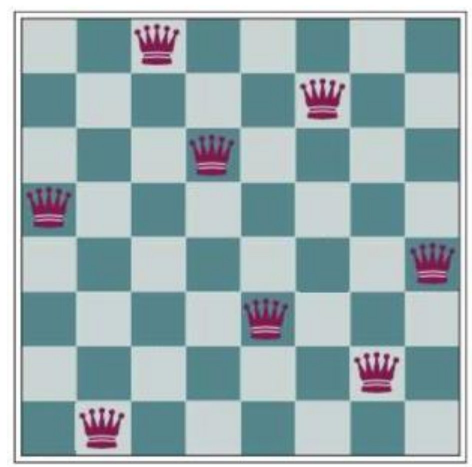
## Run 1:

| Initial population | fitness |
| --- | --- |
| 4 7 8 7 7 2 2 2 | **18** |
| 1 1 8 6 3 5 5 3 | **20** |
| 6 6 7 4 4 5 6 2 | **18** |
| 2 4 1 3 1 6 6 1 | **22** |

| Final population | fitness |
| --- | --- |
| 5 1 8 6 3 7 2 4 | **28** |
| 5 1 8 6 3 7 2 8 | **27** |
| 5 1 8 6 3 7 2 8 | **27** |
| 5 1 8 6 3 7 2 8 | **27** |

Curve change of the best fitness



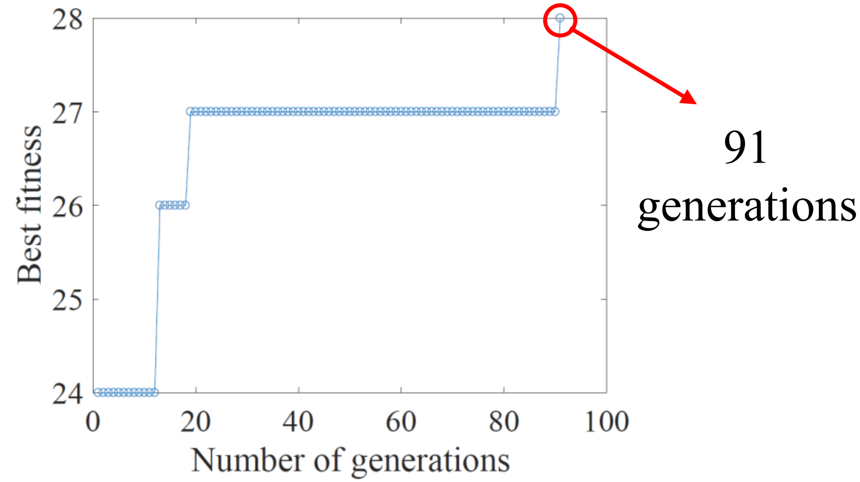
# An application to 8-queens problem

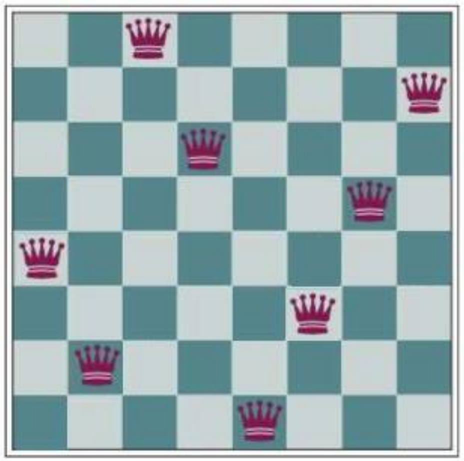
## Run 2:

| Initial population | fitness |
| --- | --- |
| 3 8 8 1 4 3 2 7 | **20** |
| 6 1 4 6 1 3 5 2 | **24** |
| 6 7 1 3 7 4 5 6 | **17** |
| 7 7 8 8 6 2 4 5 | **20** |

| Final population | fitness |
| --- | --- |
| 4 2 8 6 1 3 5 7 | **28** |
| 4 6 8 6 1 3 5 7 | **27** |
| 4 6 8 6 1 3 5 7 | **27** |
| 4 6 8 6 1 3 5 7 | **27** |

Curve change of the best fitness



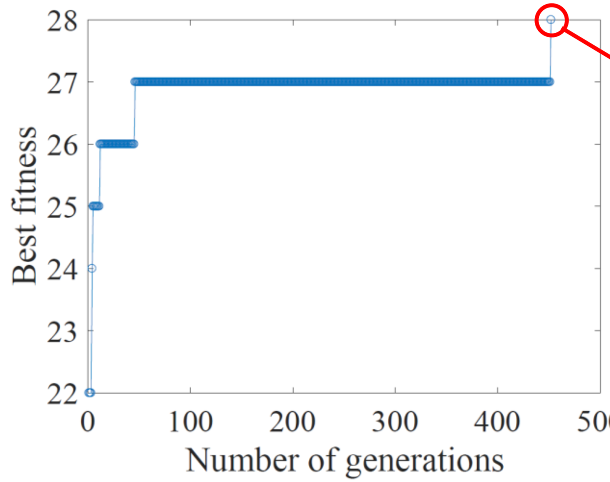
# An application to 8-queens problem

## Run 3:

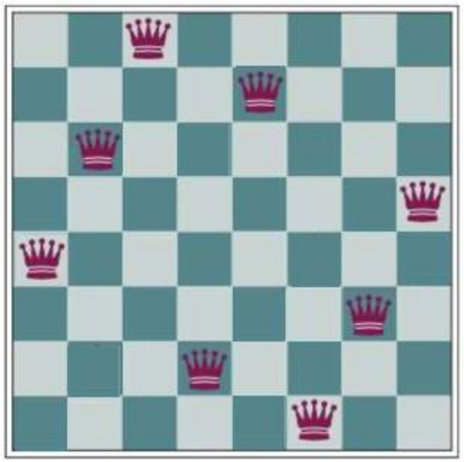
| Initial population | fitness |
|---|---|
| 4 6 5 7 2 5 1 2 | **20** |
| 2 5 7 6 4 3 3 6 | **22** |
| 5 8 7 4 3 5 4 7 | **20** |
| 4 6 2 1 4 4 6 7 | **15** |

| Final population | fitness |
|---|---|
| 4 6 8 2 7 1 3 5 | **28** |
| 4 1 8 2 7 6 3 5 | **27** |
| 4 1 8 2 7 6 3 5 | **27** |
| 4 1 8 2 7 6 3 5 | **27** |

Curve change of the best fitness



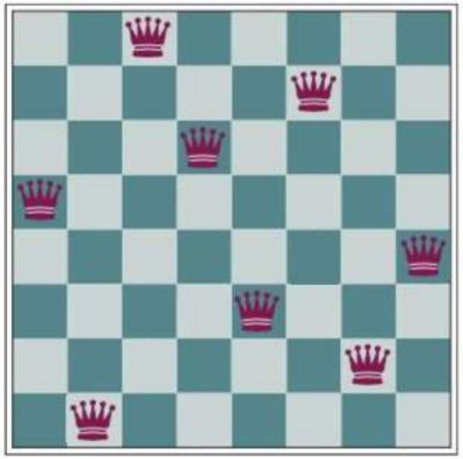
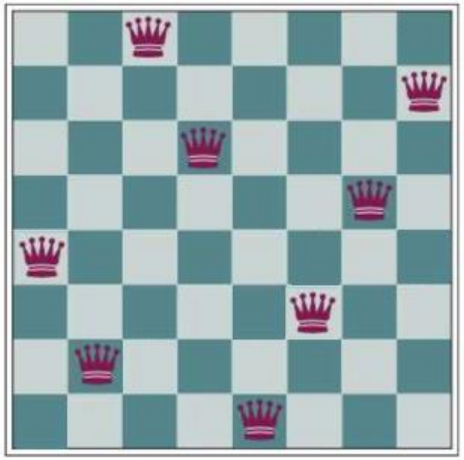
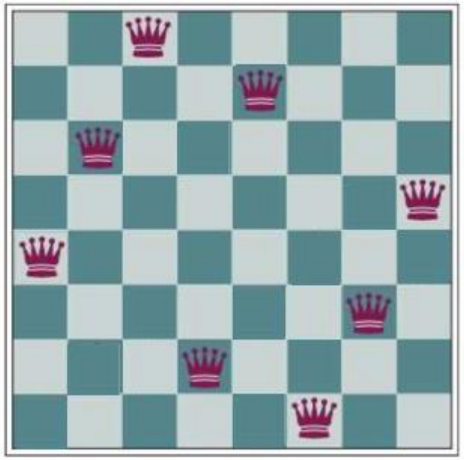

452 generations



# An application to 8-queens problem

Run 1 | Run 2 | Run 3

The generated optimal solution

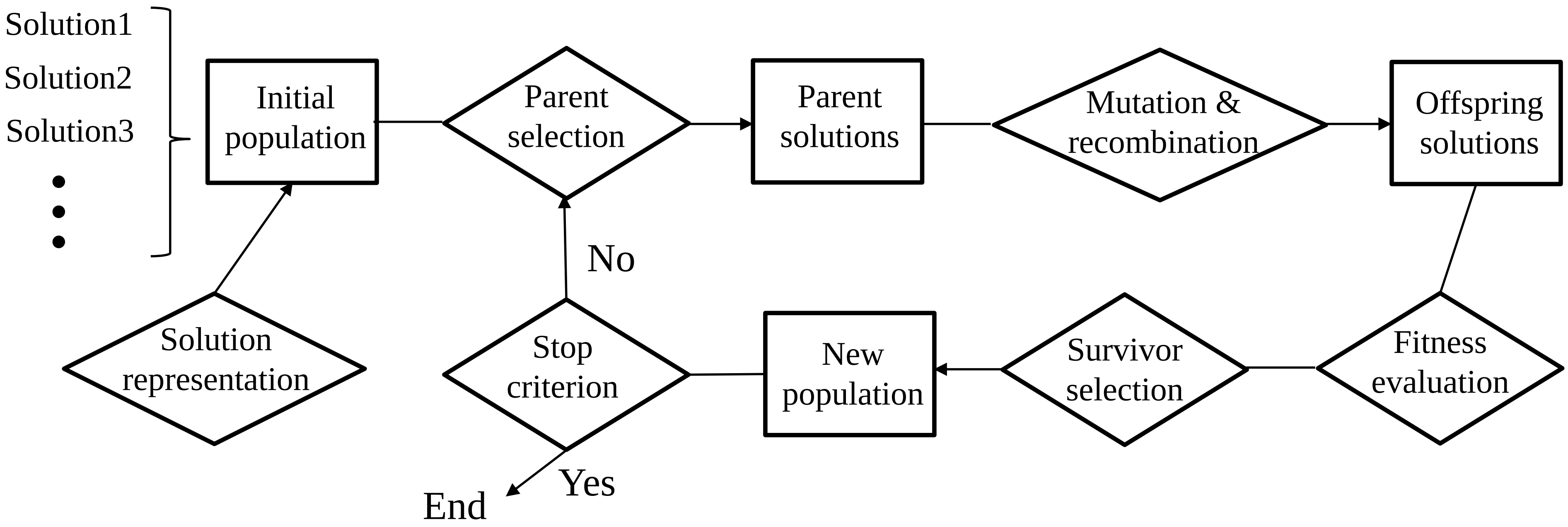




The required number of generations

15 | 91 | 452

Evolutionary algorithms are randomized algorithms

# Local search vs. Evolutionary algorithms




## Characteristics of evolutionary algorithms

- Population-based search
- Recombination
- Mutation, which can be a global search operator

# Local search vs. Evolutionary algorithms

## Advantages and disadvantages of evolutionary algorithms

- Easy to be parallelized
- Good ability of escaping from local optima
- Applicable to a wide range of problems, requiring only that the goodness of solutions can be evaluated
  - *non-differentiable problems*
  - *problems without explicit objective function formulation*
  - *problems with multiple objective functions*
- Not very efficient, but can be accelerated by
  - *utilizing modern computer facilities*
  - *combining with local search*
  - *using the machine learning techniques*

# Summary

- Hill-climbing search
- Simulated annealing
- Local beam search
- Local search for continuous spaces
- Evolutionary algorithms

(Hill-climbing search, Simulated annealing, Local beam search, Local search for continuous spaces: Local search)

# References


- S. J. Russell and P. Norvig. Artificial Intelligence: A Modern Approach. Chapter 4.1-4.2, Third edition.
- K. A. De Jong. Evolutionary Computation – A Unified Approach. Chapter 1.