Homework 3

作业要求：

提交一份pdf文档，并发送到bianc@lamda.nju.edu.cn，6月30日23:59截止。
- pdf文档命名方式："学号-姓名.pdf"，例如"MG1937000-张三.pdf"；
- 邮件标题命名："随机过程第三次作业-学号-姓名"，
  例如"随机过程第三次作业-MG1937000-张三"。

pdf可以用latex/word/markdown等方式生成，但是不要用手写证明的照片。

作业的评分主要参考以下几点：
1. 证明过程的完整性以及正确性。例如在使用之前的定理时是否充分考虑了其条件，公式推导是否完整、以及是否有错误。
2. 文档的细节。例如是否出现符号错误，文档格式是否混乱。

若发现作业出现雷同的情况，会根据相关规定给予惩罚，详情请参考课程主页中"学术诚信"的相关内容。请同学们务必独立完成作业！

Problem 1

If i is null recurrent and $i \leftrightarrow j$, prove that j is null recurrent.

Problem 2

Consider the gambler's ruin problem with $p = 0.4$ and $N = 6$. Starting in state 3, determine
- the expected amount of time spent in state 3 ($m_{3,3}$)
- the expected number of visits to state 2 ($m_{3,2}$)
- the probability of ever visiting state 4 ($f_{3,4}$)

Problem 3

Consider a branching process, suppose that $P_0 > 0$ and $P_0 + P_1 < 1$.
Prove: $\pi_0 = 1$ if and only if $\mu \leq 1$, where $\mu = \sum_{j=0}^{\infty} j P_j$ is the mean number of offspring produced by each individual

Problem 4

Consider an irreducible Markov chain with transition probabilities P_{ij}. If one can find nonnegative numbers π_i, $i \geq 0$, summing to unity, and a transition probability matrix $\boldsymbol{P}^* = [P_{ij}^*]$ such that $\pi_i P_{ij} = \pi_j P_{ji}^*$.
Prove: π_i, $i \geq 0$ are also the stationary probabilities of the reverse chain

Problem 5

For a Markov Chain $\{x_n, n \geq 0\}$, show that
$$P(X_k = i_k | X_j = i_j, \text{for all } j \neq k) = P(X_k = i_k | X_{k-1} = i_{k-1}, X_{k+1} = i_{k+1})$$