High-Order Heterogeneous Data Mining

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Why High-Order Heterogeneous?

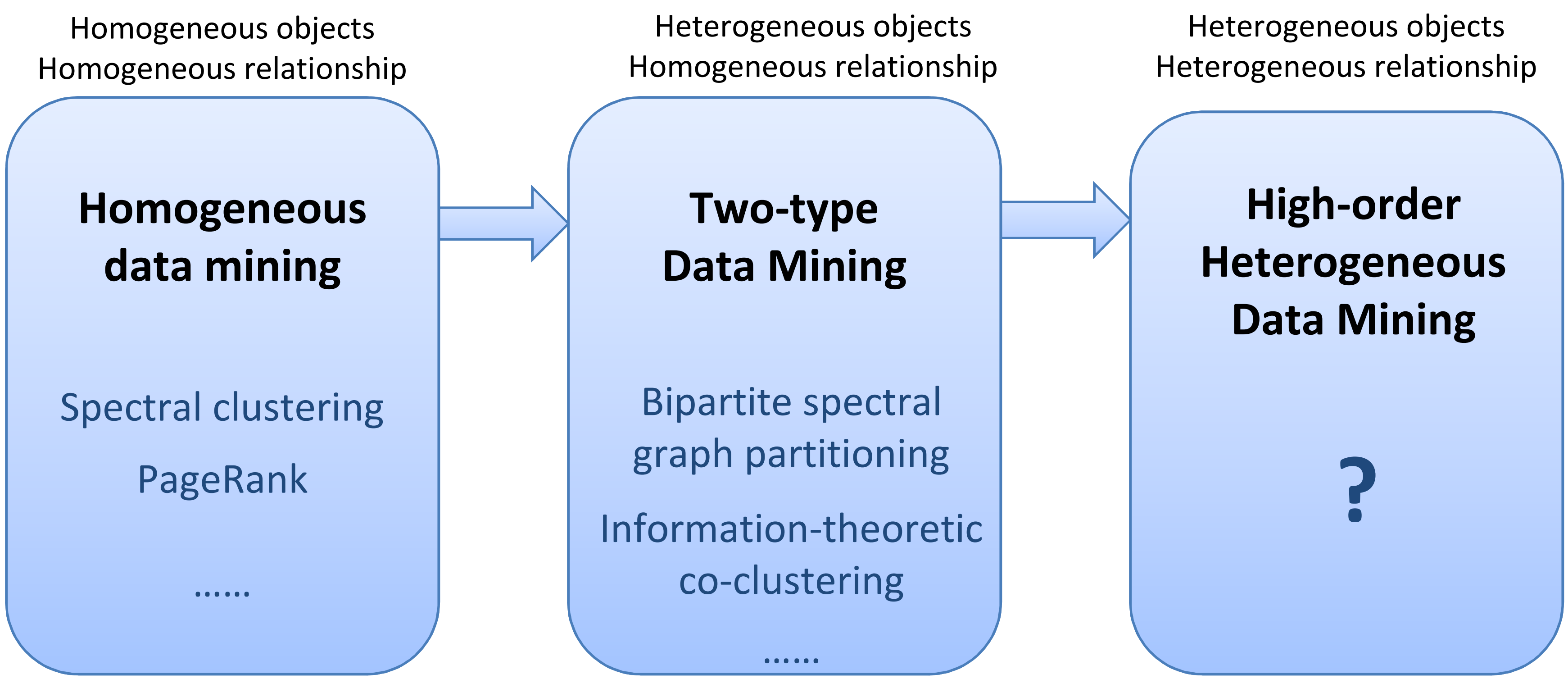
- The world is heterogeneous
  - Objects are heterogeneous:
    - (query, document…), (author, paper…)
- Many applications involve multiple types of objects
  - Web search
    - User ←→ Query ←→ Web Page
  - Academic society
    - Author ←→ Paper ←→ Conference

      ↑↓

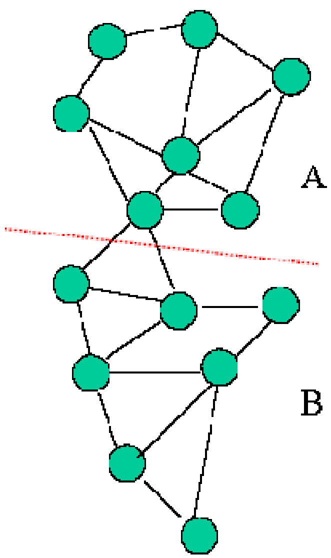
      Journal
  - Relationships among these objects are also heterogeneous: similarity, relevance, endorsement; directed, undirected…

However, …

- Most traditional ML and DM methods focus on homogeneous data, or data of no more than two types.



Related Work: Spectral Clustering
(PAMI 2000)



- Spectral clustering cuts relationship graph to cluster similar data.
- Minimize graph cut

$$obj = \frac{cut(V_1,V_2)}{weight(V_1)} + \frac{cut(V_2,V_1)}{weight(V_2)}$$

$$cut(V_1,V_2) = \sum_{i\in V_1, j\in V_2, <i,j>\in E} e_{ij}$$

and $weight(V_i) = \sum_{j\in V_i} W_j$.

$$\min \frac{q^T L q}{q^T D q}, \text{ subject to } q^T De = 0, q \neq 0$$

- Solution
  - Graph cut can be converted to a generalized eigenvalue problem by using continuous slacking: $Lq = \lambda Dq$
  - The eigenvector associated with the second smallest eigenvalue of the Laplace matrix is an optimal embedding for cut minimization.

Related Work: PageRank
(WWW 1998)

- PageRank ranks the popularity of vertices in a directed graph according to their linkage information.
- PageRank of a vertex is proportional to its parents' rank, but inversely proportional to its parents' outdegree.

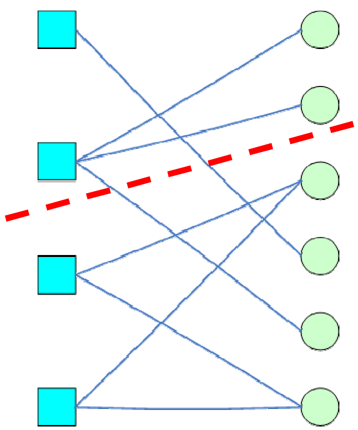
$$R(u) = d + (1-d)\sum_{v \in B_u} \frac{R(v)}{N_v}$$

$$R = (1-d)AR + d\Pi, A_{u,v} = \frac{1}{N_v}, \Pi = \frac{1}{N}[1,1,\ldots,1]'$$

- PageRank can be explained using a Markov random surfer model; or be explained as the principal eigenvector of the smoothed adjacency matrix of the Web graph.

Related Work: Bipartite Graph Partitioning

(KDD 2001)



- Cuts bipartite relationship graph to cluster two types of data simultaneously.

$$M = \begin{array}{c} \\ X \\ Y \end{array} \begin{array}{c} \begin{array}{cc} X & Y \end{array} \\ \begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix} \end{array}$$

- Due to the bipartite property of the graph, after some trivial deduction, this problem can be converted to a singular value decomposition (SVD) problem.

Related Work: Information Theoretic Co-Clustering (KDD 2003)

$C_X : \{x_1, \ldots, x_m\} \rightarrow \{\hat{x}_1, \ldots, \hat{x}_r\}$

$C_Y : \{y_1, \ldots, y_n\} \rightarrow \{\hat{y}_1, \ldots, \hat{y}_s\}$

- An optimal co-clustering minimizes $I(X,Y) - I(\hat{X},\hat{Y})$

subject to the constraints on the number of row and column clusters.

It can be proved that

$$I(X,Y) - I(\hat{X},\hat{Y}) = D(p(X,Y) \| q(X,Y))$$

where *D*(,) denotes the KL divergence, and *q*(*X*,*Y*) is a distribution of the form

$$q(x,y) = p(\hat{x},\hat{y})\,p(x \mid \hat{x})\,p(y \mid \hat{y})$$

[Step 1] Set $i = 1$. Start with (R_i, C_i), Compute $q_{[i,i]}$.

[Step 2] For every row x, assign it to the cluster $\hat{x}$ that minimizes

$$KL(p(y \mid x) \,\|\, q_{[i,i]}(y \mid \hat{x}))$$

[Step 3] We have (R_{i+1}, C_i). Compute $q_{[i+1,i]}$.

[Step 4] For every column y, assign it to the cluster $\hat{y}$ that minimizes

$$KL(p(x \mid y) \,\|\, q_{[i+1,i]}(x \mid \hat{y}))$$

[Step 5] We have (R_{i+1}, C_{i+1}). Compute $q_{[i+1,i+i]}$. Iterate 2-5.

Going Beyond…

- Modeling the relationships
  - Unified Relationship Matrix
  - Tensor
  - Collective bipartite graphs
- Designing effective data mining algorithms
  - High-order Heterogeneous Coclustering
  - High-order Heterogeneous Coranking

Unified Relationship Matrix

- Integrate pairwise relationship matrices into a unified matrix
  - L'_M : intra-type adjacency matrix
  - L'_{NM}: inter-type adjacency matrix

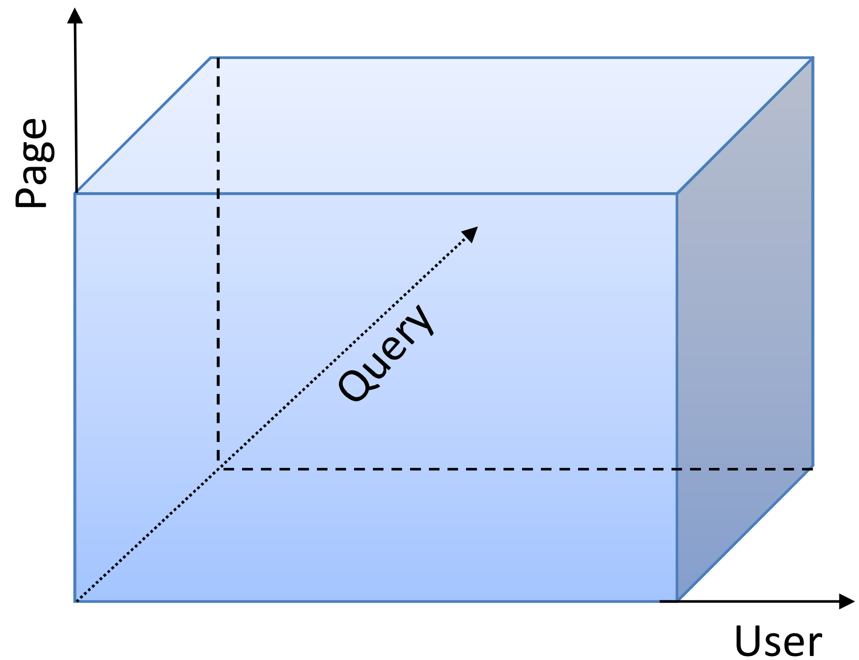
$$L = \begin{vmatrix} \lambda_{11}L_1 & \lambda_{12}L_{12} & \cdots & \lambda_{1N}L_{1N} \\ \lambda_{21}L_{21} & \lambda_{22}L_2 & \cdots & \lambda_{2N}L_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{N1}L_{N1} & \lambda_{N2}L_{N2} & \cdots & \lambda_{NN}L_N \end{vmatrix} \qquad L_{urm} = D^{-1}L$$

Combination coefficients can be manually set or learned from labeled data

- Representative Work
  - Wensi Xi, et al, Link Fusion: A Unified Link Analysis Framework for Multi-Type Interrelated Data Objects. ***WWW 2004***.
  - Zaiqing Nie, et al, Object-Level Ranking: Bringing Order to Web Objects. ***WWW 2005***.
  - Wensi Xi, et al, SimFusion: Measuring Similarity using Unified Relationship Matrix, ***SIGIR 2005***.
  - Xuanhui Wang, et al, Latent Semantic Analysis for Multiple-Type Interrelated Data Objects, ***SIGIR 2006***.

Tensor

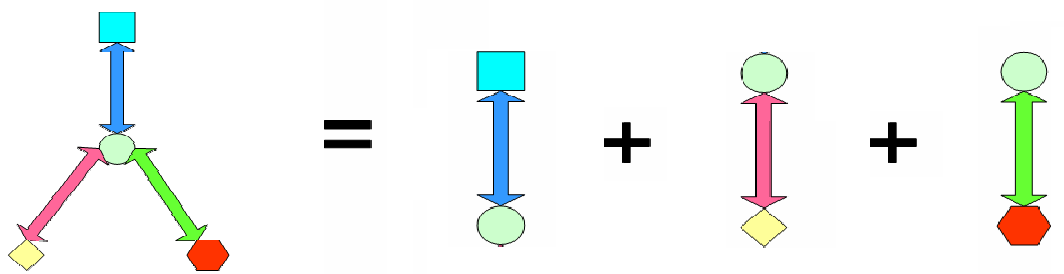
- Use multi-linear algebra to represent heterogeneous relationship.



- Representative Work
  - Jian-Tao Sun, et al, CubeSVD: A Novel Approach to Personalized Web Search, ***WWW 2005***.

Collective Bi-partite Graphs

- Decompose heterogeneous relationship into a collections of pairwise relationships.



- Representative Work
  - Bin Gao, Tie-Yan Liu, et al, Hierarchical Taxonomy Preparation for Text Categorization Using Consistent Bipartite Spectral Graph Co-partitioning, ***IEEE TKDE***.
  - Bin Gao, Tie-Yan Liu, et al, Consistent Bipartite Graph Co-Partitioning for Star-Structured High-Order Heterogeneous Data Co-Clustering, ***KDD 2005***.
  - Bin Gao, Tie-Yan Liu, et al, Star-Structured High-Order Heterogeneous Data Co-clustering based on Consistent Information Theory, ***ICDM 2006***.
  - Bo Long, Zhongfei Zhang, et al, Spectral Clustering for Multi-type Relational Data, ***ICML 2006***.

Algorithms

- Unified Relationship Matrix
  - LinkFusion (WWW 2004)
  - Object-level Ranking (WWW 2005)
  - SimFusion (SIGIR 2005)
  - Multi-type LSA (SIGIR 2006)
- Tensor
  - CubeSVD (WWW 2005)
- Collective Bipartite Graphs
  - Consistent Bipartite Graph Co-partitioning (KDD 2005)
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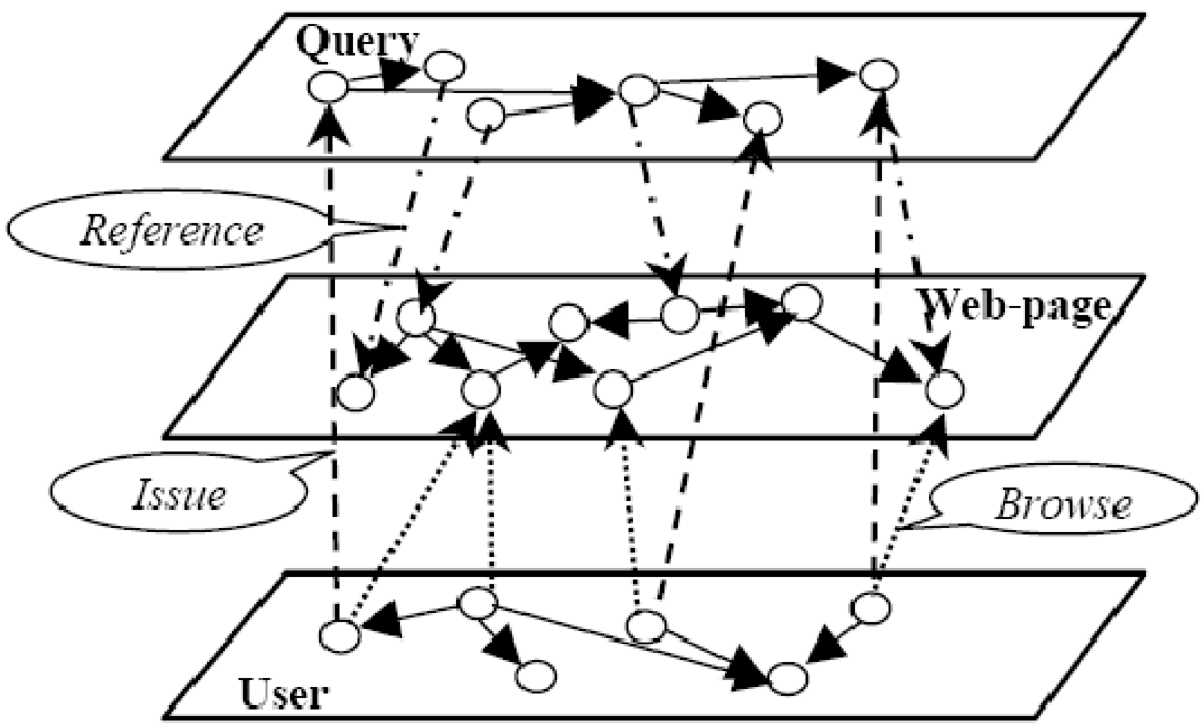
Link Fusion

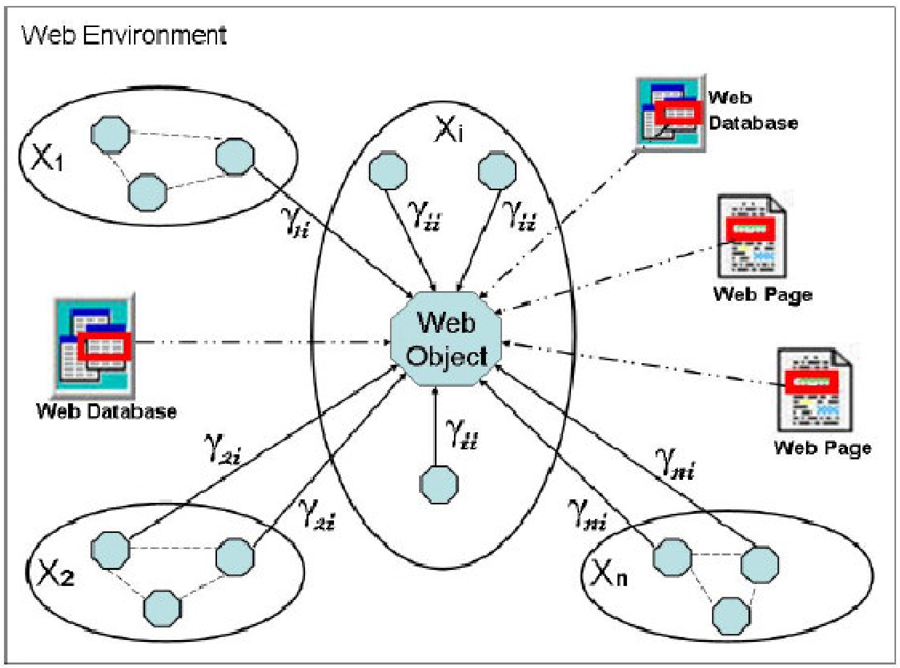
- High-order heterogeneous version of PageRank

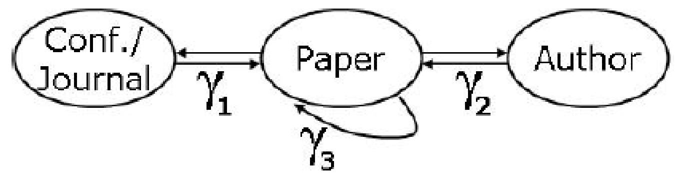


Random Walk on Heterogeneous Graph

- Construct the URM by merging pair-wise PageRank matrices with manually-set combination coefficients.
- Imagine a Markov random walk over the heterogeneous graph represented by the URM.
- Ranking over heterogeneous data will correspond to the principle eigenvector of the URM: $w = L_{urm}^T w$, and the convergence can be proven.

Object-Level Ranking





$$R_X = \varepsilon R_{EX} + (1-\varepsilon)\sum_{\forall Y} \gamma_{YX} M_{YX}^{T} R_Y$$

- Use similar URM formulation to LinkFusion
- Learn the combination coefficients with a training set.

Learning the Coefficients

Subgraph Selection

Starting with the labeling data objects, and including all other objects with less than *k*-step links from them.

→

Parameter Search

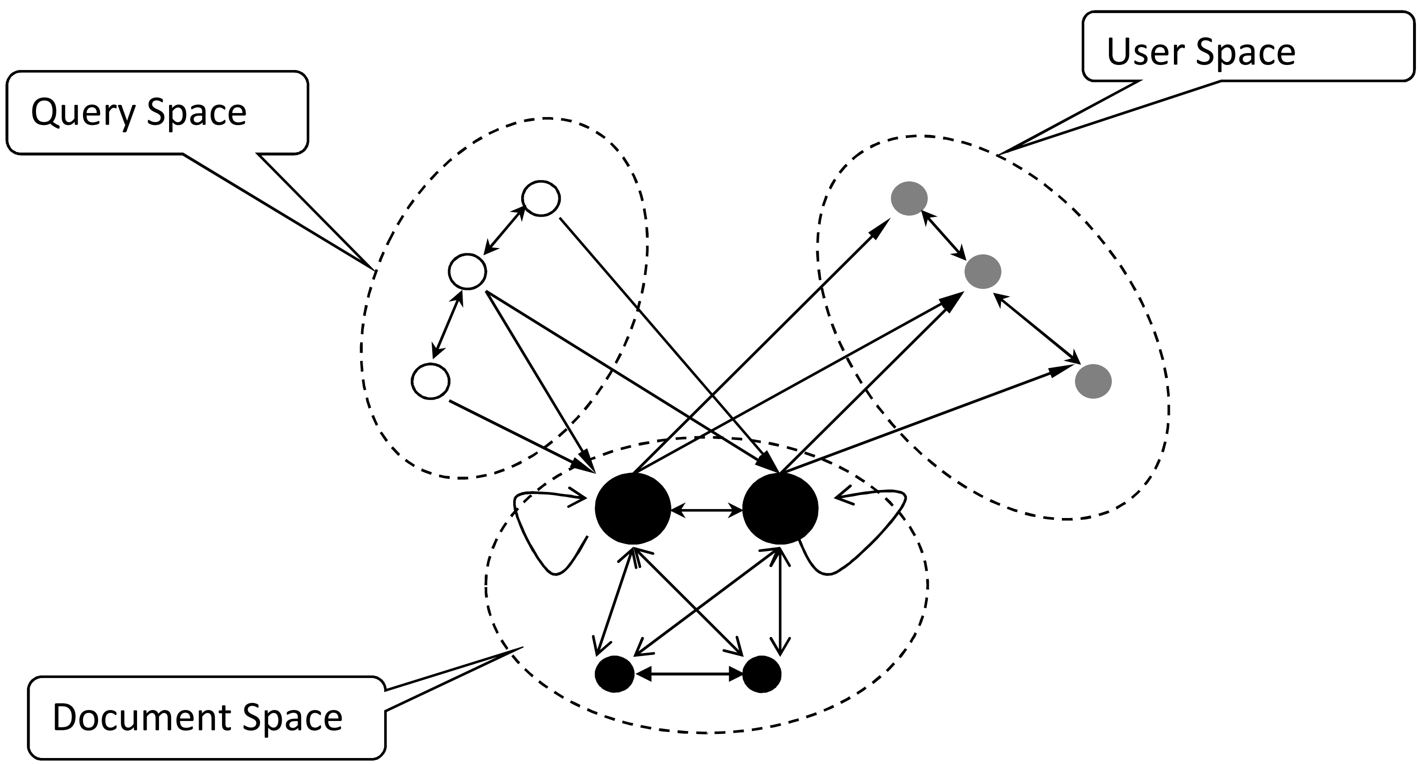
Using simulated annealing based method to search the best parameter in the selected subgraph.

Algorithm *DiameterEstimator(δ:stopping threshold)*
  for (each object type X)
    $n \leftarrow$ total number of different object types related to objects of type X;
    for (each related object type Y) $\gamma_{YX} \leftarrow \frac{1}{n}$;
  end for
  compute the *PopRank* scores over the entire graph;
  $R \leftarrow$ the ranking vector of the training objects;
  $R' \leftarrow E$;
  $k \leftarrow 0$;
  while$(||R - R'||_1 > \delta)$
    $k++$;
    compute the *PopRank* scores over the k diameter subgraph;
    $R' \leftarrow$ the ranking vector of the training objects;
  end while
  return k;
End *DiameterEstimator*;

Algorithm *SAFA(timeout: stopping condition)*
  for (each object type X)
    $n \leftarrow$ total number of different object types related to objects of type X;
    for (each related object type Y) $\gamma_{YX} \leftarrow \frac{1}{n}$;
  end for
  $t \leftarrow$ a large number;
  do
    for (each object type X)
      for (each object type Y)
        repeat
          repeat
            randomly select γ'_{YX} in $Neighbor(\gamma_{YX})$
            $diff \leftarrow f(\gamma_{YX}) - f(\gamma'_{YX})$;
            if $diff > 0$ **then** $\gamma_{YX} \leftarrow \gamma'_{YX}$;
            else generate random x in (0,1)
              if $x < exp(-diff/t)$ **then** $\gamma_{YX} \leftarrow \gamma'_{YX}$;
          until iteration count = $max\_number\_iteration$;
          $t \leftarrow 0.9t$;
        until iteration count = $max\_number\_iteration$;
      end for
    end for
  until timeout;
  return the best combination of $\gamma_{YX}s$;
End *SAFA*;

SimFusion

The similarity of two data objects in one data type can be reinforced by the similarity value of other data objects they are related to.

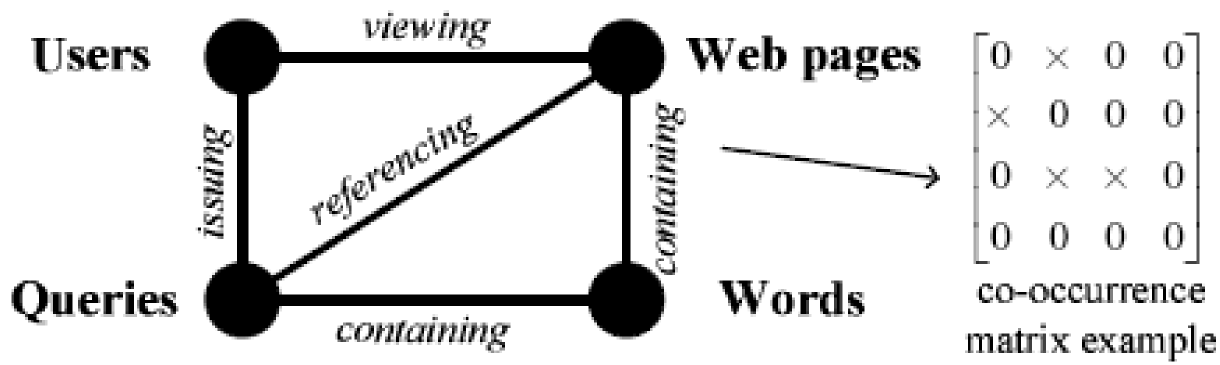


Mathematical Formulation

- The similarity reinforcement assumption can be represented as:
  - $S^{new} = L_{urm} S^{original} L_{urm}^T$
  - $S^n = L_{urm} S^{n-1} L_{urm}^T = L_{urm}^n S^0 (L_{urm}^n)^T$
  - Convergence can be proven.
- The so-calculated similarity can be used for many applications such as object clustering and information retrieval.

Multi-type LSA

- The Mutual Reinforcement Principle of LSA
  - On a multiple-type graph G with N vertices and a number of pairwise co-occurrence relationships, *important* objects of a type co-occur with *important objects* of other types.



Low Rank Approximation

- Conduct EVD on the URM
- Apply similar ideas to principal component analysis, we can regard top k eigenvectors as representing the top k important concepts, and use them to span a k-dimensional semantic space to represent all the objects.
- Use the low-rank approximation of the URM to capture latent semantics, just as classical LSA does.

Discussions on URM

- Pros
  - By building URM, traditional methods for homogeneous data can be easily used.
  - Linear algebra might be the most mature mathematical tool in data mining.
- Cons
  - Basic assumption in these approaches is questionable: is it really reasonable that heterogeneous relationship can become homogeneous with linear scaling?

Algorithms

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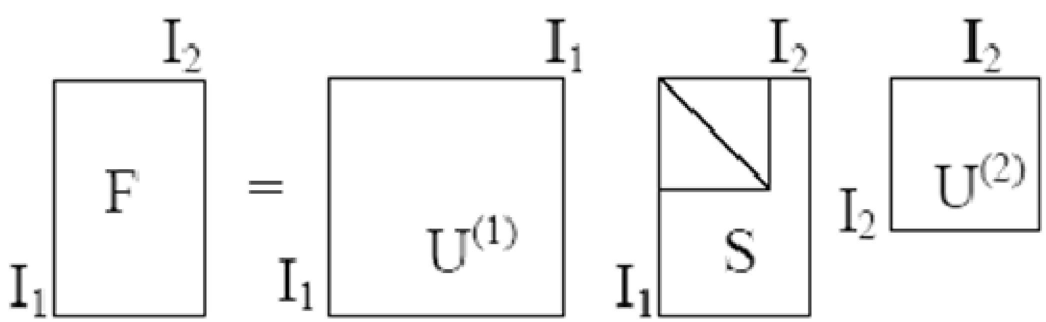
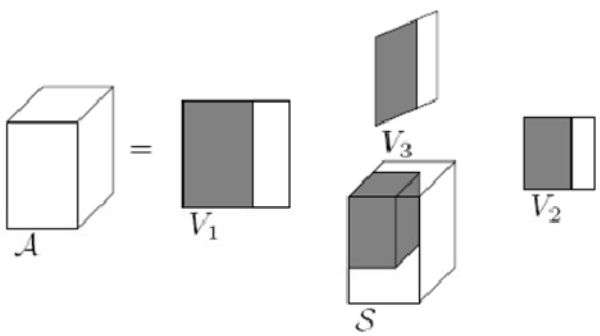
CubeSVD

- Matrix Singular Value Decomposition (SVD)
  - Latent Semantic Indexing (LSI)
    - Apply SVD on document-term matrix
  - In Recommender System
    - Apply SVD on user-item preference matrix



CubeSVD (cont.)

- Tensor Singular Value Decomposition (High-order SVD)
  - Higher-Order SVD might also capture the latent factors that govern the relations among multi-type objects.
  - These semantic relationships can be used to get better clustering.



$$\mathcal{A} = \mathcal{S} \times_1 V_1 \times_2 V_2 \cdots \times_N V_N$$

1. Construct tensor $\mathcal{A}$ from the clickthrough data. Suppose the numbers of user, query and Web page are m, n, k respectively, then $\mathcal{A} \in R^{m \times n \times k}$. Each tensor element measures the preference of a $\langle user, query \rangle$ pair on a Web page.
2. Calculate the matrix unfolding A_u, A_q and A_p from tensor $\mathcal{A}$. A_u is calculated by varying user index of tensor $\mathcal{A}$ while keeping query and page index fixed. A_q and A_p are computed in a similar way. Thus A_u, A_q, A_p is a matrix of $m \times nk$, $n \times mk$, $k \times mn$ respectively.
3. Compute SVD on A_u, A_q and A_p, set V_u, V_q and V_p to be the left matrix of the SVD respectively.
4. Select $m_0 \in [1, m]$, $n_0 \in [1, n]$ and $k_0 \in [1, k]$. Remove the right-most $m - m_0$, $n - n_0$ and $k - k_0$ columns from V_u, V_q and V_p , then denote the reduced left matrix by W_u, W_q and W_p respectively. Calculate the core tensor as follows:

$$\mathcal{S} = \mathcal{A} \times_1 W_u^T \times_2 W_q^T \times_3 W_p^T$$

5. Reconstruct the original tensor by:

$$\hat{\mathcal{A}} = \mathcal{S} \times_1 V_u \times_2 V_q \times_3 V_p$$

Discussions on Tensor

- Pros
  - Using tensor to represent heterogeneous data objects is more natural than URM
- Cons
  - Multi-linear algebra is in its initial stage, and many basic operations for tensor have not been reasonably define.
    - Tensors cannot always be "diagonalized"
    - k successive rank-1 approximations to tensors do not necessarily result in the best rank-k approximation
    - Eight factors about tensor
  - Complexity of tensor operator is very high, thus tensor based methods are difficult to scale up.

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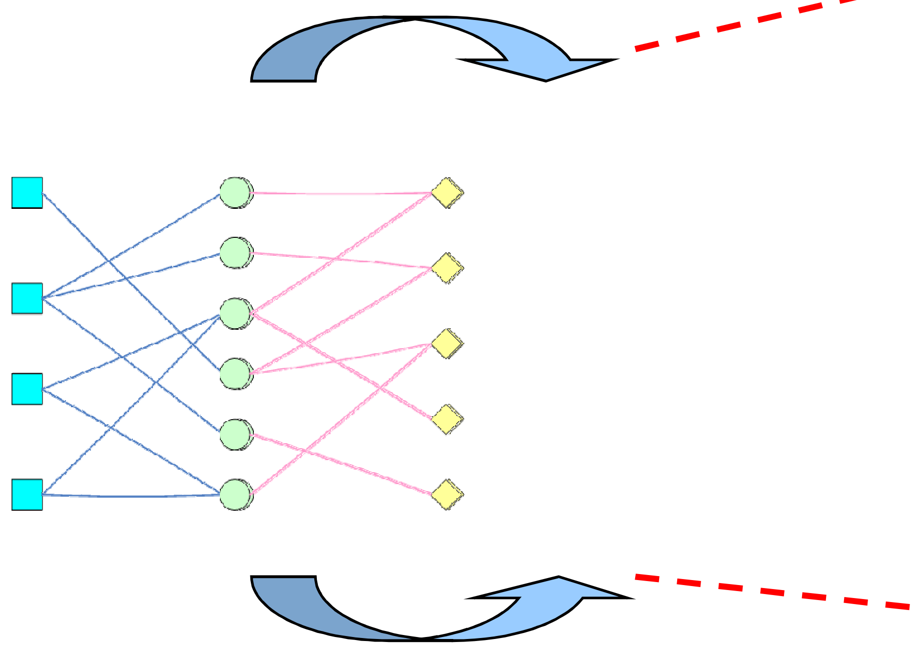
Consistent Bipartite Graph Copartitioning

- User graphs to represent the heterogeneous relationship.
- Divide the heterogeneous graph into a collection of bipartite graphs.
- Conduct spectral co-clustering on each bipartite graph, provided that the partitioning of the shared part of two bipartite graphs should be the same or almost the same.
- Develop an SDP-based solution to get the consistent partitioning results.

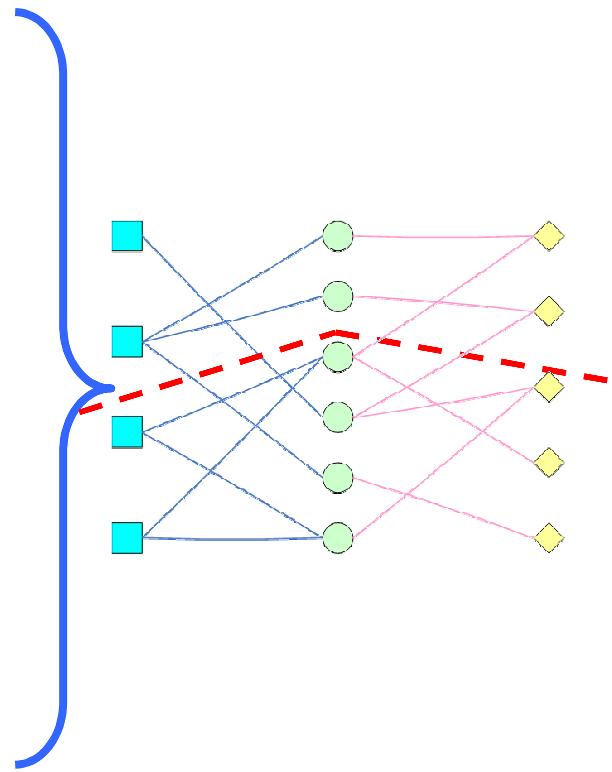
Consistent Partitioning



divide this tripartite graph into
two bipartite graphs

$$A_{m\times n}$$
$$M_1 = \begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix}$$
$$D^{(1)} = diag(M_1 e)$$
$$L^{(1)} = D^{(1)} - M_1$$
$$q = \begin{pmatrix} x \\ y \end{pmatrix}_{(m+n)\times 1}$$

$$B_{n\times t}$$
$$M_2 = \begin{bmatrix} 0 & B \\ B^T & 0 \end{bmatrix}$$
$$D^{(2)} = diag(M_2 e)$$
$$L^{(2)} = D^{(2)} - M_2$$
$$p = \begin{pmatrix} y \\ z \end{pmatrix}_{(n+t)\times 1}$$



partition these two graphs
simultaneously and consistently

Formulating the Optimization Problem

- Minimize the cuts of the two bipartite graphs, with the constraints that their partitioning results on the central type of objects are the same.
- Objective Function:

$$\min \quad \frac{q^T L^{(1)} q}{q^T D^{(1)} q}$$

$$\min \quad \frac{p^T L^{(2)} p}{p^T D^{(2)} p}$$

$$\text{subject to} \quad q^T D^{(1)} e = 0,\ q \neq 0$$

$$p^T D^{(2)} e = 0,\ p \neq 0$$

$$0 < \beta < 1$$

$$q = \begin{pmatrix} x \\ y \end{pmatrix}_{(m+n)\times 1}$$

$$p = \begin{pmatrix} y \\ z \end{pmatrix}_{(n+t)\times 1}$$

How to Solve the Optimization Problem #1: Convert it to a QCQP Problem

Simplify the original Problem to single-objective programming

$$\begin{cases} \min \beta \dfrac{q^T L^{(1)} q}{q^T D^{(1)} q} + (1-\beta) \dfrac{p^T L^{(2)} p}{p^T D^{(2)} p} \\ \text{subject to } \; q^T D^{(1)} e = 0, q \neq 0 \\ \qquad\qquad\quad p^T D^{(2)} e = 0, \; p \neq 0 \\ \qquad\qquad\quad 0 < \beta < 1 \end{cases}$$

Assistant Notations

$$\Gamma_1 = \begin{bmatrix} L^{(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}_{s \times s}, \quad \Gamma_2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & L^{(2)} \end{bmatrix}_{s \times s}$$

$$\Pi_1 = \begin{bmatrix} D^{(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}_{s \times s}, \quad \Pi_2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & D^{(2)} \end{bmatrix}_{s \times s}$$

Sum-of-ratios Quadratic Fractional Programming

$$\begin{cases} \min \left(\beta \dfrac{\omega^T \Gamma_1 \omega}{\omega^T \Pi_1 \omega} + (1-\beta) \dfrac{\omega^T \Gamma_2 \omega}{\omega^T \Pi_2 \omega} \right) \\ \text{subject to } \; \omega^T \Pi_1 e = 0 \\ \qquad\qquad\quad \omega^T \Pi_2 e = 0 \\ \qquad\qquad\quad \omega \neq 0, 0 < \beta < 1 \end{cases}$$

Quadratically Constrained Quadratic Programming (QCQP)

$$\begin{cases} \min \omega^T \Gamma \omega \\ \text{subject to } \; \omega^T \Pi_1 \omega = e^T \Pi_1 e \\ \qquad\qquad\quad \omega^T \Pi_2 \omega = e^T \Pi_2 e \\ \qquad\qquad\quad \omega^T \Pi_1 e = 0 \\ \qquad\qquad\quad \omega^T \Pi_2 e = 0 \end{cases}$$

$$\Gamma = \frac{\beta}{e^T \Pi_1 e} \Gamma_1 + \frac{1-\beta}{e^T \Pi_2 e} \Gamma_2, \; 0 < \beta < 1$$

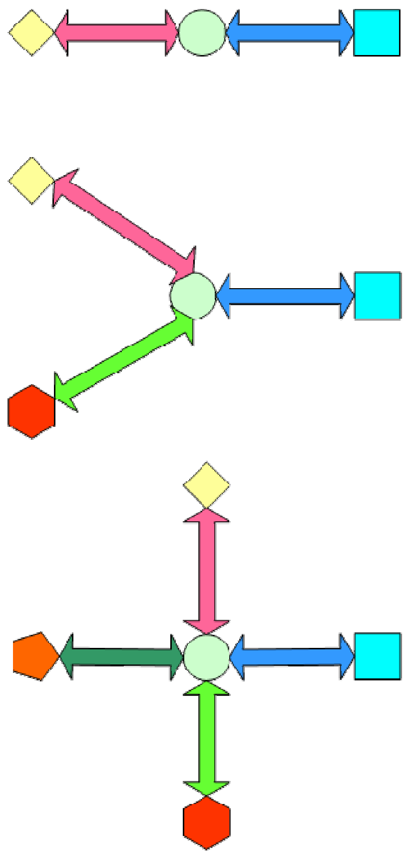
How to Solve the Optimization Problem #2: Convert QCQP to SDP

Semi-definite Programming (SDP)

$$
\begin{cases}
\min_{\omega,\Omega} \begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \Gamma \end{bmatrix} \bullet \begin{bmatrix} 1 & \omega^T \\ \omega & \Omega \end{bmatrix} \\
\text{subject to } \begin{bmatrix} -e^T\Pi_1 e & \mathbf{0} \\ \mathbf{0} & \Pi_1 \end{bmatrix} \bullet \begin{bmatrix} 1 & \omega^T \\ \omega & \Omega \end{bmatrix} = 0 \\
\begin{bmatrix} -e^T\Pi_2 e & \mathbf{0} \\ \mathbf{0} & \Pi_2 \end{bmatrix} \bullet \begin{bmatrix} 1 & \omega^T \\ \omega & \Omega \end{bmatrix} = 0 \\
\begin{bmatrix} 0 & e^T\Pi_1/2 \\ \Pi_1 e/2 & \mathbf{0} \end{bmatrix} \bullet \begin{bmatrix} 1 & \omega^T \\ \omega & \Omega \end{bmatrix} = 0 \\
\begin{bmatrix} 0 & e^T\Pi_2/2 \\ \Pi_2 e/2 & \mathbf{0} \end{bmatrix} \bullet \begin{bmatrix} 1 & \omega^T \\ \omega & \Omega \end{bmatrix} = 0 \\
\begin{bmatrix} 1 & \omega^T \\ \omega & \Omega \end{bmatrix} \succeq 0
\end{cases}
$$

$$
\begin{cases}
\min_W \begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \Gamma \end{bmatrix} \bullet W \\
\text{subject to } \begin{bmatrix} -e^T\Pi_1 e & \mathbf{0} \\ \mathbf{0} & \Pi_1 \end{bmatrix} \bullet W = 0 \\
\begin{bmatrix} -e^T\Pi_2 e & \mathbf{0} \\ \mathbf{0} & \Pi_2 \end{bmatrix} \bullet W = 0 \\
\begin{bmatrix} 0 & e^T\Pi_1/2 \\ \Pi_1 e/2 & \mathbf{0} \end{bmatrix} \bullet W = 0 \\
\begin{bmatrix} 0 & e^T\Pi_2/2 \\ \Pi_2 e/2 & \mathbf{0} \end{bmatrix} \bullet W = 0 \\
\begin{bmatrix} 1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \bullet W = 1, \\
\begin{bmatrix} 0 & e \\ e & \mathbf{0} \end{bmatrix} \bullet W = \theta_1, \\
\begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{0} & E \end{bmatrix} \bullet W = \theta_2 \\
W \succeq 0
\end{cases}
$$

Extension to More Complex Heterogeneous Graphs



$$\begin{cases} \min \sum_{i=1}^{k-1} \beta_i \dfrac{q_i^T L^{(i)} q_i}{q_i^T D^{(i)} q_i} \\ \text{subject to } \; q_i^T D^{(i)} e = 0, q_i \neq 0, i = 1, \ldots, k-1 \\ \qquad \sum_{i=1}^{k-1} \beta_i = 1, \; 0 < \beta_i < 1 \end{cases}$$

Consistent Information-theoretic Co-clustering

The consistency concept

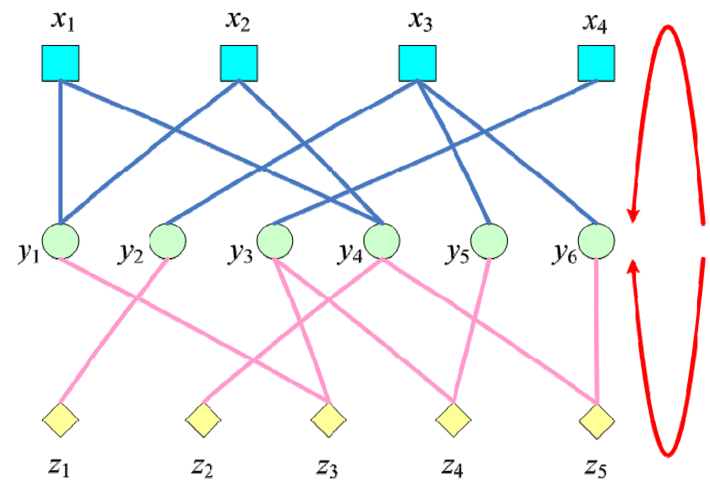
+

Information-theoretic coclustering (KDD 2003)

Mathematical Formulation

- Co-clustering

$$C_X : \{x_1, \ldots, x_m\} \rightarrow \{\hat{x}_1, \ldots, \hat{x}_r\}$$
$$C_Y : \{y_1, \ldots, y_n\} \rightarrow \{\hat{y}_1, \ldots, \hat{y}_s\}$$
$$C_Z : \{z_1, \ldots, z_l\} \rightarrow \{\hat{z}_1, \ldots, \hat{z}_t\}$$



- A consistent co-clustering minimizes the following objective functions

$$(i)\ F(X,Y,Z) = \alpha D(p_1(X,Y) \| q_1(X,Y)) + (1-\alpha) D(p_2(Y,Z) \| q_2(Y,Z)),$$
$$\quad where\ 0 < \alpha < 1$$

$$(ii)\ F(X,Y,Z) = \min_{X,Y,Z} \left\{ \max\left\{ D(p_1(X,Y) \| q_1(X,Y)), D(p_2(Y,Z) \| q_2(Y,Z)) \right\} \right\}$$

- Similar iterative method can be used to optimize *F(X,Y,Z),* and the convergence can be proved.

Generalized SVD for Co-clustering

- Rather than integrating heterogeneous relationship in a unified matrix or using tensor, we try to connect heterogeneous relationships using generalized SVD.
- While SVD corresponds to the optimal embedding of bipartite graph, GSVD might correspond to tripartite graph.

Theorem 1 If we have $\hat{A} \in R^{m\times n}$ and $\hat{B} \in R^{n\times t}$, $m \le n \le t$, then there exists unitary matrices $U \in R^{m\times m}, V \in R^{t\times t}$ and reversible matrix $X \in R^{n\times n}$ such that:

$$\begin{cases} \hat{A} = UCX^T \\ \hat{B} = XSV^T \end{cases}, \qquad (11)$$

where $C=diag(c_1,c_2,\ldots,c_m)$, $c_i \ge 0$ and $S=diag(s_1,s_2,\ldots,s_n)$, $s_i \ge 0$.

Generalized SVD for Co-clustering

1. Given A and B, form P_1, P_2, R_1, R_2, and $\hat{A}, \hat{B}$.
2. Compute GSVD of $\hat{A}, \hat{B}$ to get U, X, V, C, and S.
3. Form $H = CX^T XS$ and compute SVD of it to get U_H, V_H.
4. Form $U^* = UU_H, V^* = VV_H$ and take the second column vectors of them, u_2 and v_2, to form the normalized embedding vector

$$\omega_2 = [P_1^{-1/2} u_2 \;\; R_2^{-1/2} v_2]^T.$$

5. Cluster on the one-dimensional data $P_1^{-1/2} u_2$ and $R_2^{-1/2} v_2$ to obtain the desired bipartition of categories and terms, respectively.

No mathematical proof yet, since generalized SVD has no explicit objective function.

Spectral Clustering for Multi-type Relational Data

- Handling both pairwise relations and features

$$L = \sum_{1\le i<j\le m} w_a^{(ij)} ||R^{(ij)} - C^{(i)} A^{(ij)} (C^{(j)})^T||^2 + \sum_{1\le i\le m} w_b^{(i)} ||F^{(i)} - C^{(i)} B^{(i)}||^2$$

$$\max_{\substack{\{(C^{(i)})^T C^{(i)} \\ = I_{k_i}\}_{1\le i\le m}}} \sum_{1\le i\le m} w_b^{(i)} tr((C^{(i)})^T F^{(i)} (F^{(i)})^T C^{(i)}) + \sum_{1\le i<j\le m} w_a^{(ij)} tr((C^{(i)})^T R^{(ij)} C^{(j)} (C^{(j)})^T (R^{(ij)})^T C^{(i)})$$

$$\max_{(C^{(p)})^T C^{(p)} = I_{k_p}} \mathrm{tr}((C^{(p)})^T M^{(p)} C^{(p)})$$

$$M^{(p)} = w_b^{(p)} (F^{(p)} (F^{(p)})^T) + \sum_{p<j\le m} w_a^{(pj)} (R^{(pj)} C^{(j)} (C^{(j)})^T (R^{(pj)^T})) + \sum_{1\le j<p} w_a^{(jp)} ((R^{(jp)})^T C^{(j)} (C^{(j)})^T (R^{(jp)})).$$

Optimization Steps

- It can be proved the final equivalent optimization problem has close-form solution.
- The following algorithm is used to approximate this solution.

Algorithm 1 Spectral Relational Clustering

Input: Relation matrices $\{R^{(ij)} \in \mathbb{R}^{n_i \times n_j}\}_{1 \leq i < j \leq m}$, feature matrices $\{F^{(i)} \in \mathbb{R}^{n_i \times f_i}\}_{1 \leq i \leq m}$, numbers of clusters $\{k_i\}_{1 \leq i \leq m}$, weights $\{w_a^{(ij)}, w_b^{(i)} \in R_-\}_{1 \leq i < j \leq m}$.
Output: Cluster indicator matrices $\{C^{(p)}\}_{1 \leq p \leq m}$.
Method:

```
Initialize {C^(p)}_{1≤p≤m} with othonormal matrices.
repeat
    for p = 1 to m do
        Compute the matrix M^(p) as in Eq. (9).
        Update C^(p) by the leading k_p eigenvectors of M^(p).
    end for
until convergence
for p = 1 to m do
    transform C^(p) into a cluster indicator matrix by the k-means.
end for
```

Discussions on Collective Graphs

- Pros
  - It is more natural to decompose heterogeneous relationships into homogenous relationships, than to combine homogeneous relationships to heterogeneous relationships.
- Cons
  - Complexity of graph processing is relatively high than power method.
  - Graph fusion has not been well studied yet.

Summary

	URM	Tensor	Consistent Bipartite Graph
Clustering	SimFusion Multi-type LSA	CubeSVD	Consistent Bipartite Graph Copartitioning Consistent Information-Theoretic Coclustering … for Multi- … l Data
Ranking	LinkFusion Object-level Ranking	?	?

CubeRank ?

Consistent Rank?

Future Work

- Modeling the heterogeneous relationship more effectively.
  - Matrix, tensor, graphs, …
  - What is the next?
- Develop more efficient algorithms for high-order heterogeneous data mining.
  - Scalability is an issue for most of the algorithms mentioned in this talk.
  - Large-scale (multi-)linear algebra and large scale optimization
  - Supervised or semi-supervised learning for high-order heterogeneous data (i.i.d is not a reasonable assumption).

Further Discussions

- Although data objects are heterogeneous, they can be regarded as sampled from the same probability space.
  - The heterogeneity just comes from different views of the space.
- Can we recover the unified probability space and solve this problem from the root?
  - Reference paper
    - Ying Liu, Tao Qin, Tie-Yan Liu, et al, Similarity Space Projection: A Novel Framework for Web Image Search and Annotation. MIR 2005.

Thanks!

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