

Last class

- What is search
- Problem complexity: P, NP, NP-hard, NP-complete
- Tree-search and graph-search
- Performance evaluation criteria
- Asymptotic notations



南 京 大 学
人 工 智 能 学 院

SCHOOL OF ARTIFICIAL INTELLIGENCE, NANJING UNIVERSITY



Heuristic Search and Evolutionary Algorithms

Lecture 2: Uninformed Search

Chao Qian (钱超)

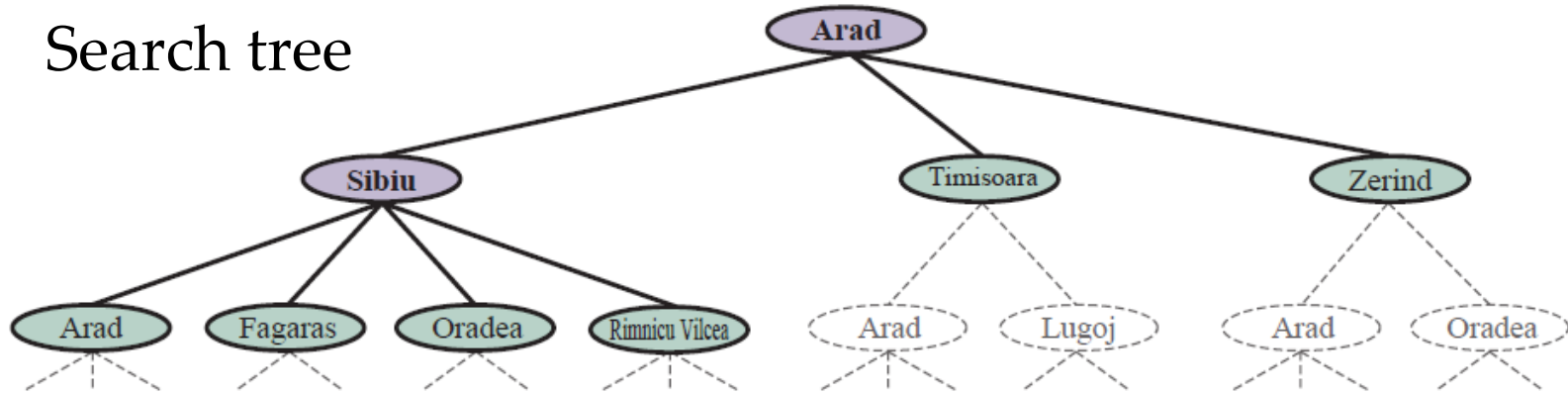
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Email: qianc@nju.edu.cn

Homepage: <http://www.lamda.nju.edu.cn/qianc/>

Search algorithms

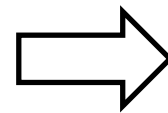
Search tree



Explored set: Arad, Sibiu

Frontier: Fagaras, Oradea, Rimnicu Vilcea, Timisoara, Zerind

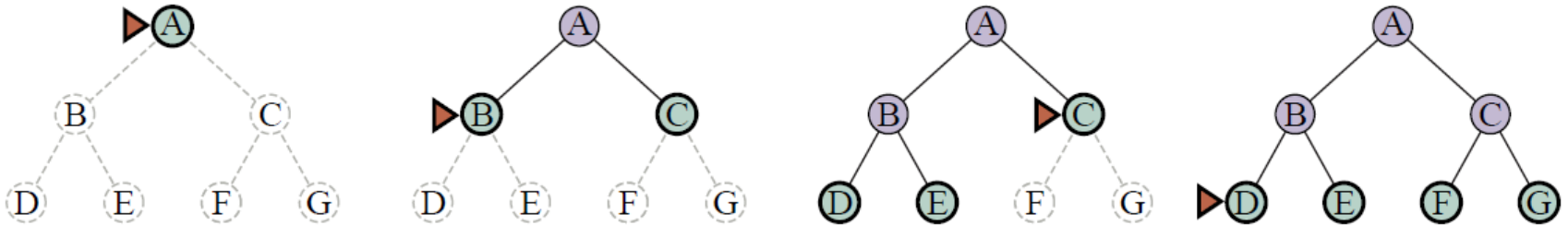
How to select one node from
the frontier for expansion?



Different search
algorithms

Breadth-first search

Main idea: expand the root node first, then all the successors of the root node, then their successors, and so on



function BREADTH-FIRST-SEARCH(*problem*) **returns** a solution node or *failure*

node $\leftarrow$ NODE(*problem*.INITIAL)

if *problem*.IS-GOAL(*node*.STATE) **then return** *node*

frontier $\leftarrow$ a FIFO queue, with *node* as an element

reached $\leftarrow$ {*problem*.INITIAL}

while not IS-EMPTY(*frontier*) **do**

node $\leftarrow$ POP(*frontier*)

for each *child* **in** EXPAND(*problem*, *node*) **do**

s $\leftarrow$ *child*.STATE

if *problem*.IS-GOAL(*s*) **then return** *child*

if *s* is not in *reached* **then**

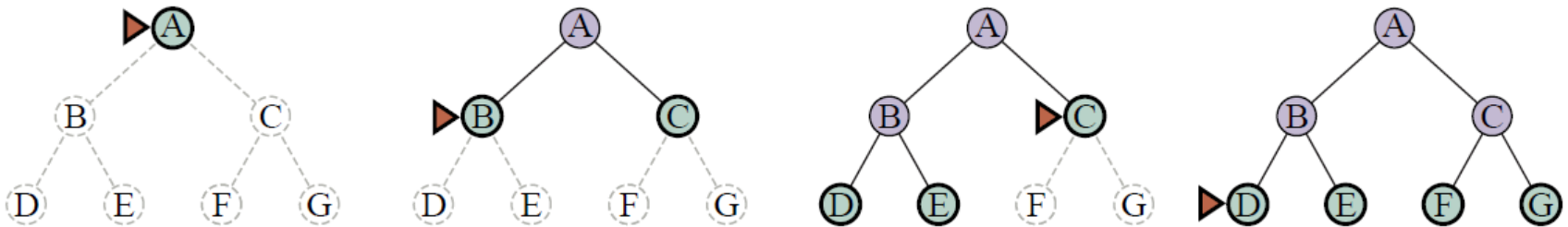
add *s* to *reached*

add *child* to *frontier*

return *failure*

Breadth-first search

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if *problem*.IS-GOAL(*s*) **then return** *child*

if *s* is not in *reached* **then**

add *s* to *reached*

add *child* to *frontier*

return *failure*

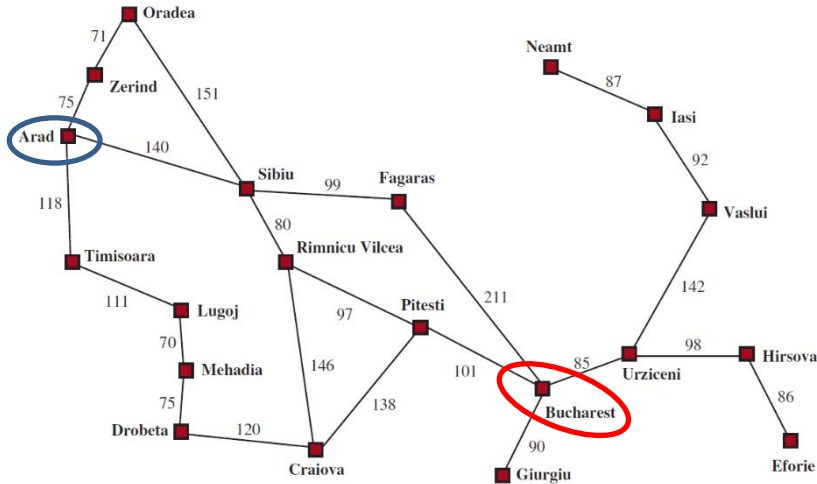
Expand the
shallowest node

Goal test is applied
when a node is
generated rather than
when it is expanded

The union of explored
set and frontier

Graph-search

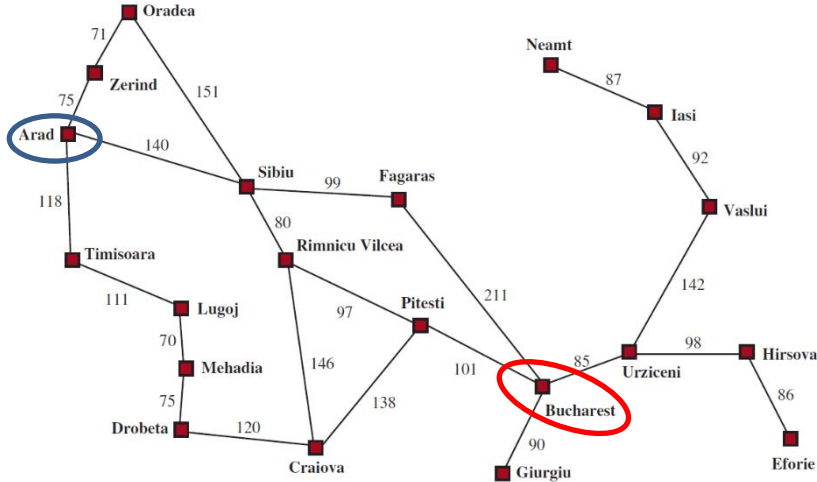
Breadth-first search - example



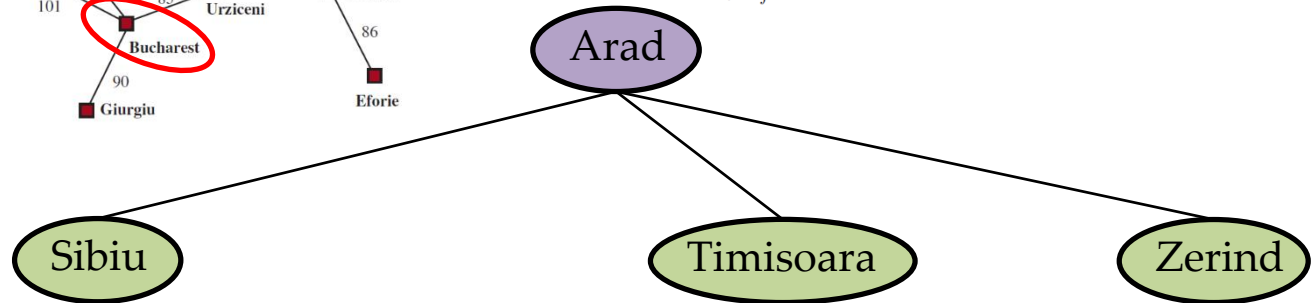
```
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    for each child in EXPAND(problem, node) do
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Arad

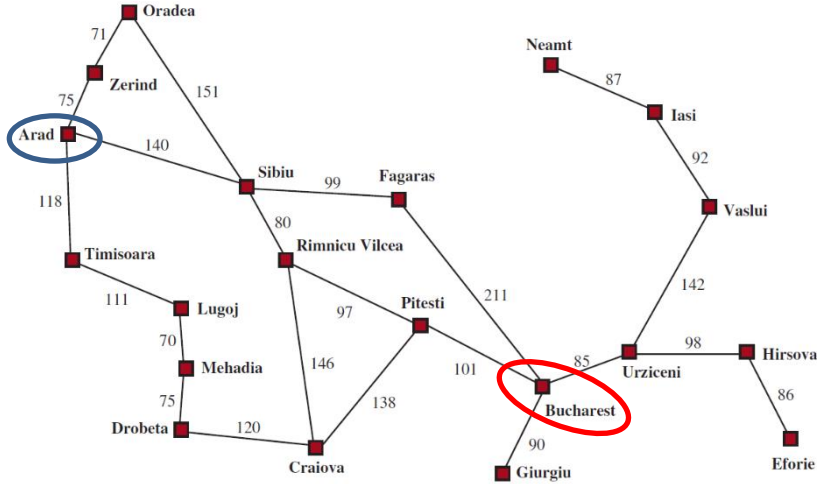
Breadth-first search - example



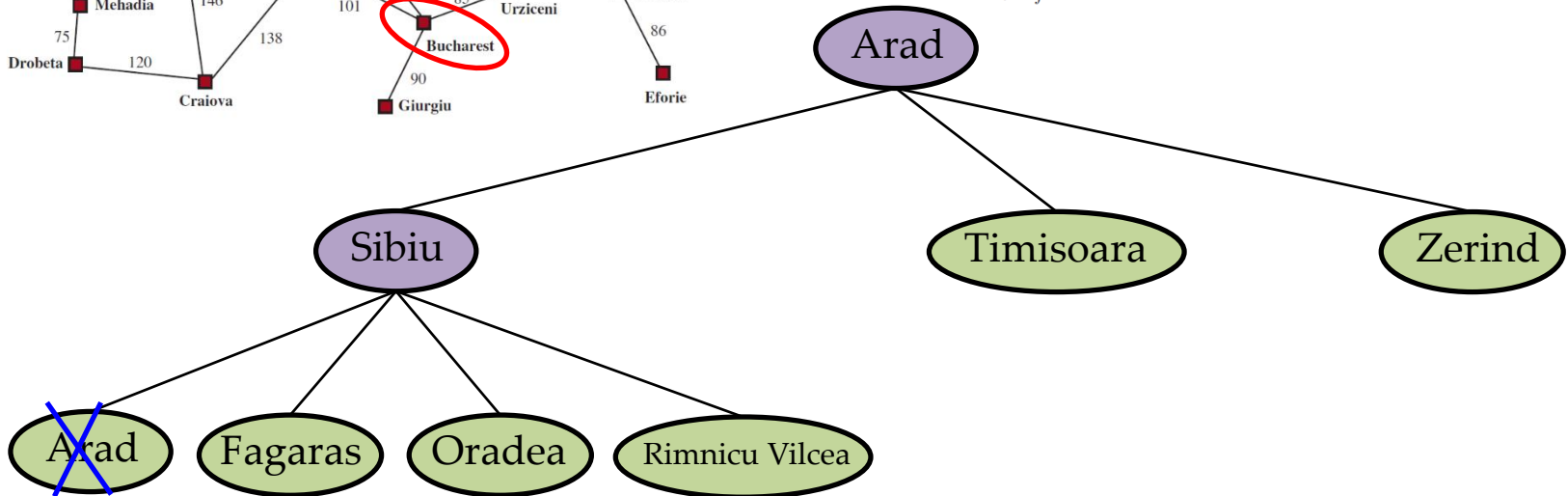
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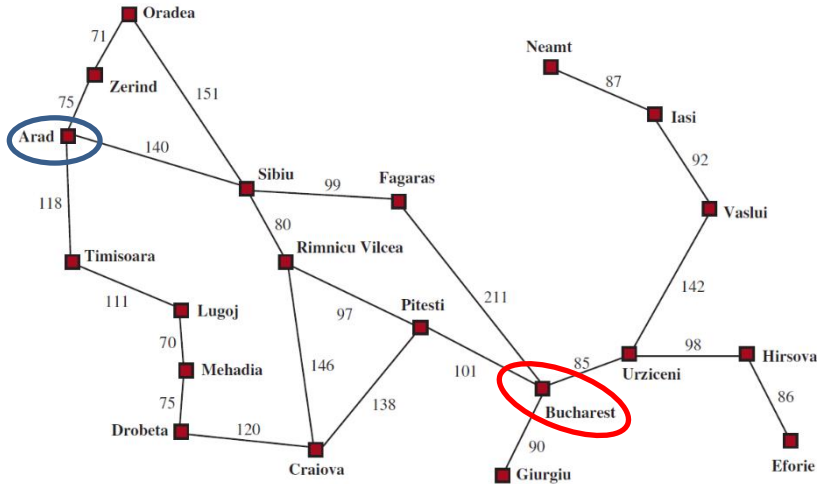
Breadth-first search - example



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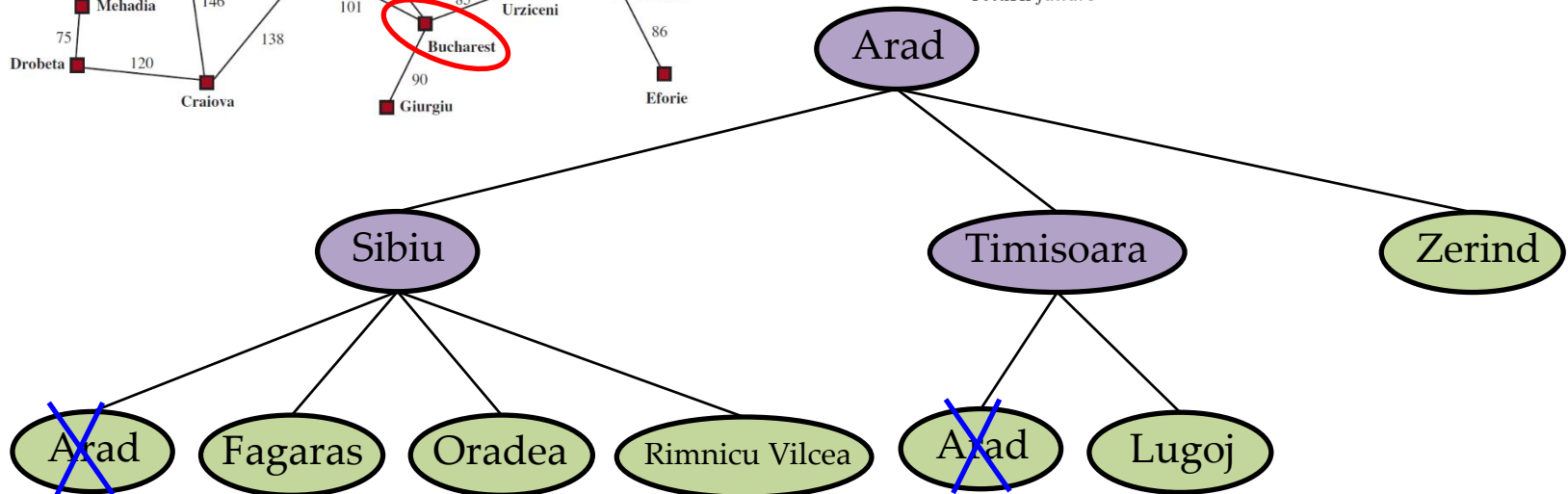


Breadth-first search - example

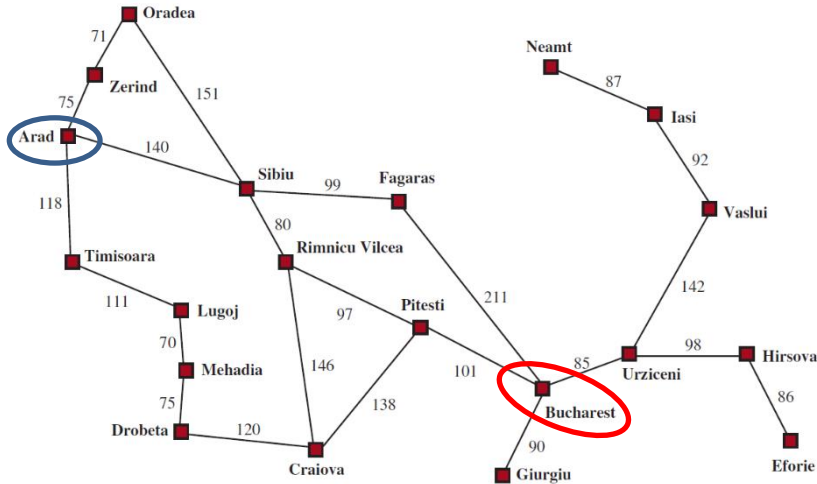


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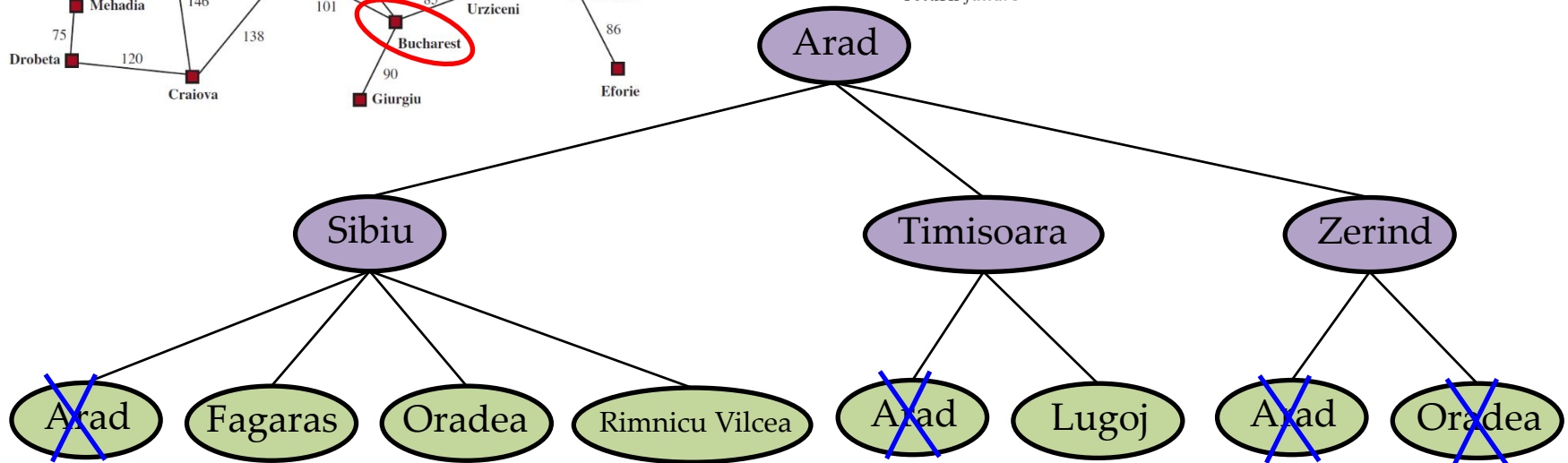
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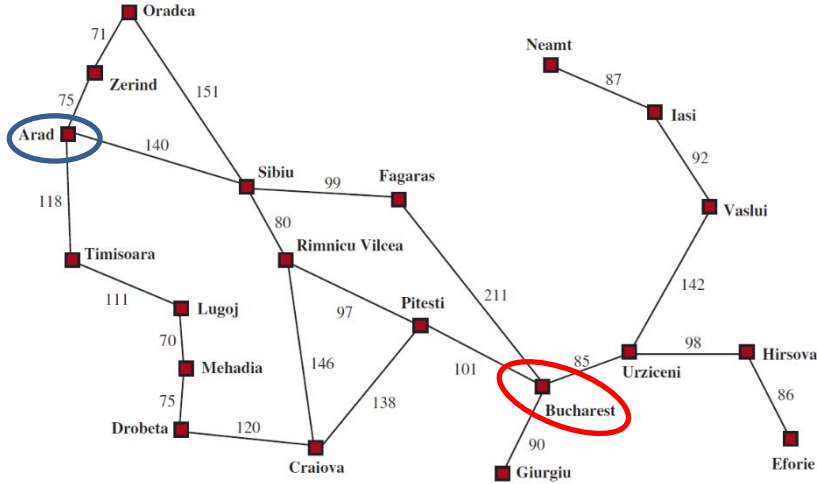
Breadth-first search - example



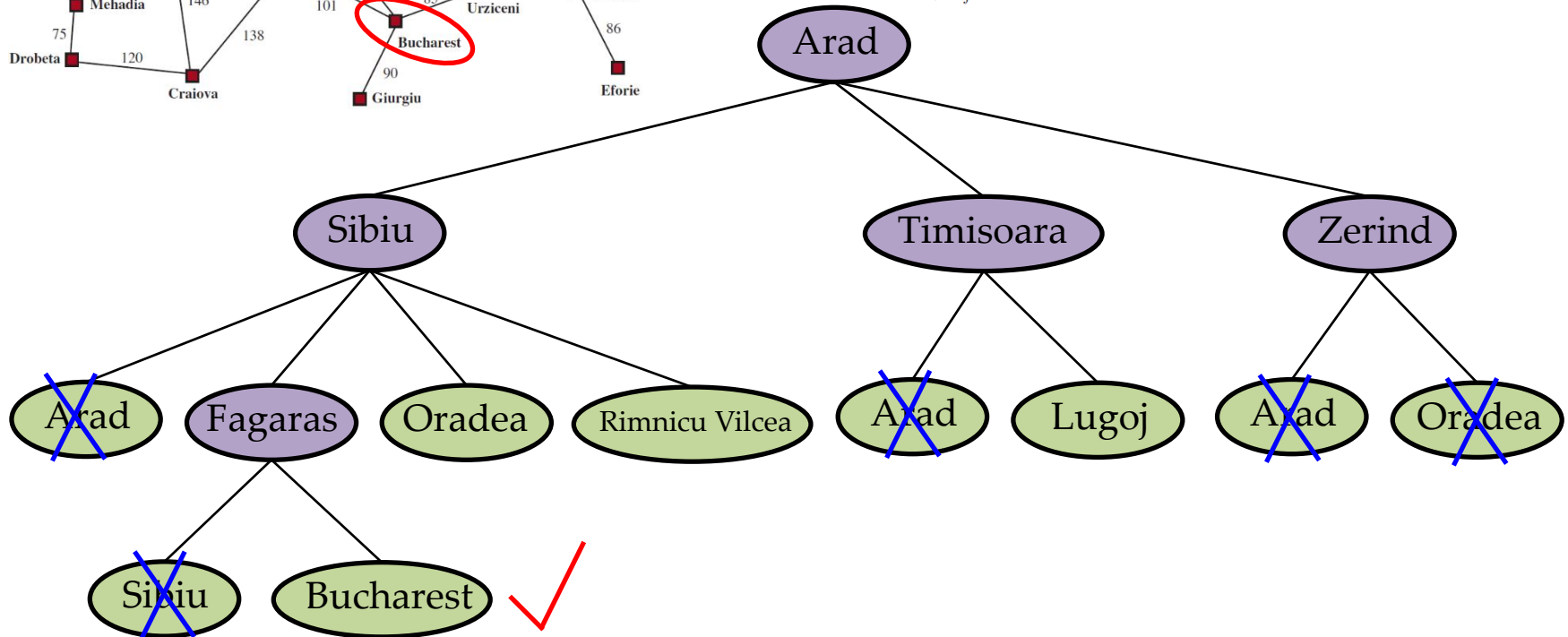
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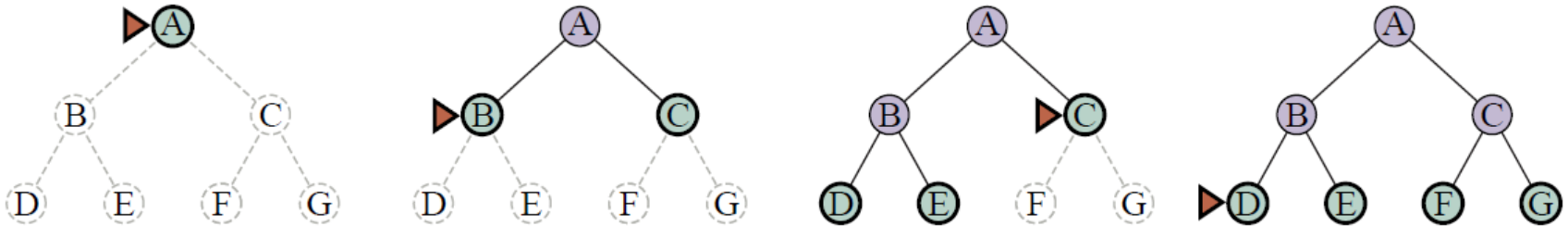
Breadth-first search - example



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  return failure
```



Breadth-first search - performance



- **Complete**

if the depth d of the shallowest goal node is finite

- **Optimal**

if all actions have the same cost

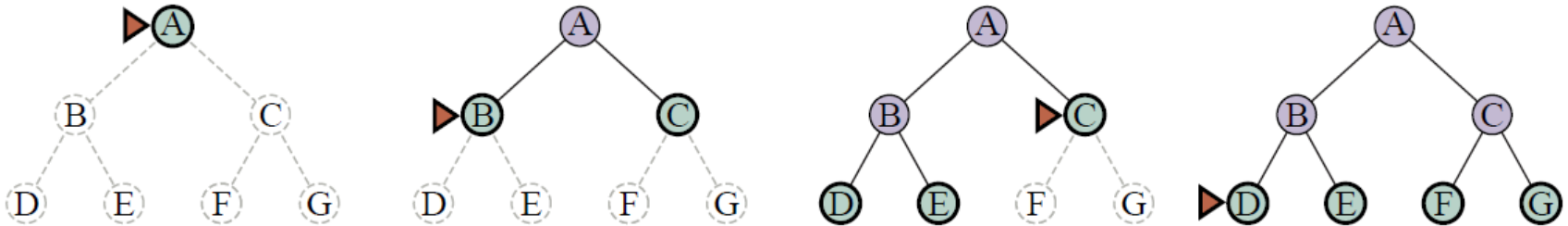
- **Time complexity**

$b + b^2 + b^3 + \dots + b^d = O(b^d)$, where b is the branching factor

If goal test is applied when a node is expanded

$$b + b^2 + b^3 + \dots + b^d + b^{d+1} = O(b^{d+1})$$

Breadth-first search - performance



- **Time complexity**

$b + b^2 + b^3 + \dots + b^d = O(b^d)$, where b is the branching factor

If goal test is applied when a node is expanded

$$b + b^2 + b^3 + \dots + b^d + b^{d+1} = O(b^{d+1})$$

- **Space complexity**

$b + b^2 + b^3 + \dots + b^{d-1} = O(b^{d-1})$ in the explored set
 b^d in the frontier

} $O(b^d)$

If using tree-search, $O(b^d)$

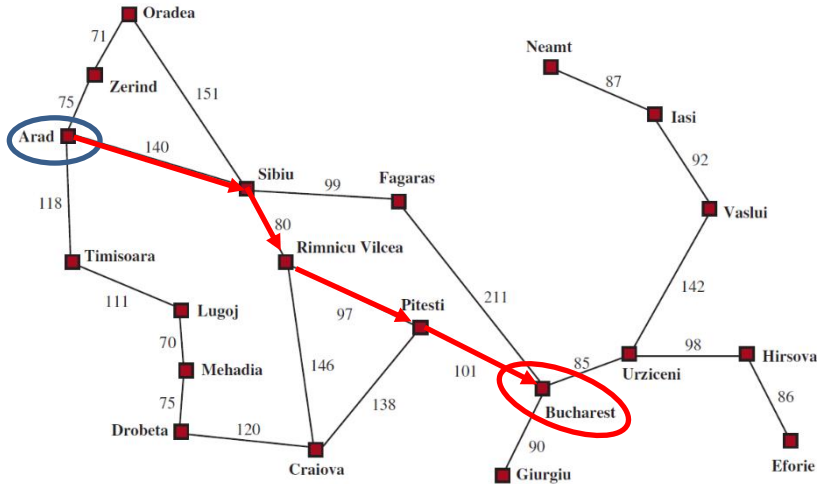
Breadth-first search - performance

$b = 10$ 1 million nodes/second 1000 bytes/node

Depth	Nodes	Time	Memory
2	110	.11 milliseconds	107 kilobytes
4	11,110	11 milliseconds	10.6 megabytes
6	10^6	1.1 seconds	1 gigabyte
8	10^8	2 minutes	103 gigabytes
10	10^{10}	3 hours	10 terabytes
12	10^{12}	13 days	1 petabyte
14	10^{14}	3.5 years	99 petabytes
16	10^{16}	350 years	10 exabytes

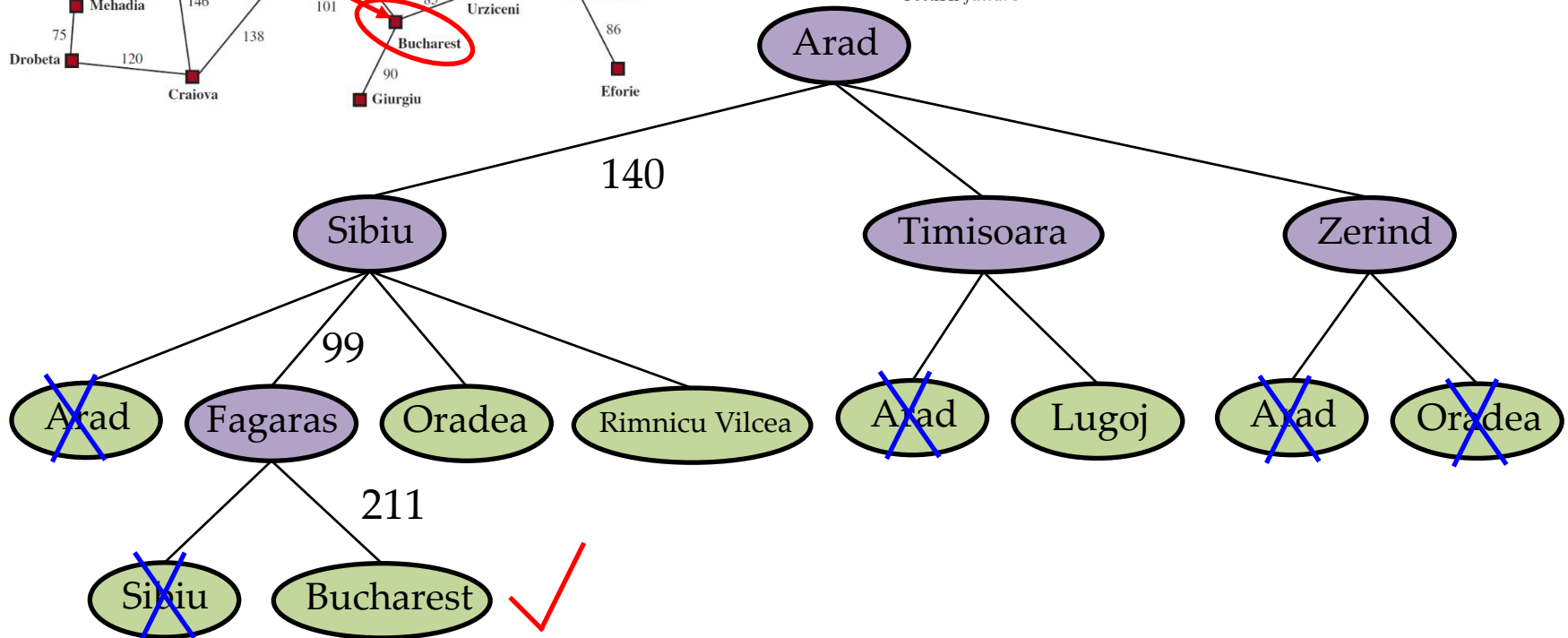
The time and space complexity of BFS: not acceptable

Breadth-first search - example



```

function BREADTH-FIRST-SEARCH(problem) returns a solution node or failure
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  while not IS-EMPTY(frontier) do
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    for each child in EXPAND(problem, node) do
      s ← child.STATE
      if problem.IS-GOAL(s) then return child
      if s is not in reached then
        add s to reached
        add child to frontier
  return failure
    
```



Uniform-cost search

Main idea: expand the node n with the lowest cost $g(n)$

function BEST-FIRST-SEARCH(*problem*, *f*) **returns** a solution node or *failure*

$node \leftarrow \text{NODE}(\text{STATE}=\text{problem.INITIAL})$

$frontier \leftarrow$ a priority queue ordered by f with $node$ as an element

$reached \leftarrow$ a lookup table, with one entry with key problem.INITIAL and value $node$

while not IS-EMPTY($frontier$) **do**

$node \leftarrow \text{POP}(frontier)$

if $\text{problem.IS-GOAL}(node.STATE)$ **then return** $node$

for each $child$ **in** EXPAND($problem, node$) **do**

$s \leftarrow child.STATE$

if s is not in $reached$ **or** $child.PATH-COST < reached[s].PATH-COST$ **then**

$reached[s] \leftarrow child$

add $child$ to $frontier$

return $failure$

Path cost $g(n)$ from the initial node to the current node

Goal test is applied when a node is expanded rather than when it is generated

The union of explored set and frontier

Graph-search

function EXPAND(*problem*, *node*) **yields** nodes

$s \leftarrow node.STATE$

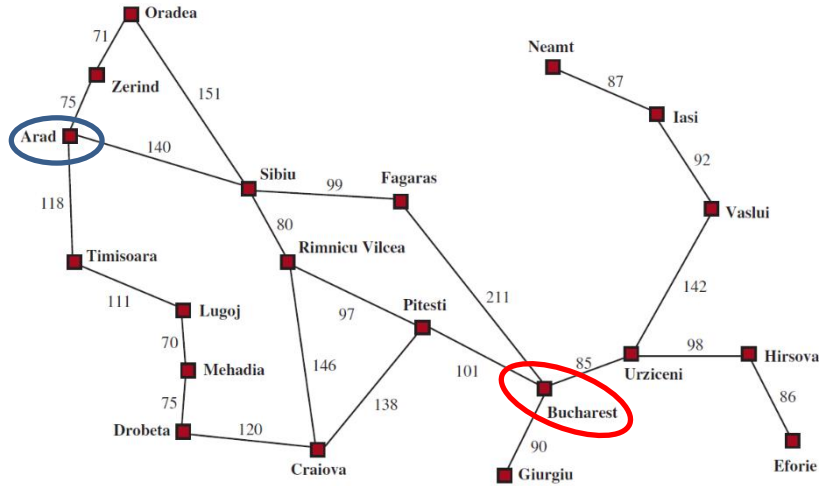
for each $action$ **in** $problem.ACTIONS(s)$ **do**

$s' \leftarrow problem.RESULT(s, action)$

$cost \leftarrow node.PATH-COST + problem.ACTION-COST(s, action, s')$

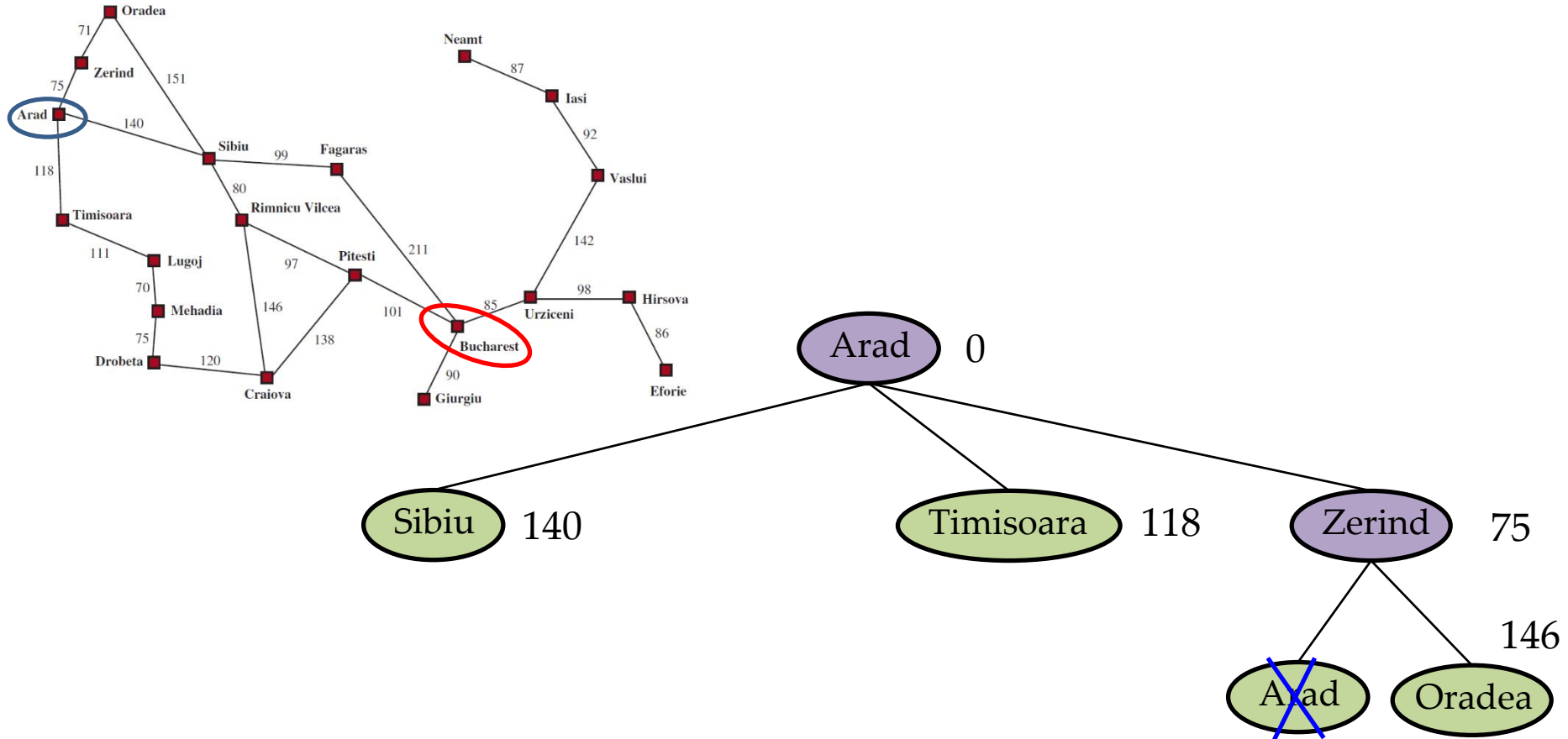
yield $\text{NODE}(\text{STATE}=s', \text{PARENT}=node, \text{ACTION}=action, \text{PATH-COST}=cost)$

Uniform-cost search - example

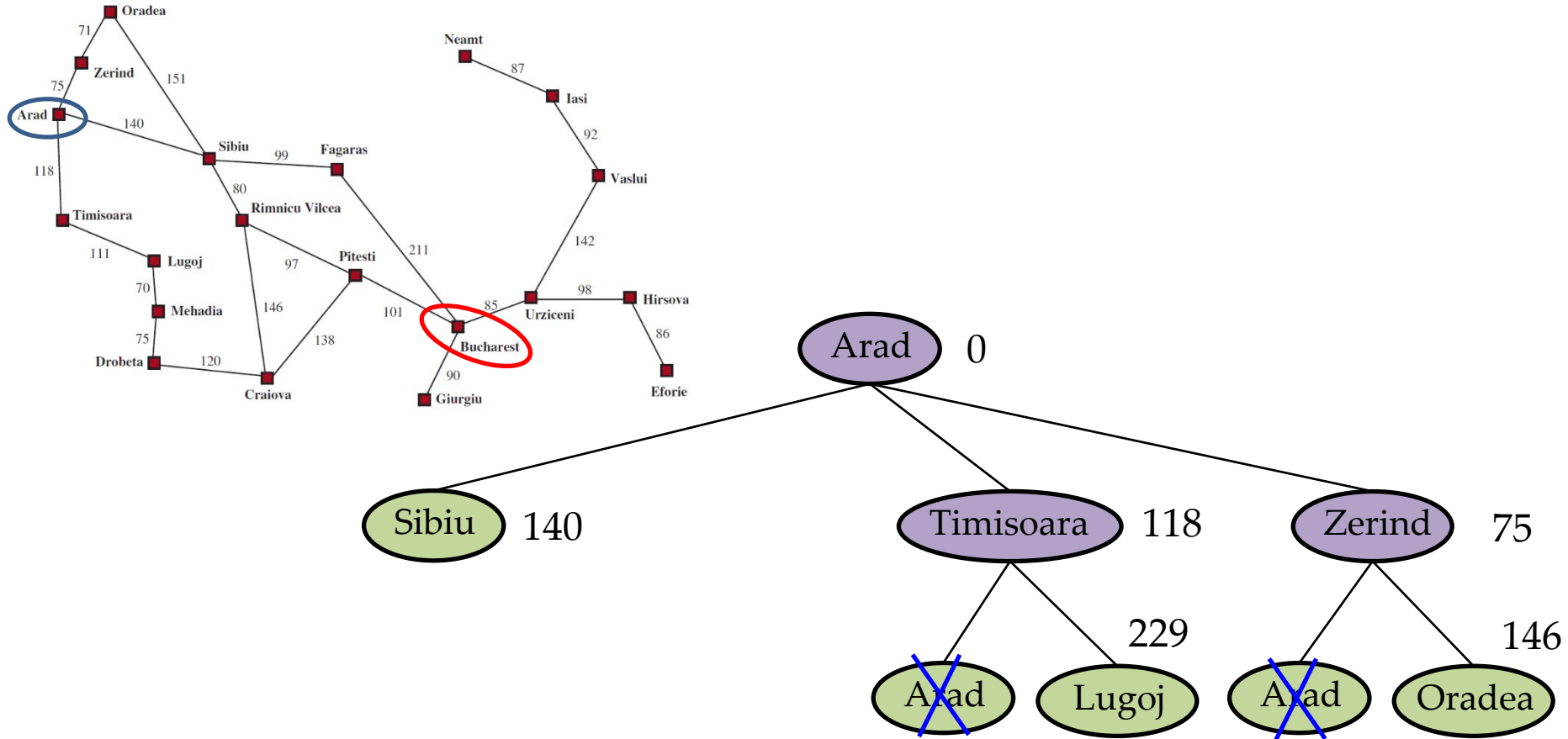


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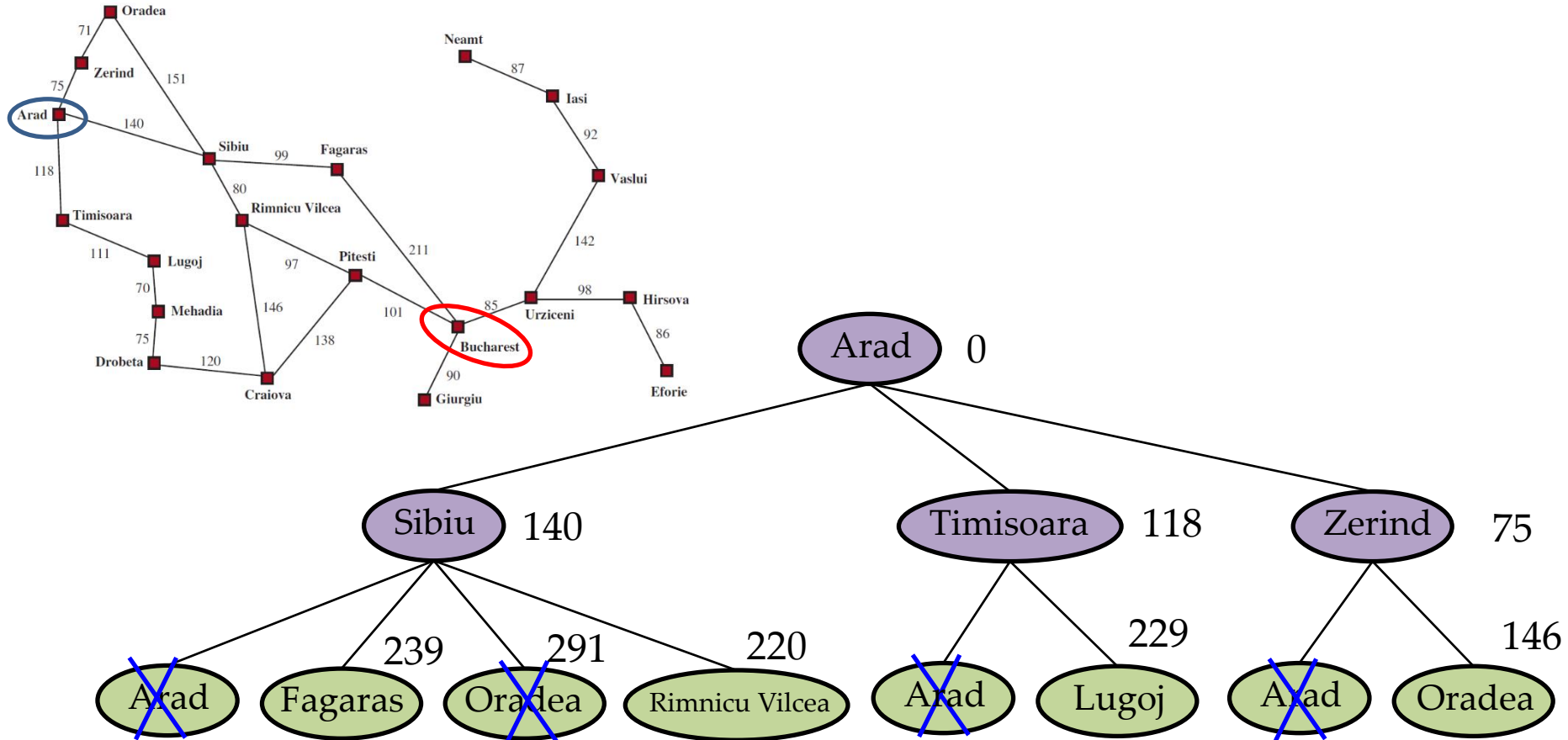
Uniform-cost search - example



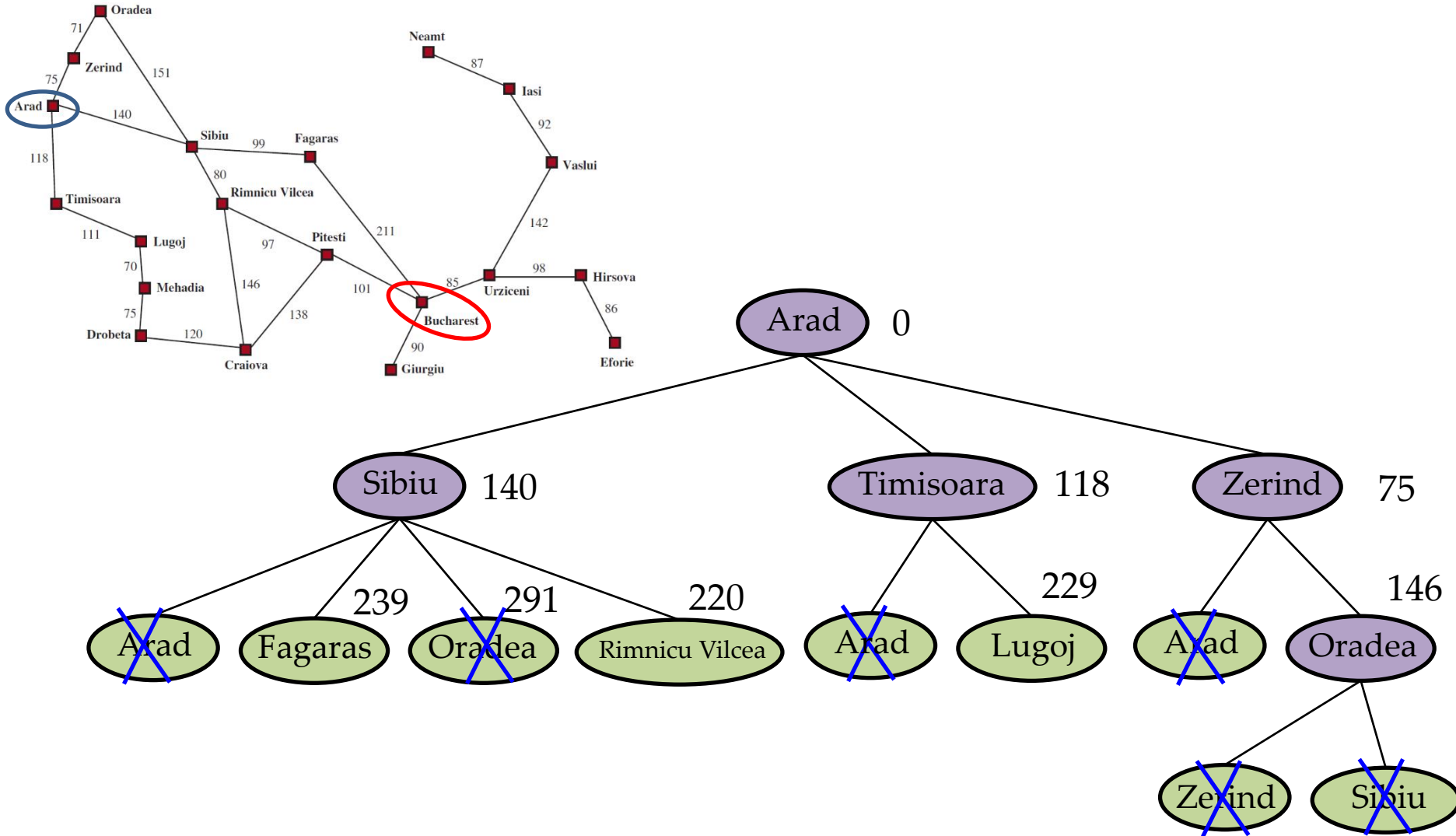
Uniform-cost search - example



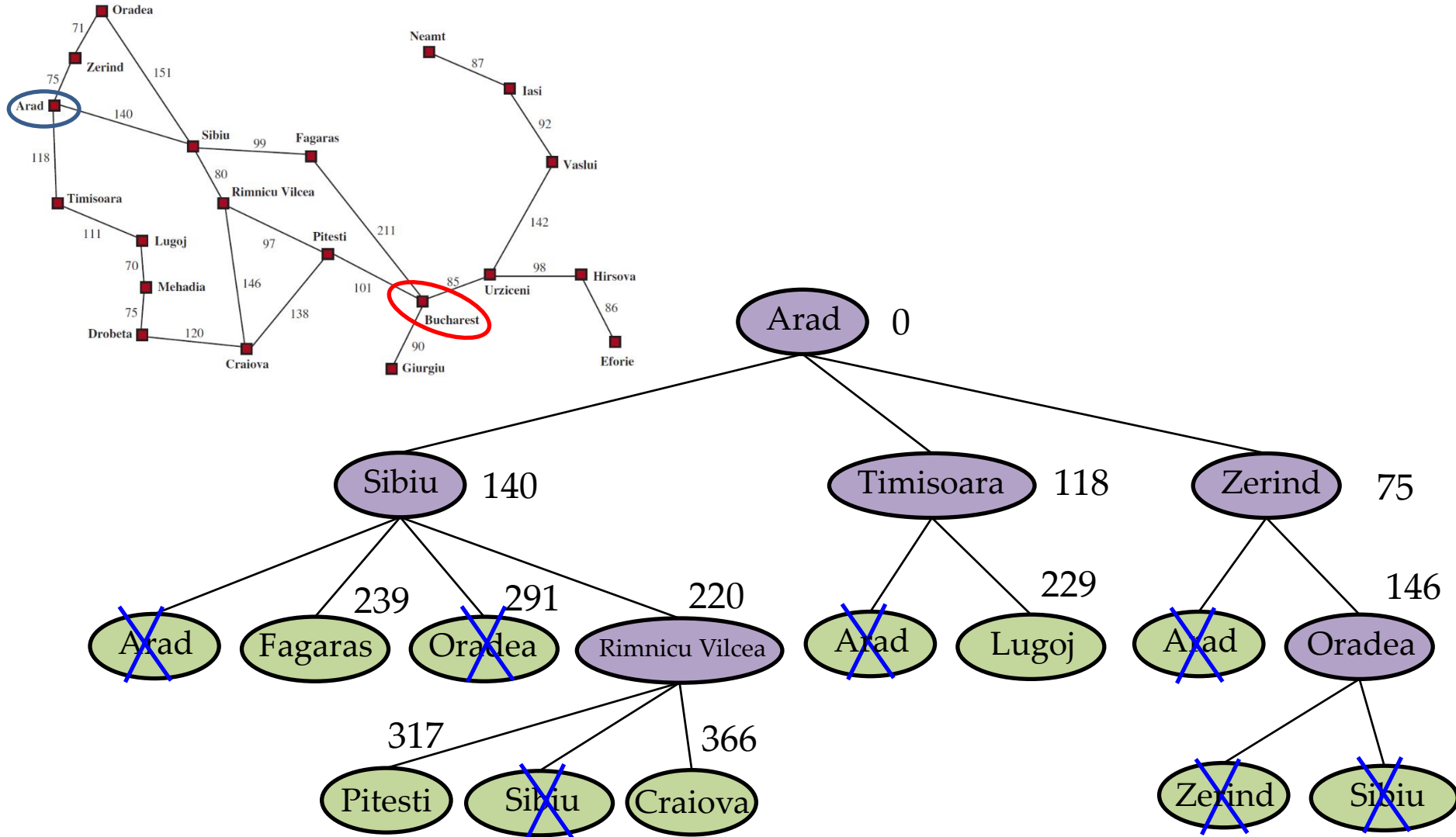
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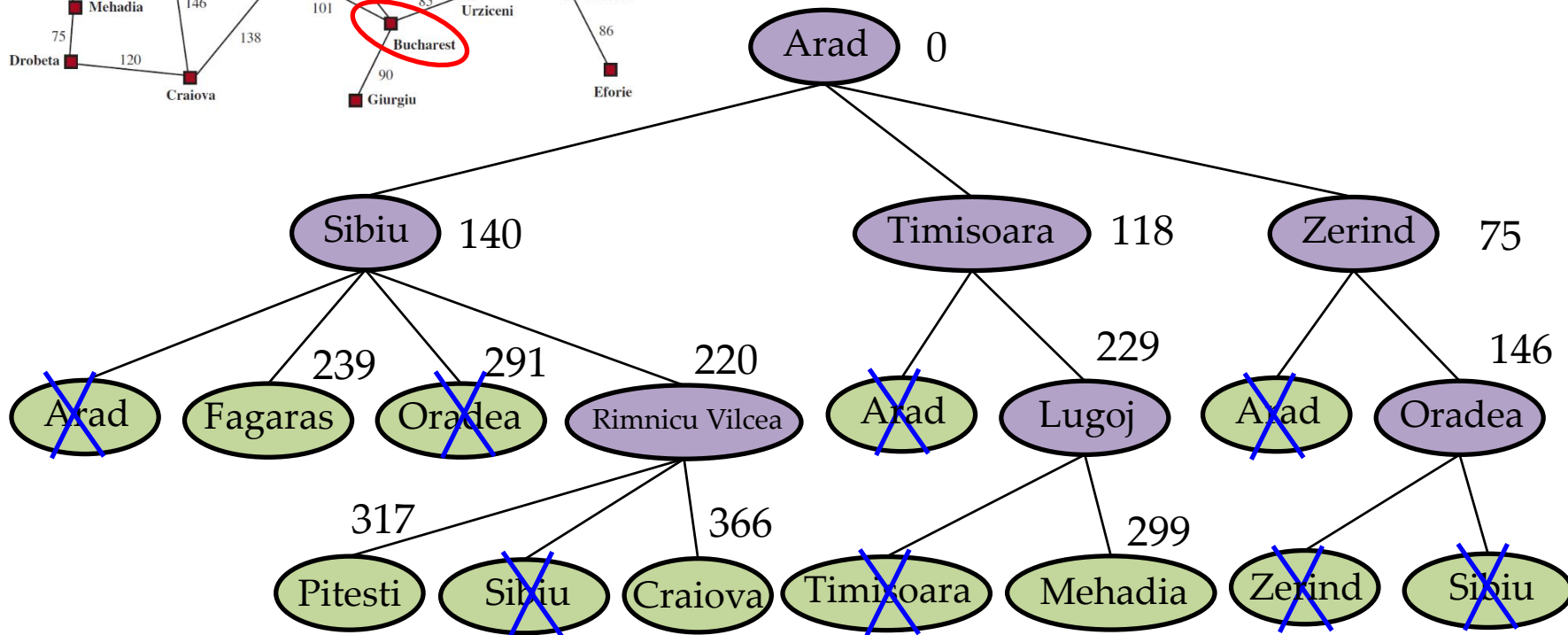
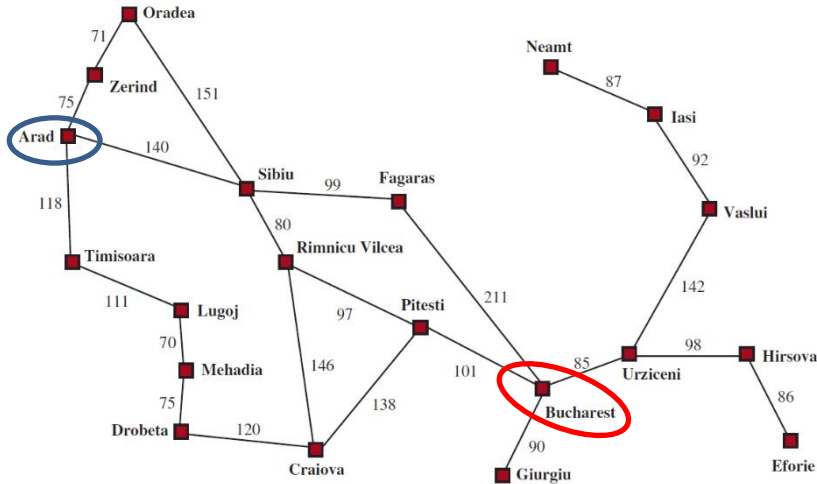
Uniform-cost search - example



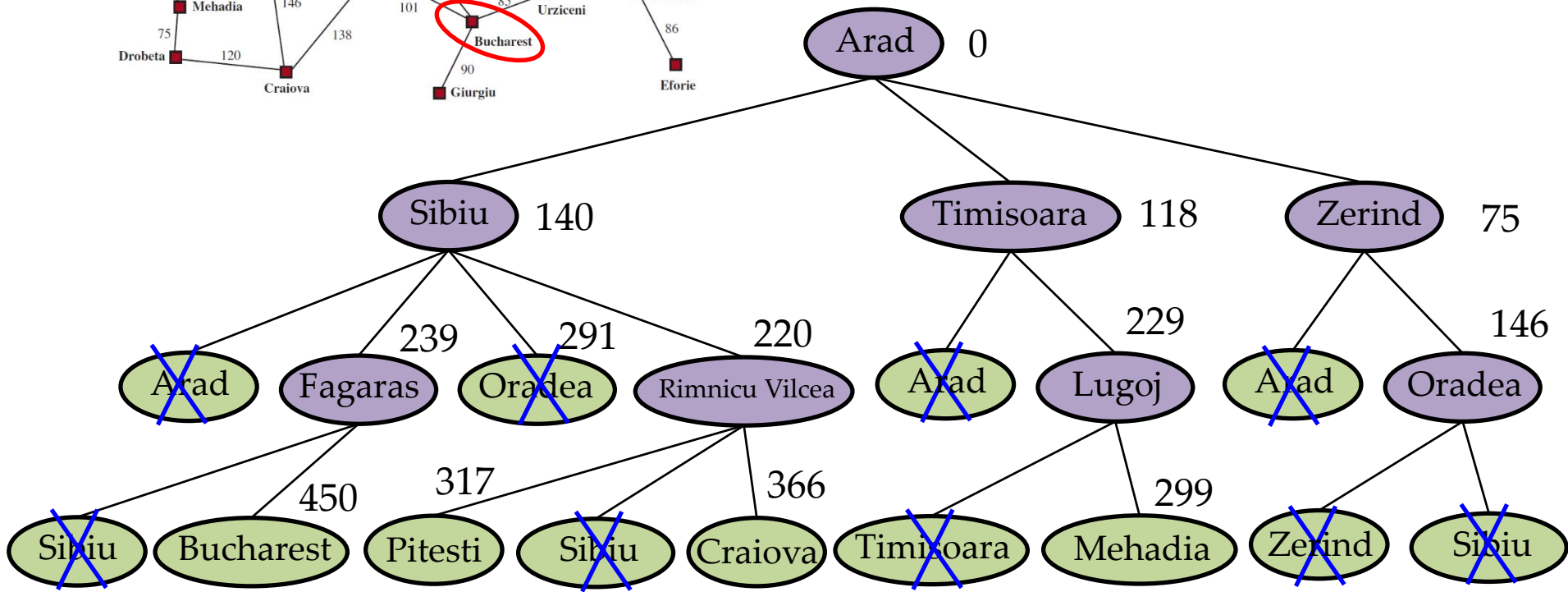
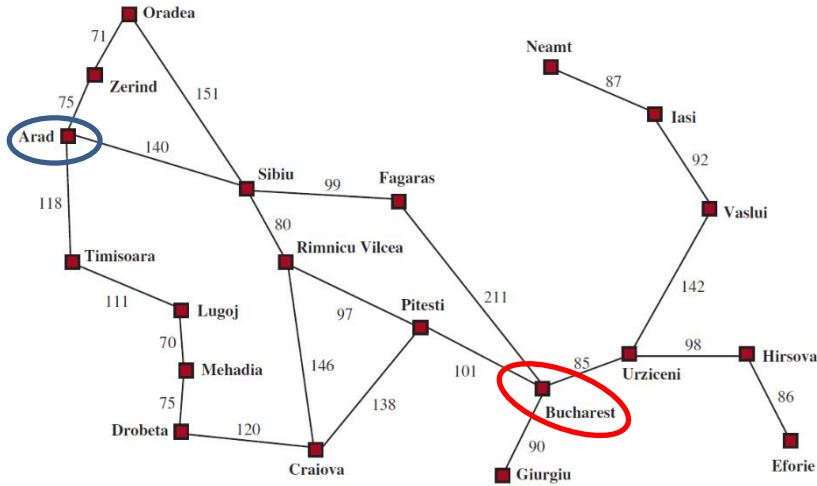
Uniform-cost search - example



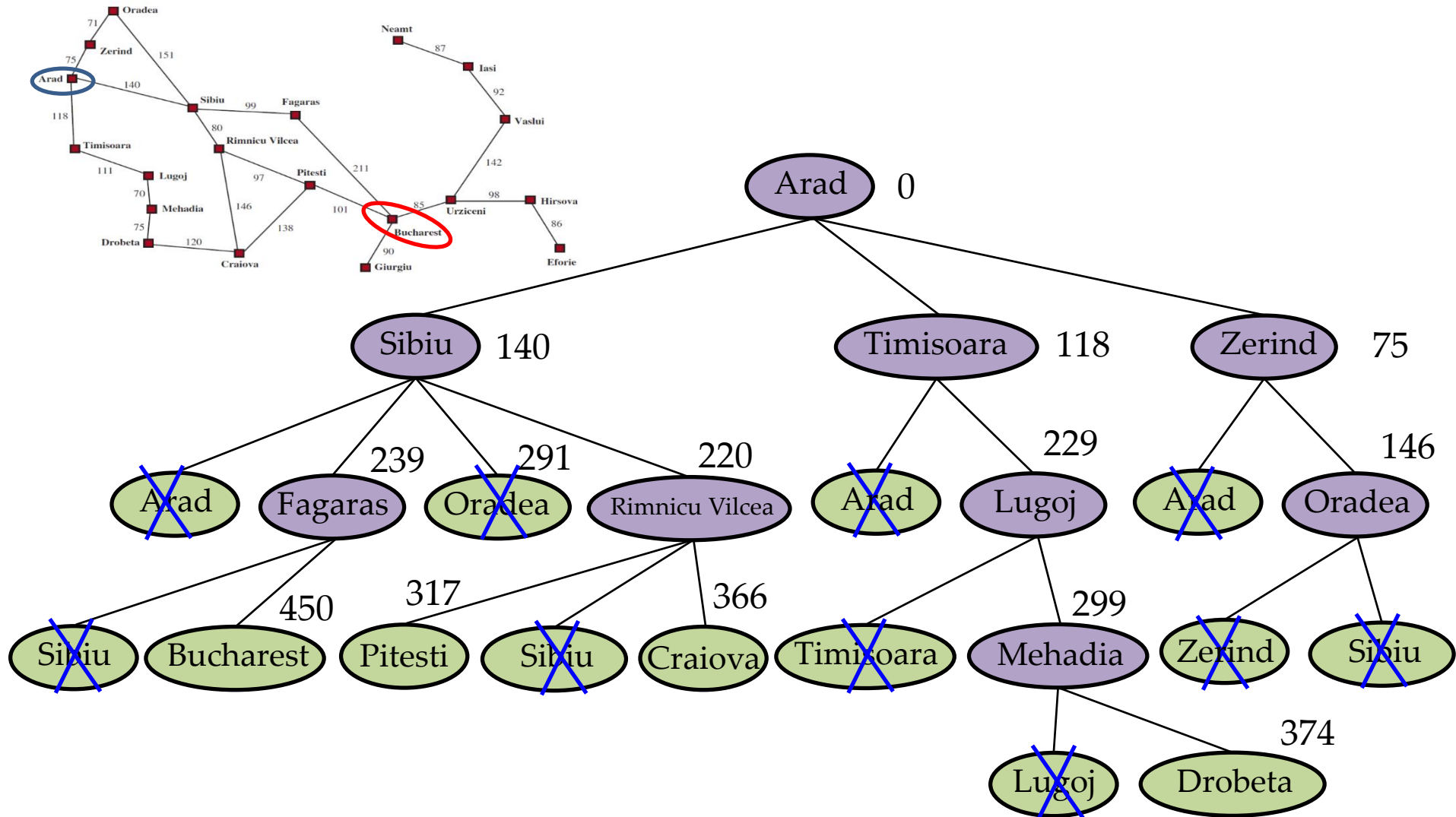
Uniform-cost search - example



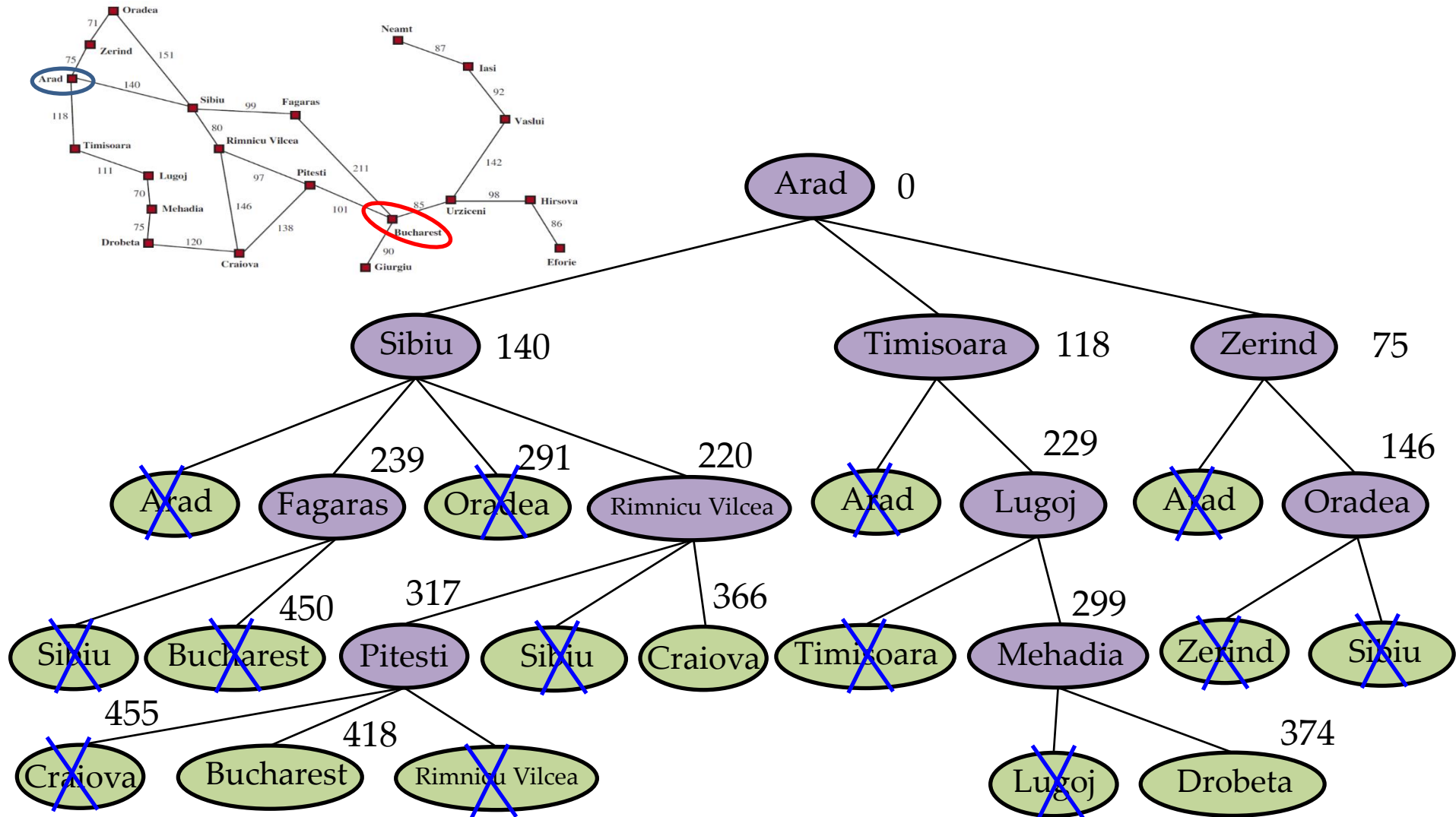
Uniform-cost search - example



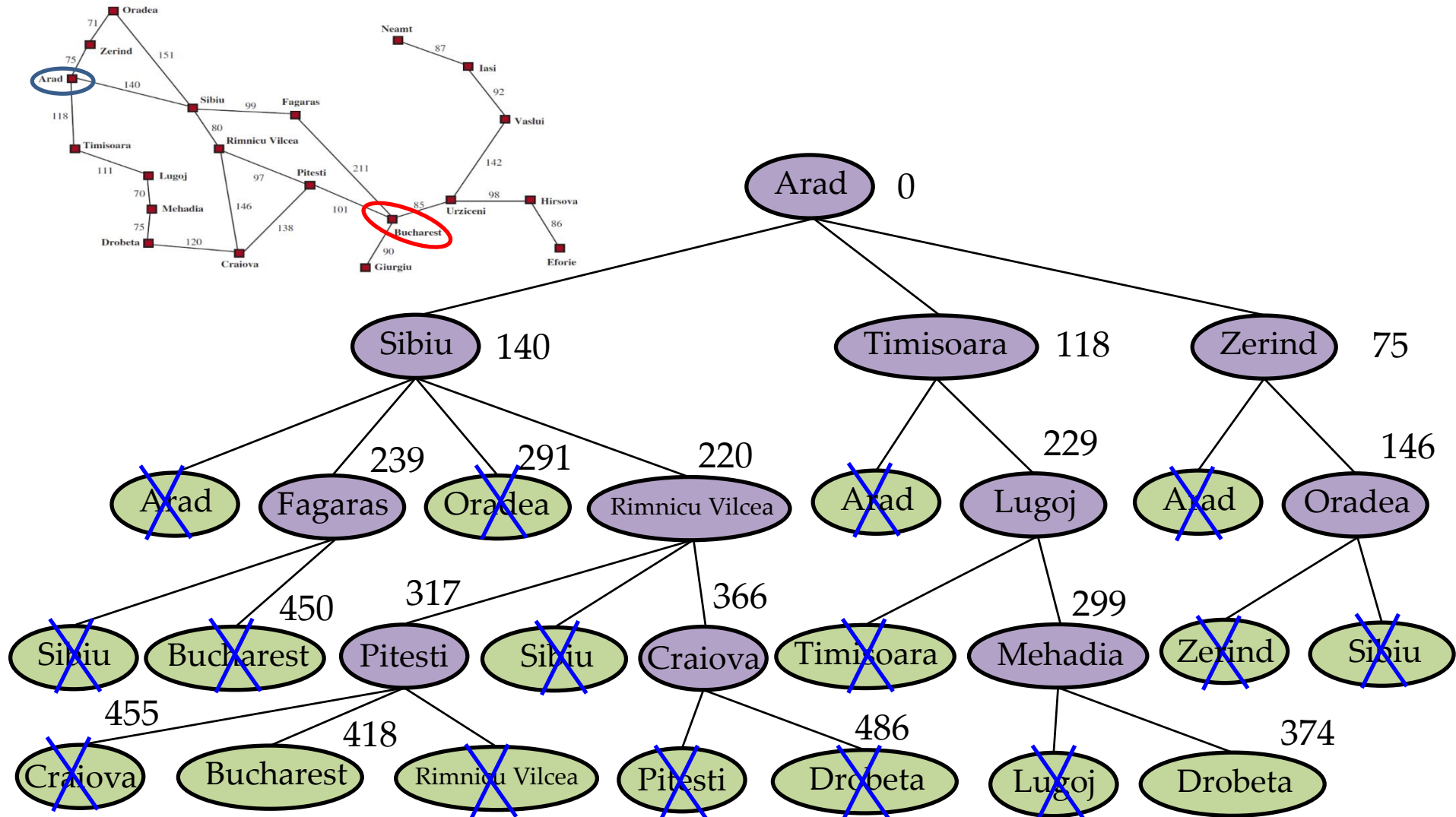
Uniform-cost search - example



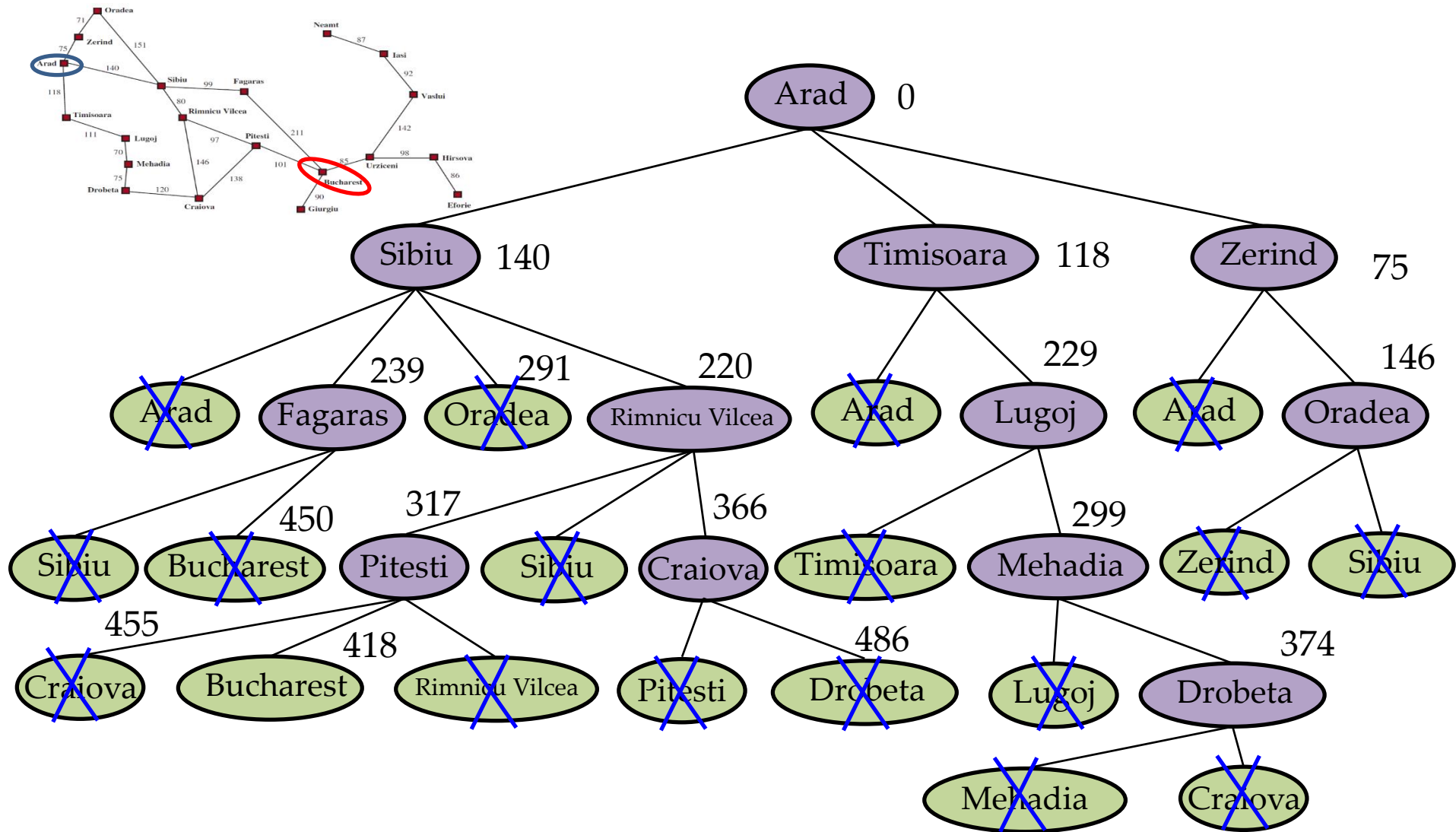
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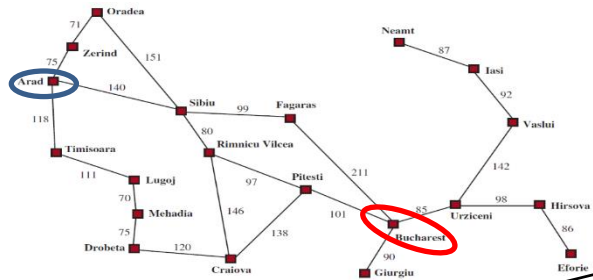
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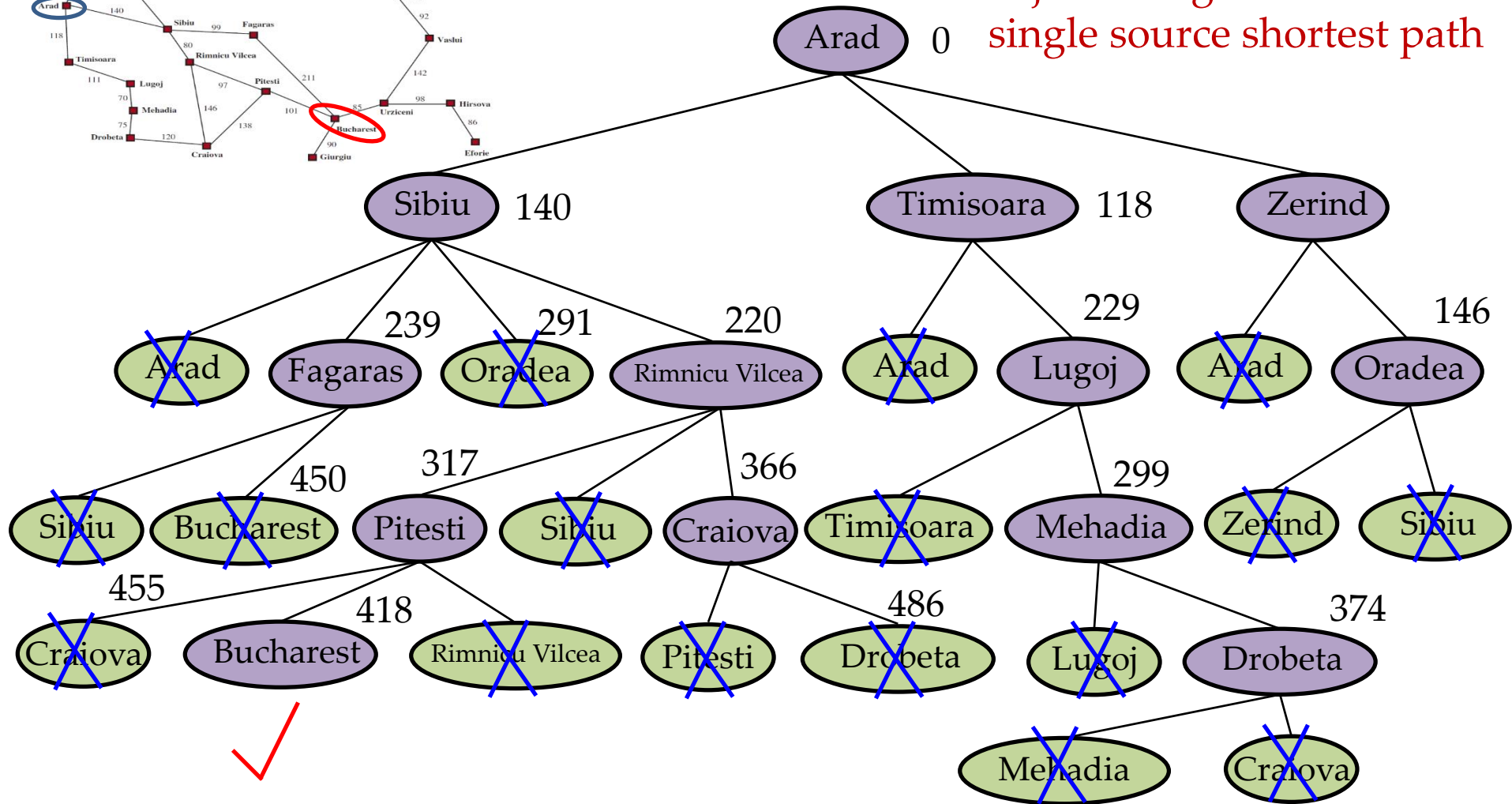
Uniform-cost search - example



Uniform-cost search - example



Dijkstra's Algorithm for
single source shortest path



Uniform-cost search - performance

Main idea: expand the node n with the lowest cost $g(n)$

- **Complete**

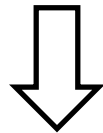
if the cost of every step exceeds some small positive constant ϵ

- **Optimal**

when a node n is expanded, the optimal path to $n.state$ has been found

Otherwise, there is another frontier node n' on the optimal path to $n.state$, whose corresponding node is denoted as n''

$$g(n') \leq g(n'') < g(n)$$



n' will be expanded before n , making a contradiction

Uniform-cost search - performance

Main idea: expand the node n with the lowest cost $g(n)$

- **Complete**

if the cost of every step exceeds some small positive constant ϵ

- **Optimal**

when a node n is expanded, the optimal path to n .state has been found

- **Time complexity**

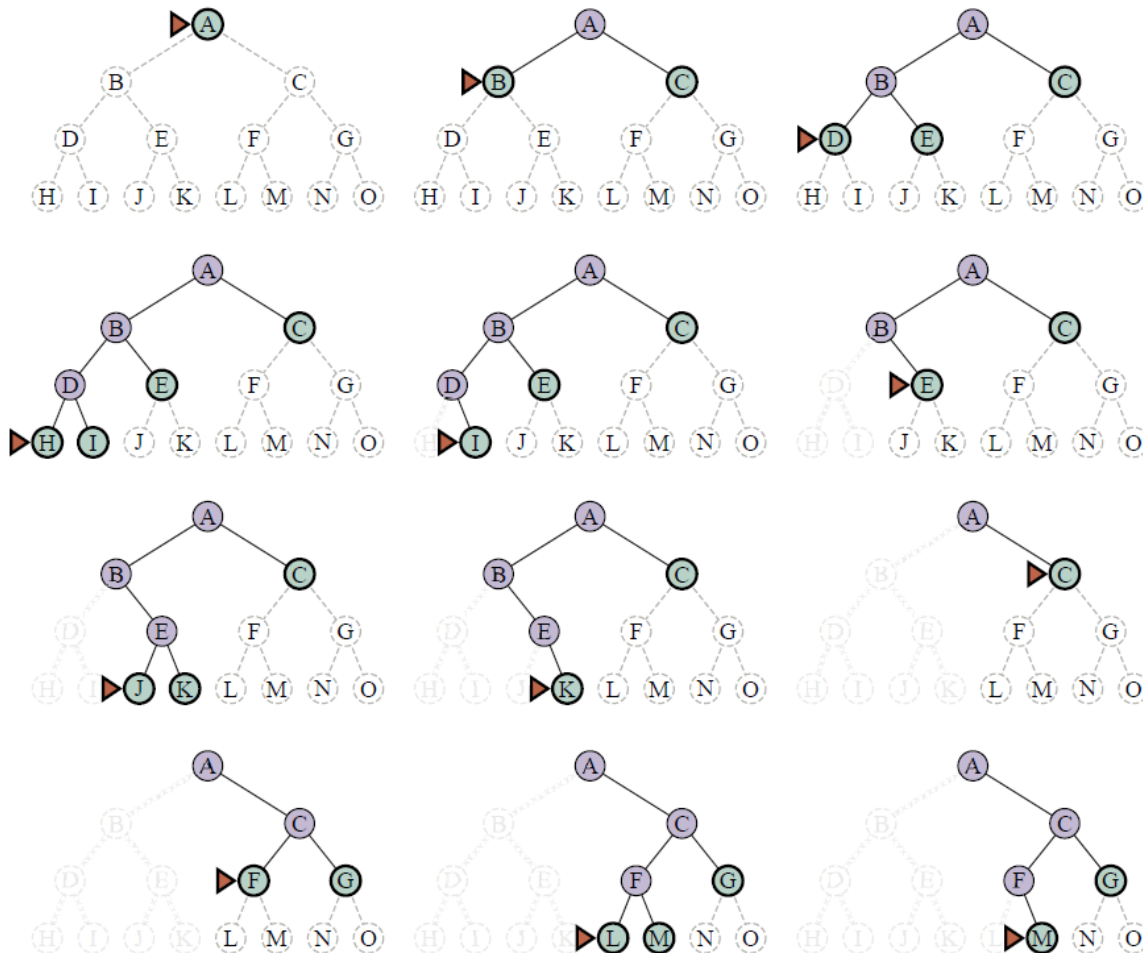
$O(b^{1+\lceil C^/\epsilon \rceil})$, where C^* is the cost of the optimal path*

- **Space complexity**

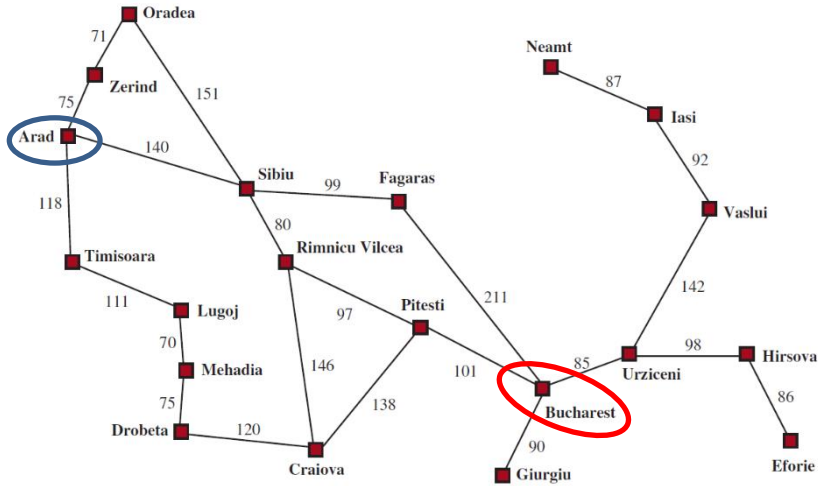
$O(b^{1+\lceil C^/\epsilon \rceil})$, where C^* is the cost of the optimal path*

Depth-first search

Main idea: expand the deepest node in the frontier

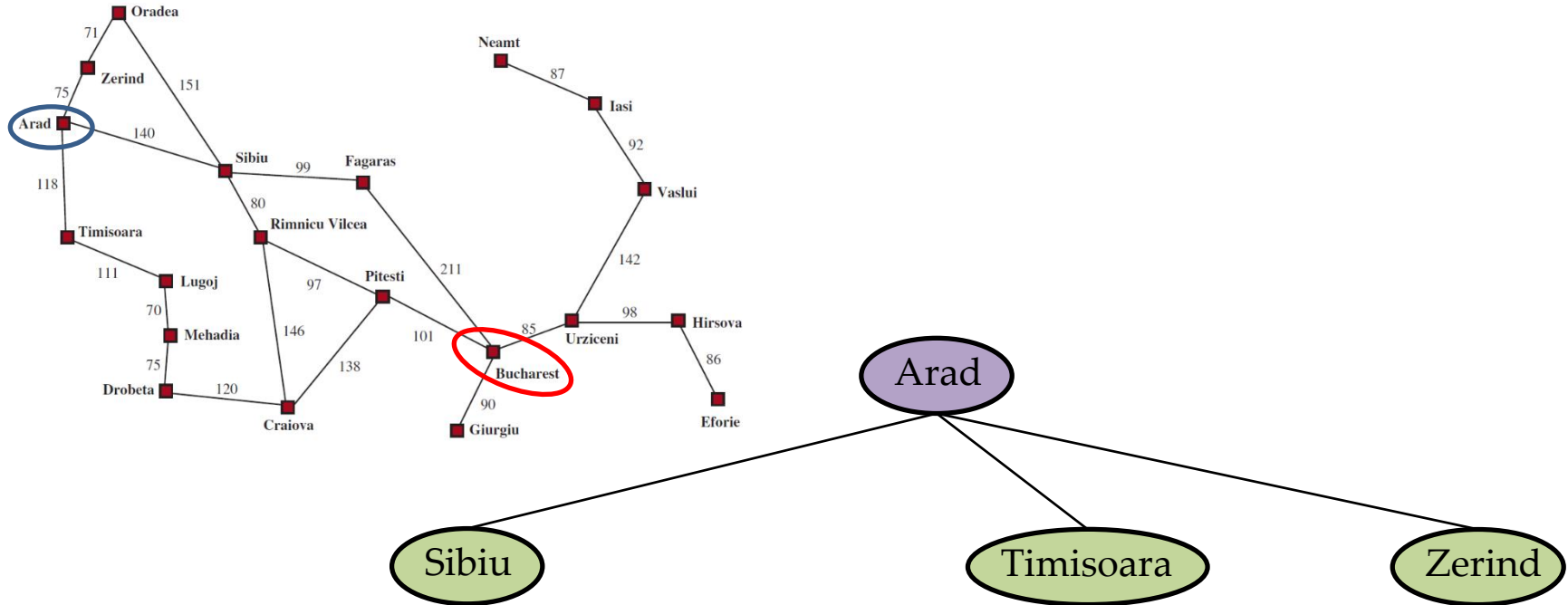


Depth-first search - example

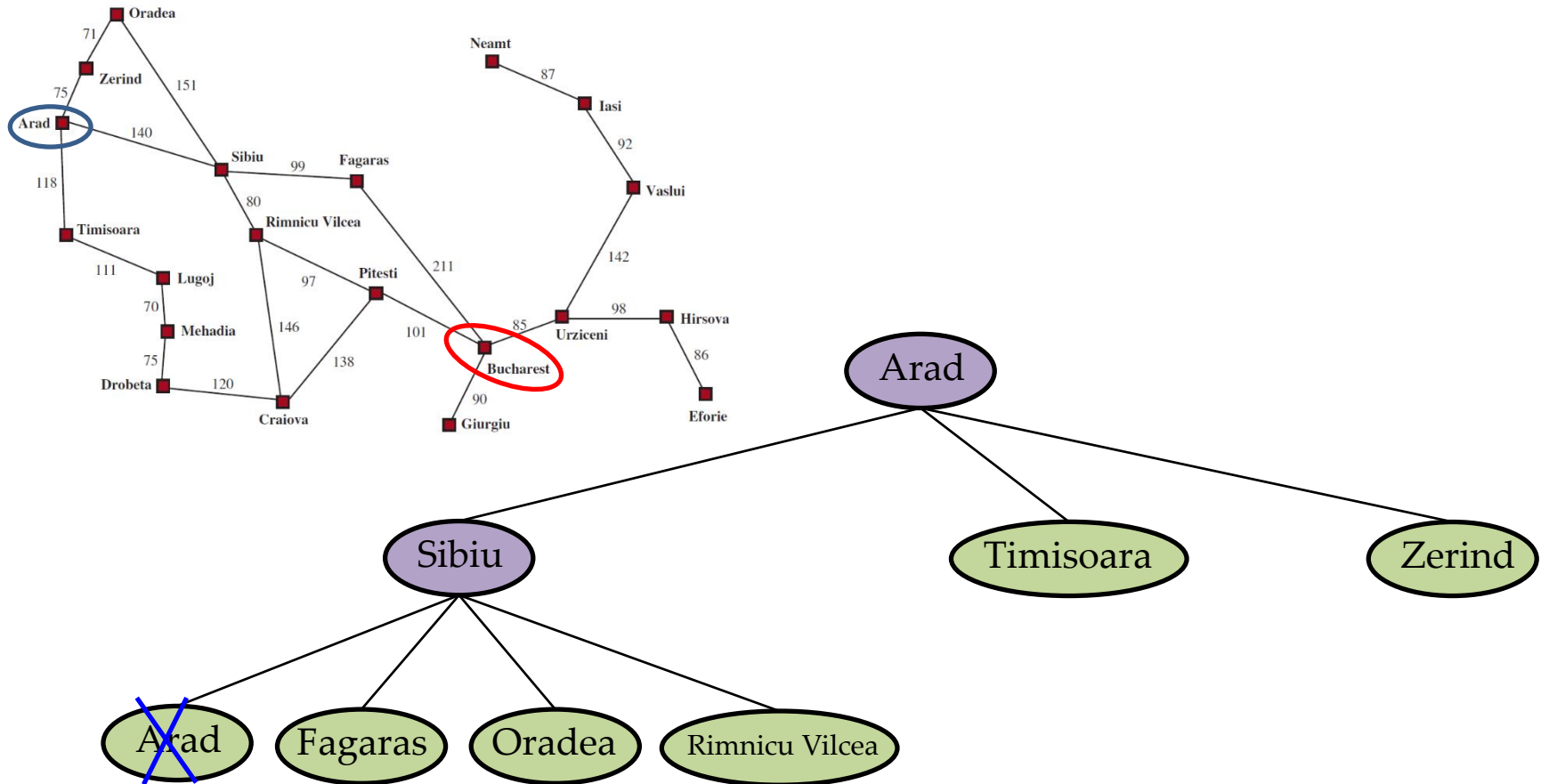


Arad

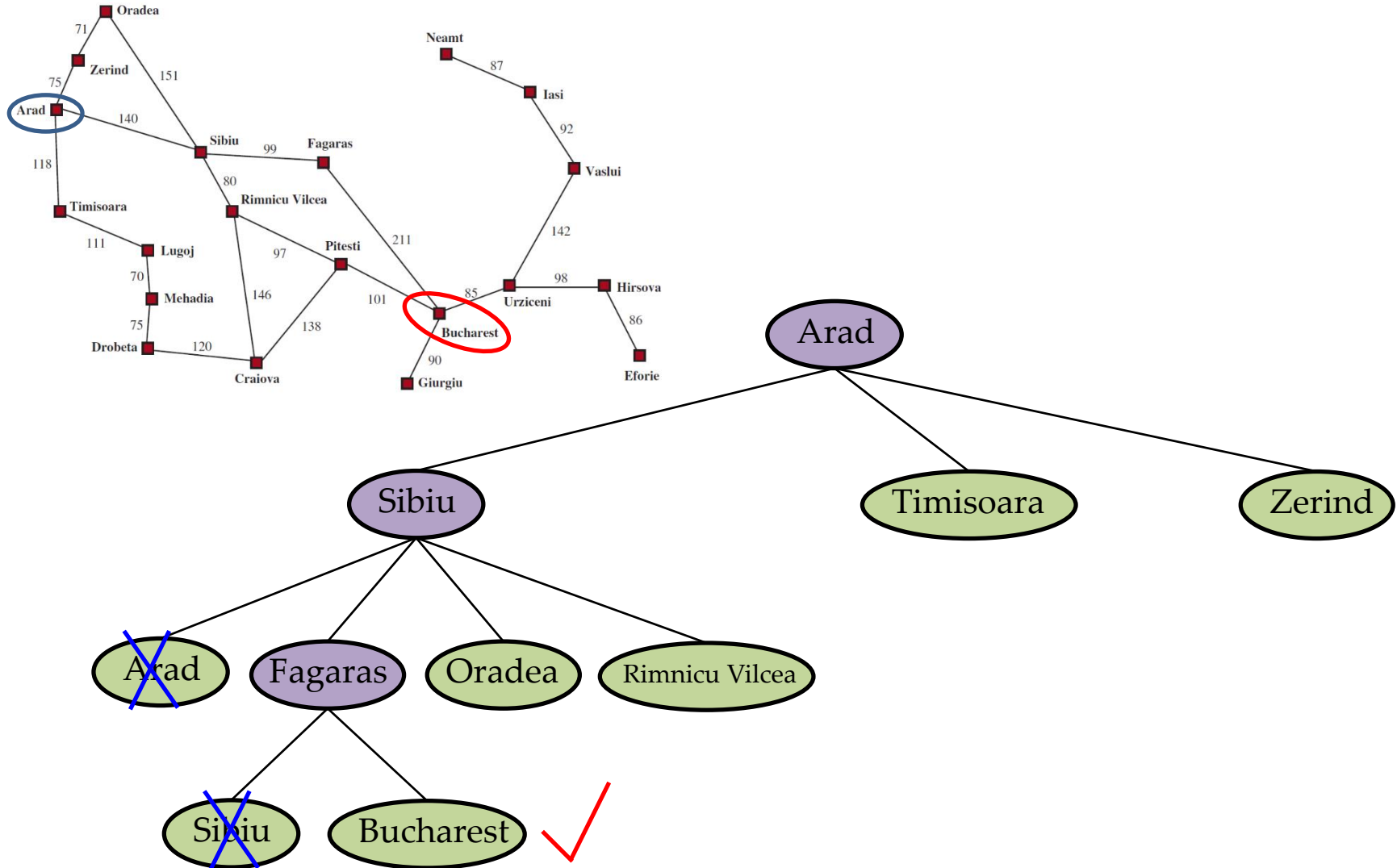
Depth-first search - example



Depth-first search - example



Depth-first search - example



Depth-first search - performance

Main idea: expand the deepest node in the frontier

- **Complete**

if using graph-search and the state space is finite

- **Optimal** X

- **Time complexity**

*$O(b^m)$, where m is the maximum depth of any node, if using tree-search
the size of the state space, if using graph-search*

- **Space complexity**

$O(bm)$, if using tree-search

the size of the state space, if using graph-search

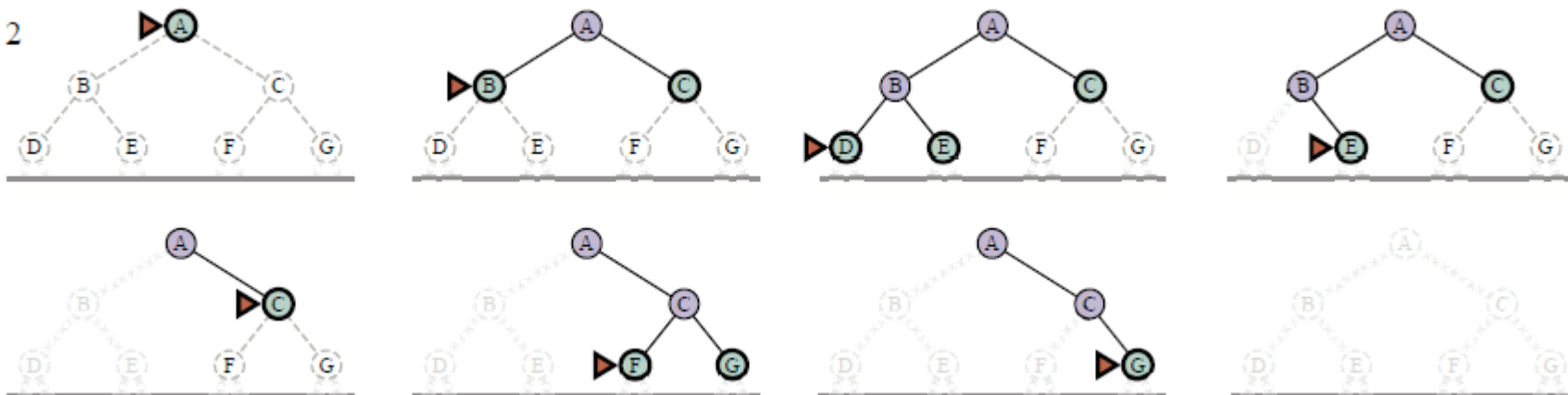
Depth-limited search

Main idea: depth-first search with a depth limit l

limit: 1



limit: 2



Depth-limited search - performance

Main idea: depth-first search with a depth limit l

- **Complete**

if $l \geq d$, the depth of the shallowest goal node

- **Optimal**

if $l > d$, no; if $l = d$ and all actions have the same cost, yes

- **Time complexity**

$O(b^l)$

- **Space complexity**

$O(bl)$, if using tree-search

Iterative deepening depth-first search

Main idea: iteratively apply depth-limited search with gradually increasing depth limit l

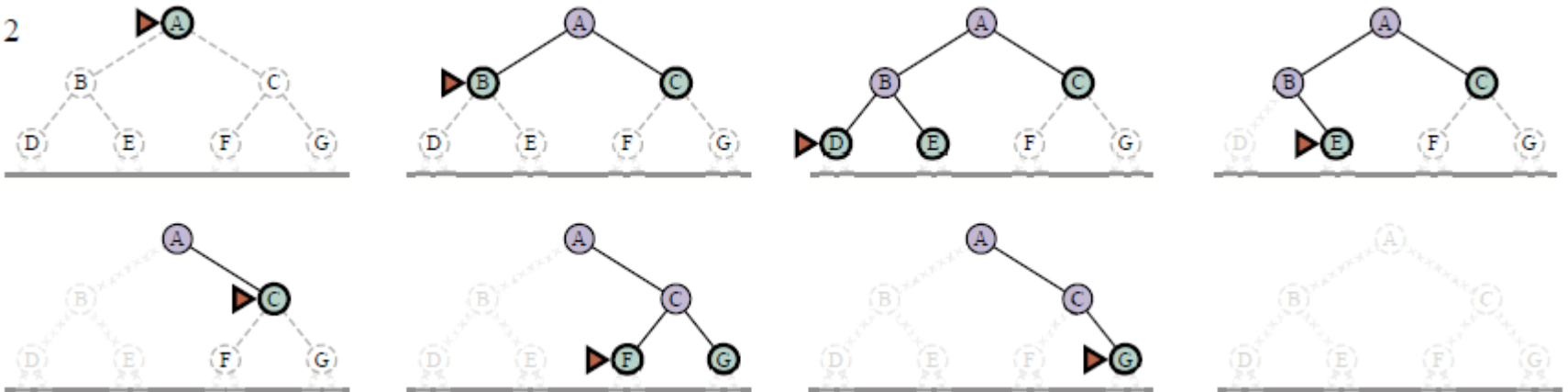
limit: 0



limit: 1



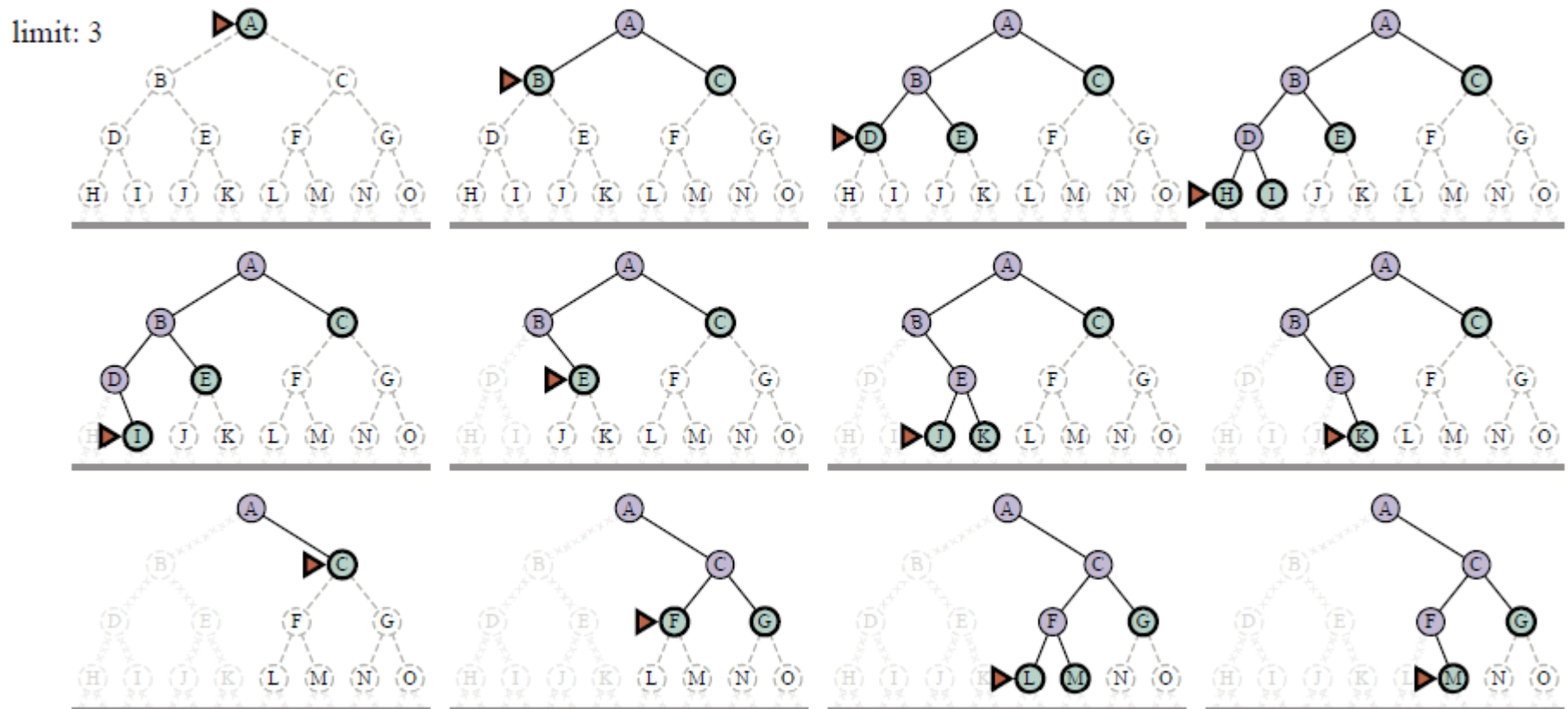
limit: 2



Iterative deepening depth-first search

Main idea: iteratively apply depth-limited search with gradually increasing depth limit l

Stop until $l = d$



Iterative deepening depth-first search – performance

Main idea: iteratively apply depth-limited search with gradually increasing depth limit l Stop until $l = d$

- **Complete**

if the depth d of the shallowest goal node is finite

- **Optimal**

if all actions have the same cost

Combine the benefit of breadth-first search and depth-first search

- **Time complexity**

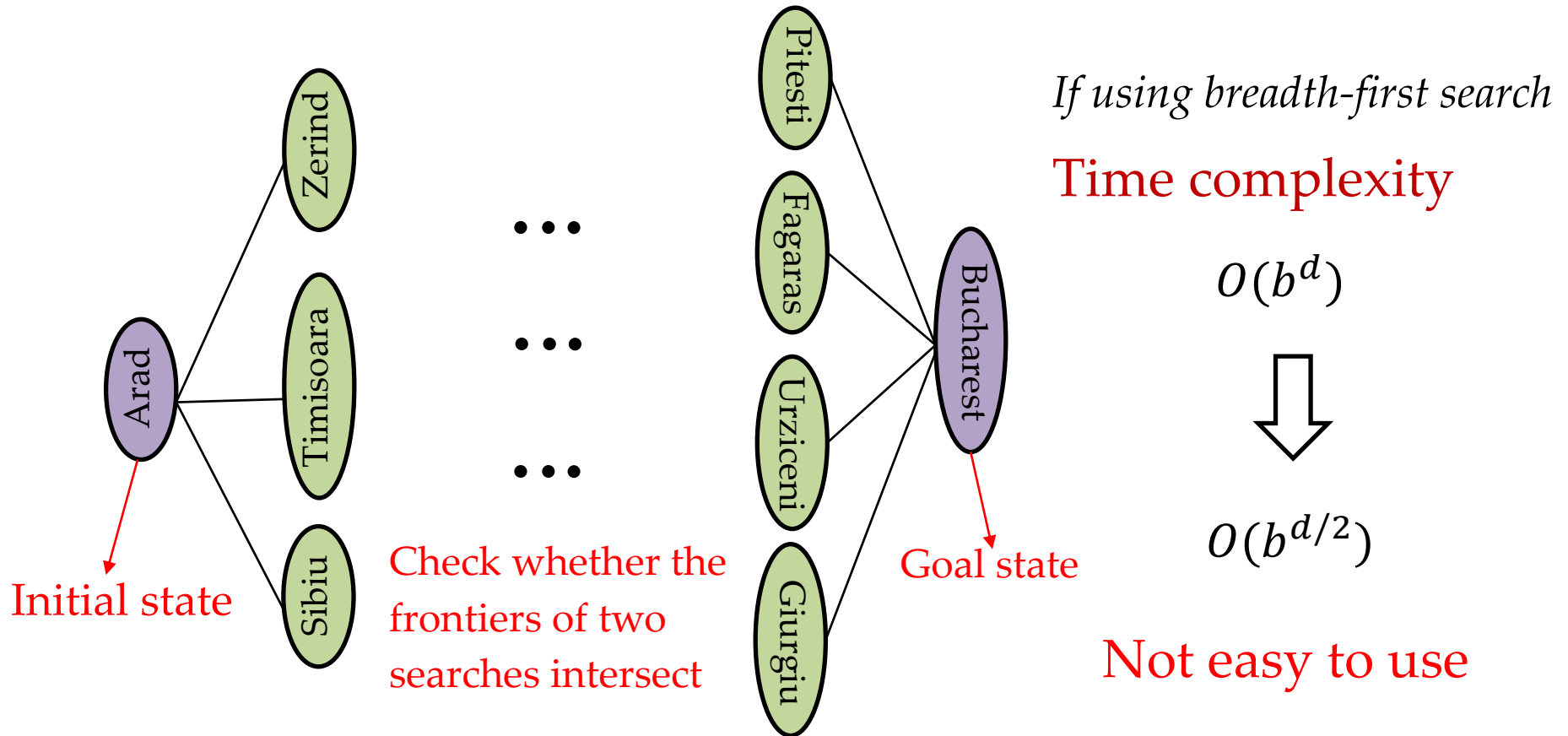
$$db + (d - 1)b^2 + (d - 2)b^3 + \cdots + (1)b^d = O(b^d)$$

- **Space complexity**

$O(bd)$, if using tree-search

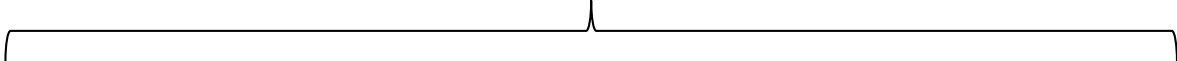
Bidirectional search

Main idea: run two simultaneous searches, one forward from the initial state and the other backward from the goal



Performance comparison

Tree-search versions



Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?	Yes ¹	Yes ^{1,2}	No	No	Yes ¹	Yes ^{1,4}
Optimal cost?	Yes ³	Yes	No	No	Yes ³	Yes ^{3,4}
Time	$O(b^d)$	$O(b^{1+\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{1+\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(b\ell)$	$O(bd)$	$O(b^{d/2})$

Figure 3.15 Evaluation of search algorithms. b is the branching factor; m is the maximum depth of the search tree; d is the depth of the shallowest solution, or is m when there is no solution; ℓ is the depth limit. Superscript caveats are as follows: ¹ complete if b is finite, and the state space either has a solution or is finite. ² complete if all action costs are $\geq \epsilon > 0$; ³ cost-optimal if action costs are all identical; ⁴ if both directions are breadth-first or uniform-cost.

Summary

- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening depth-first search
- Bidirectional search

**Uninformed
search**

*No additional
information beyond
the problem definition*

References

- S. J. Russell and P. Norvig. Artificial Intelligence: A Modern Approach. Chapter 3.4, Third edition.