From Tetris to Relational Reinforcement Learning

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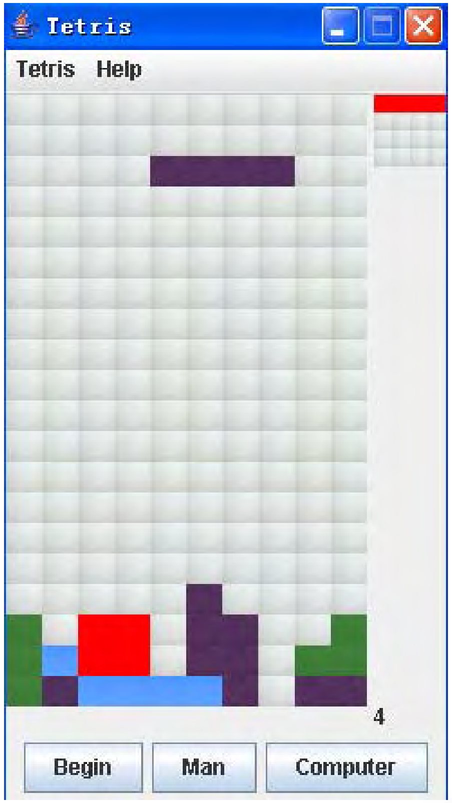
Outline

- Tetris
- Learn optimal policy by reinforcement learning (RL)
- RL + function approximation is enough?
- Features of Tetris
- Towards first order logic
- Markov logic networks
- Conclusion

Tetris (1)

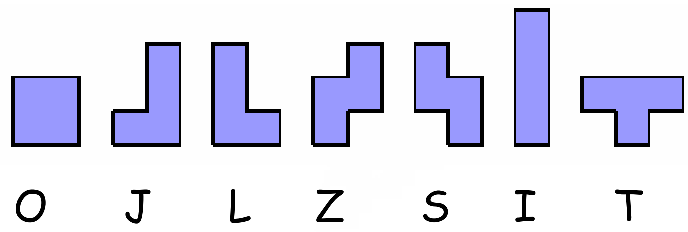
- Rewards (scores) = number of cleared lines



Tetris is a falling-blocks puzzle video game originally designed and programmed by **Alexey Pajitnov** in 1985.

Tetris (2)

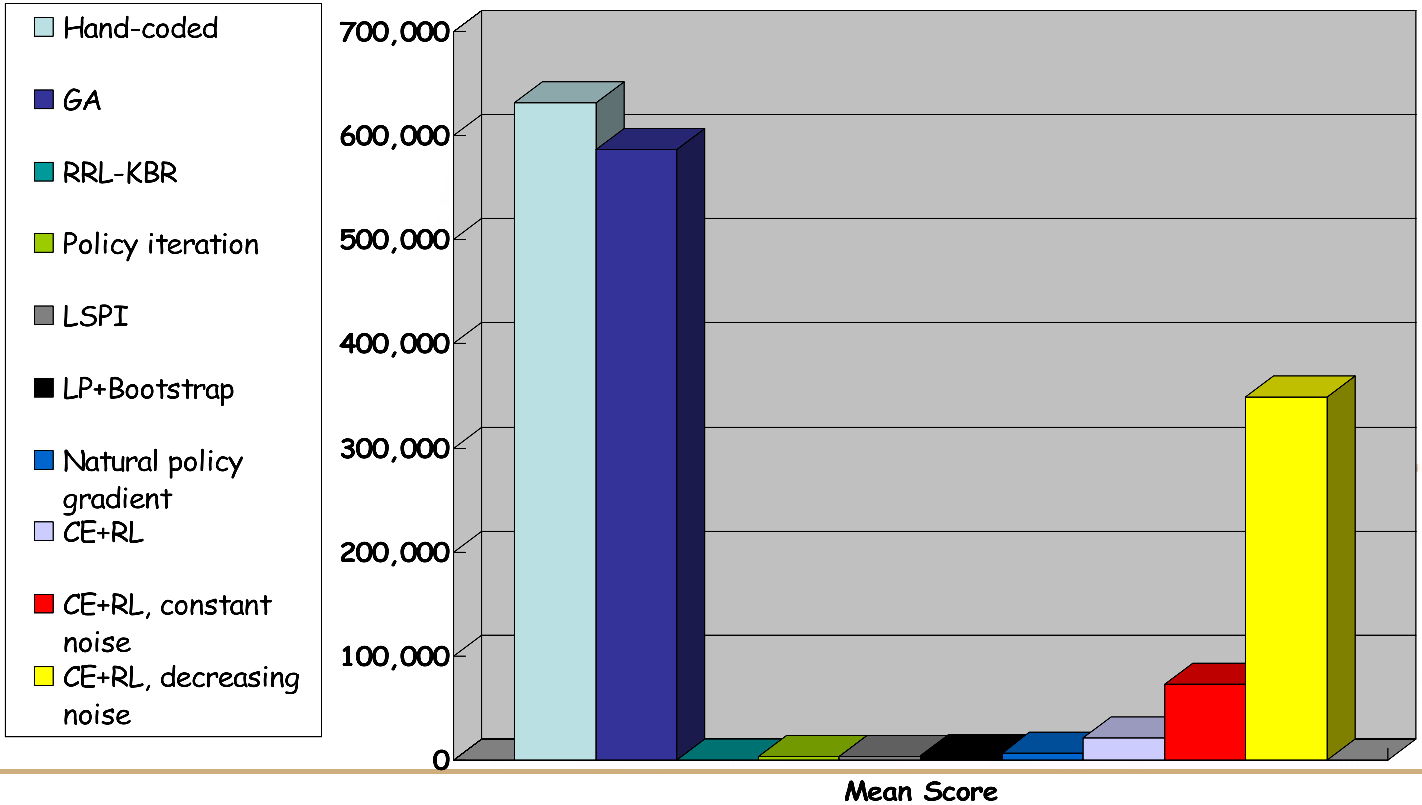
- Play the "offline" version of Tetris, where the initial board and piece sequence are known, is NP-hard. [Demaine et al., 2003]



- Artificial Tetris player [Ramon and Driessens, 2004]
  - 500,000 lines when they only include information about the falling block.
  - 5,000,000 lines when the next block is considered.

Known algorithms

- Average scores of various algorithms [Szita and Lorincz, 2006]
  - Non-reinforcement learning algorithms
  - Reinforcement learning algorithms

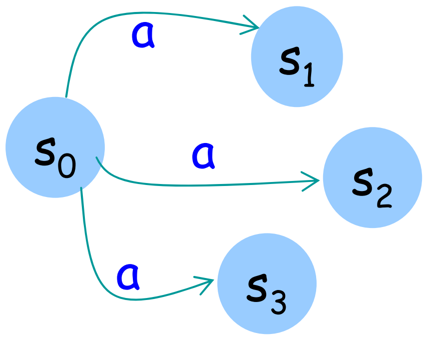


Abstract of Tetris

- State space (S):
  - $2^{200}*7*4*10(7) > 10^{60}$
- Action (A):
  - Drop, turn, right, left
- Goal:
  - Maximize the expected rewards (scores).

Sequence decision problem.

Modeling Tetris



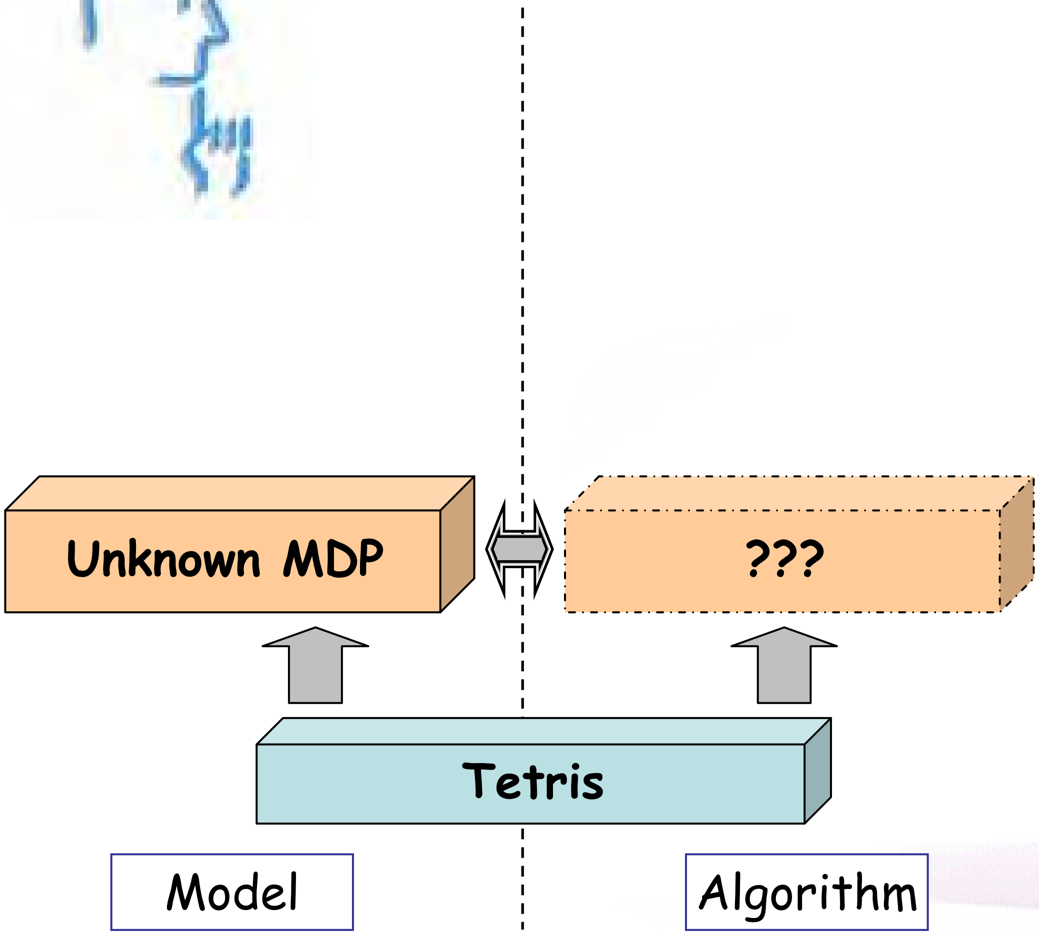
- Markov Decision Process (MDP)
  - A set of States: S
  - A set of Actions: A
  - Reward function: $r: S \times A \rightarrow \Re$

  and

  - State transition function: The next block's shape is undetermined.

  $$P: S \times A \rightarrow S$$

- However, the model of Tetris is unknown in advance. Planning (or optimizing) is infeasible in Tetris.


Unknown MDP
???
Tetris
Model
Algorithm

Learn model or learn optimal policy?

- Learn model
  - By Monte Carlo sampling, can learn (or estimate) the model.
  - Given the estimated model, use planning technology to obtain the optimal policy.
- Learn optimal policy
  - By trial-and-error, get some experiences (or samples) $\langle s, a, s', r \rangle$
  - Learn the optimal policy from experiences directly.

Key question: how to predict the long term rewards

- Return function

discounted - parameter $\gamma<1$. $return = \sum_{i=0}^{\infty} \gamma^i r(s_i, a_i)$

undiscounted or average reward

$$return = \lim_{N \to \infty} \frac{1}{N} \sum_{i=0}^{N-1} r(s_i, a_i)$$

- Bellman equation
  - Using iterative method to compute the return (value) function

Bellman equation given the determined policy Π

The basic idea (in one episode):

$$\begin{aligned} R_t &= r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \gamma^3 r_{t+4} \cdots \\ &= r_{t+1} + \gamma \left(r_{t+2} + \gamma r_{t+3} + \gamma^2 r_{t+4} \cdots \right) \\ &= r_{t+1} + \gamma R_{t+1} \end{aligned}$$

So, in many episodes :

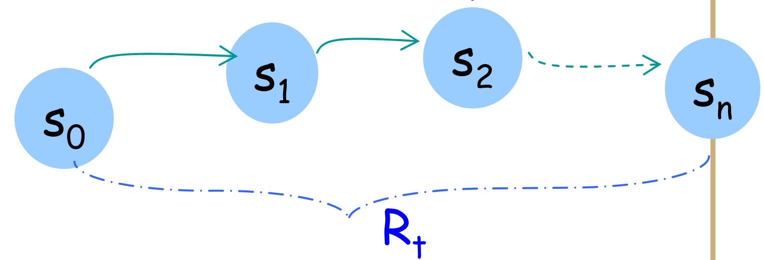
$$\boxed{\begin{aligned} V^{\pi}(s) &= E_{\pi}\left\{ R_t \,\middle|\, s_t = s \right\} \\ &= E_{\pi}\left\{ r_{t+1} + \gamma V\left(s_{t+1}\right) \,\middle|\, s_t = s \right\} \end{aligned}}$$

Or, without the expectation operator:

$$V^{\pi}(s) = \sum_{s'} P_{ss'}^{a} \left[R_{ss'}^{a} + \gamma V^{\pi}(s') \right]$$

is unknown

Temporal-Difference learning



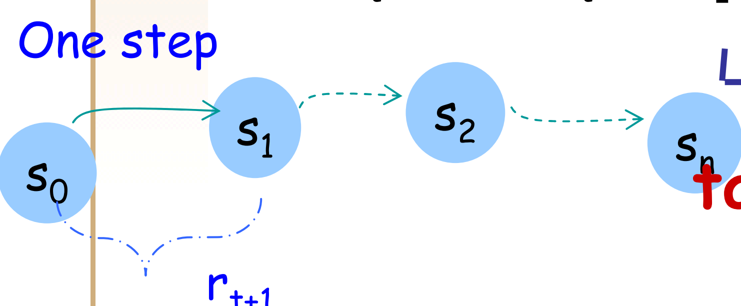
Simple Monte Carlo method:

$$V(s_t) \leftarrow V(s_t) + \alpha \left[R_t - V(s_t) \right]$$

target: the actual return after time *t*.

The simplest TD method, TD(0) :

$$V(s_t) \leftarrow V(s_t) + \alpha \left[\underbrace{r_{t+1} + \gamma V(s_{t+1})}_{\text{target}} - V(s_t) \right]$$



target: an estimate of the return.

The detail materials can be found in the talk of MLA'04.

Q-learning

- For each $s \xrightarrow{a}$, calculate/predict the Q values.

$$Q(s,a) \leftarrow Q(s,a) + \alpha\left[r_{s,s'}^{a} + \gamma \max_{a'} Q(s',a') - Q(s,a)\right]$$

- The optimal policy:

$$\Pi^{*}(s) \leftarrow \arg\max_{a} Q(s,a)$$

Average reward reinforcement learning algorithm

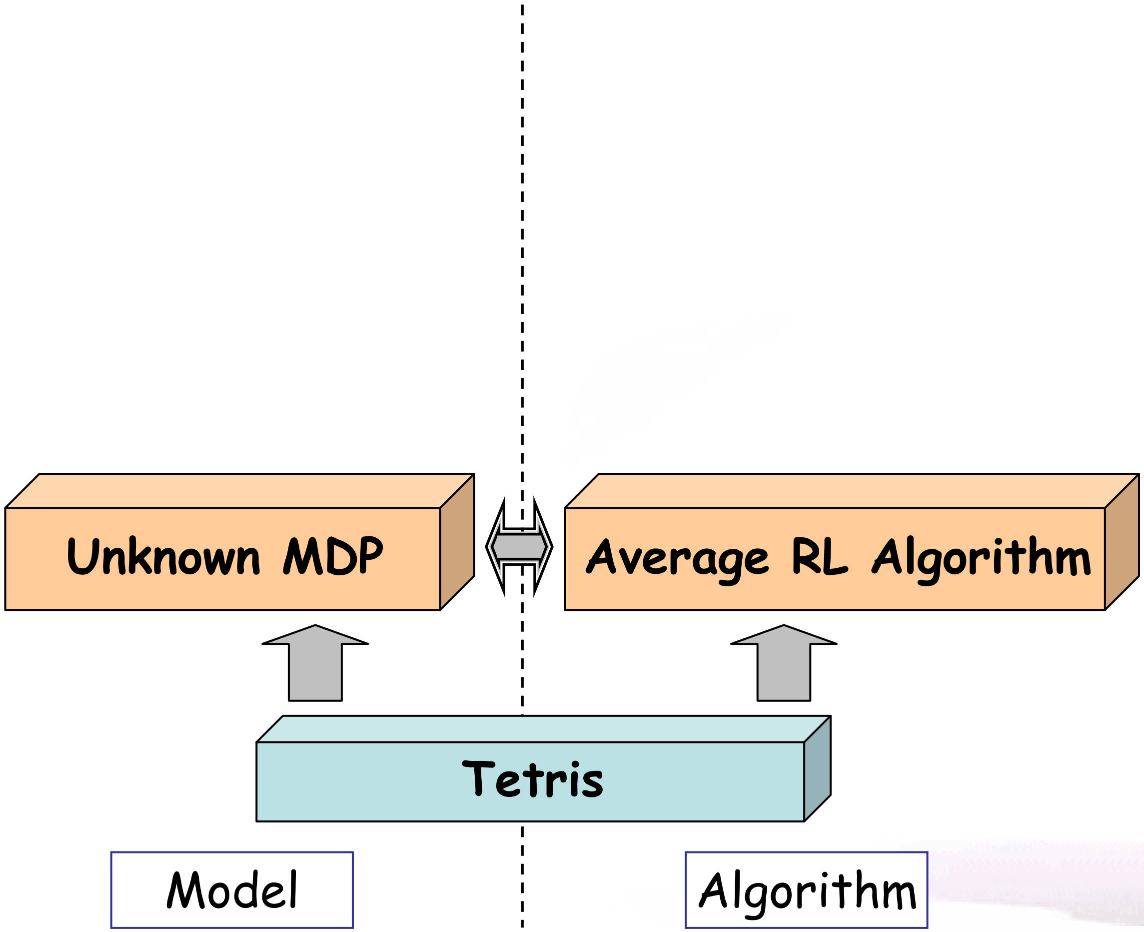
- Average reward G-learning algorithm

$$G(s,a) \leftarrow G(s,a) + \alpha\left[r_{s,s'}^{a} - g(s_0) + \max_{a'} G(s',a') - G(s,a)\right]$$

$$if \quad s = s_0, g(s_0) \leftarrow \max_{a} G(s_0, a)$$

reference state

The detail materials can be found in the talk of MLA'06.


Unknown MDP
Average RL Algorithm
Tetris
Model
Algorithm

How to speed up the learning process

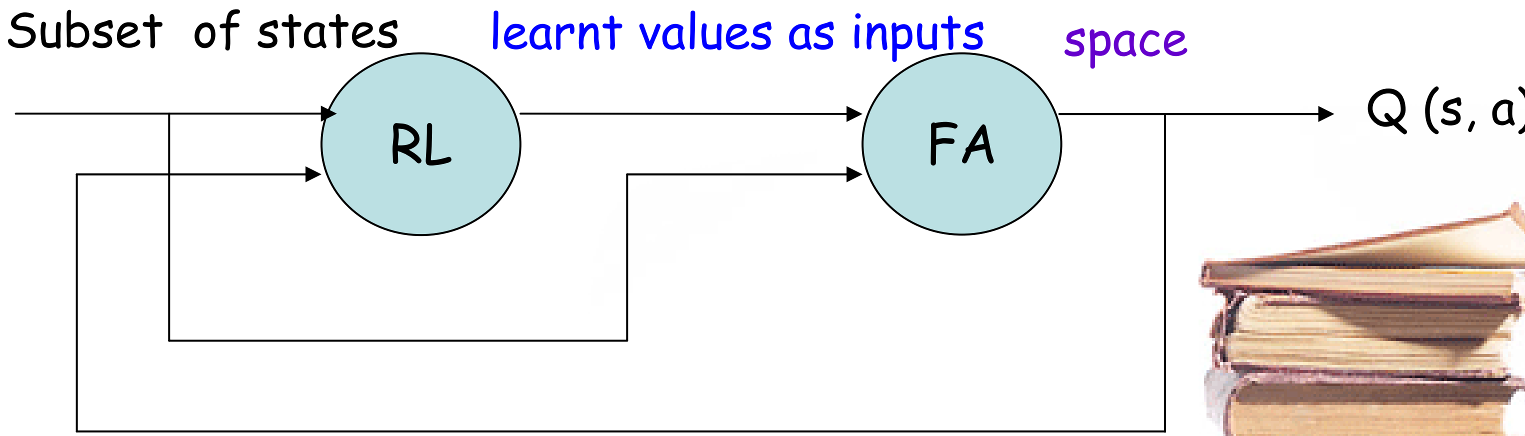
- Problem: large state space
  - State space: 10*20 grids, 7 shapes and 10 locations.
  - Action space: 4 actions.
- Solution: in similar state-action pairs, the Q-value may be similar.
- Technical points: using function approximation to general the Q-values.

Question: after learn a Q(s,a), when will visit the state 's' again?

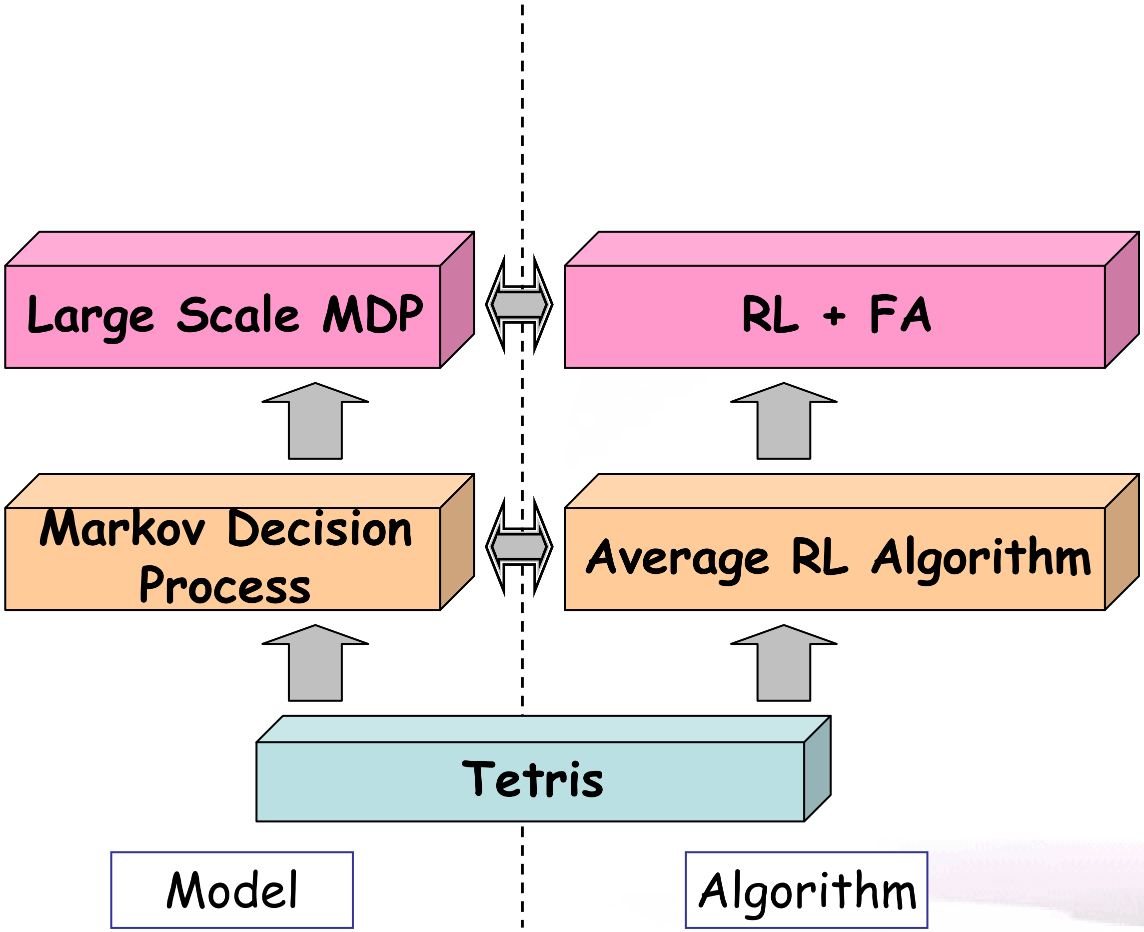
RL + function approximation

- Neural network et al.

Generalization of the value function to the entire state space



$$Q_0 \to M(Q_0) \to \Gamma\big(M(Q_0)\big) \to M\Big(\Gamma\big(M(Q_0)\big)\Big)$$
$$\to \Gamma\Big(M\Big(\Gamma\big(M(Q_0)\big)\Big)\Big) \to \cdots$$


Large Scale MDP
RL + FA
Markov Decision Process
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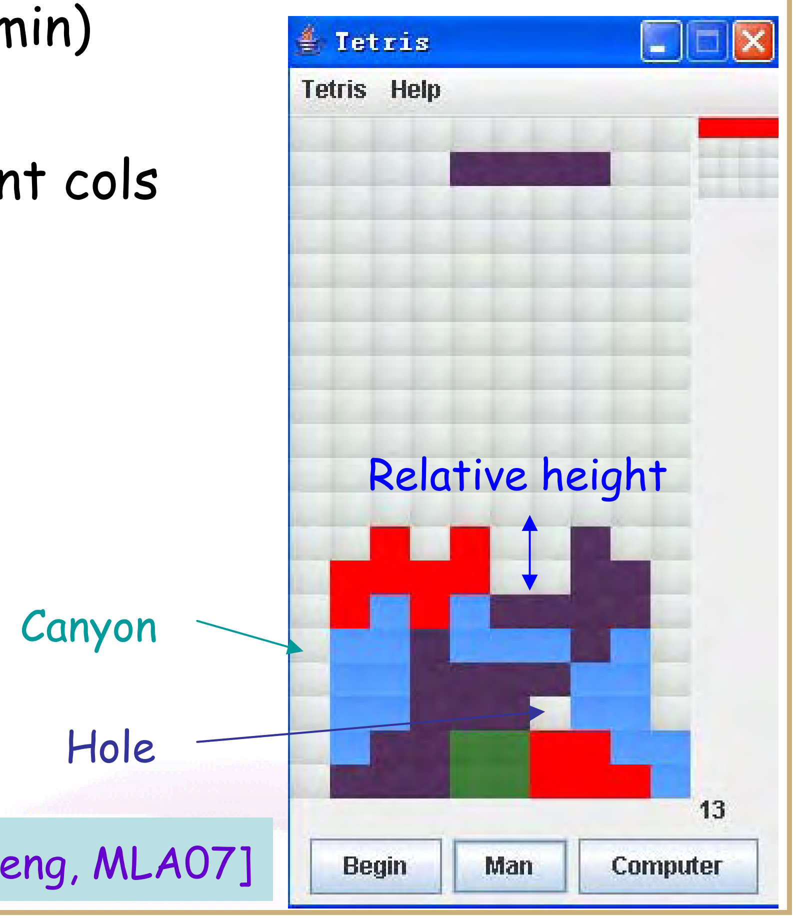
Features of states & actions

- Relative features
  - Height of wall (max, avg, min)
  - Number of Holes
  - Height difference adjacent cols
  - Canyon (width, height)
  - ...

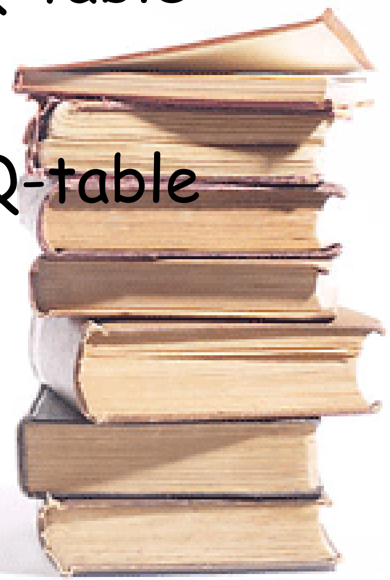
- Macro actions
  - Fits
  - Increasesheight, ...
  - Number of deleted lines



Good features beat good learning! [Feng, MLA07]

Some discussions and thinking…

- Classical RL
  - Use look-up table
- RL + FA
  - Use function to generalize the Q-table
- Relative features
  - Use features to generalize the Q-table



Is it enough?

Relational domain

- Challenges [Tadepalli et al., 2004]
  - Function approximation
  - Prior Knowledge
  - Generalization across objects
  - Transfer learning across tasks
  - Run-time planning and reasoning

Relational reinforcement learning

- RRL
  - Reinforcement learning + relational representation
- Relational representation
  - Represents value function as a first order logic regression tree
- Algorithms
  - TG algorithm [Driessens et al, 2001]
  - RIB (instance based algorithm) [Driessens and Ramon, 2003]
  - KBR (kernel based algorithm) [Gartner et al, 2003]

Decision Tree

- Each internal node of a decision tree contains a test.
- Decision trees partition the whole example space and assign class values to each example.
- Make prediction
  - Starts in the root of the tree
  - Applies a test to the example
  - Propagates the example to the corresponding subtree
  - Leaf is the prediction

First order logical decision tree

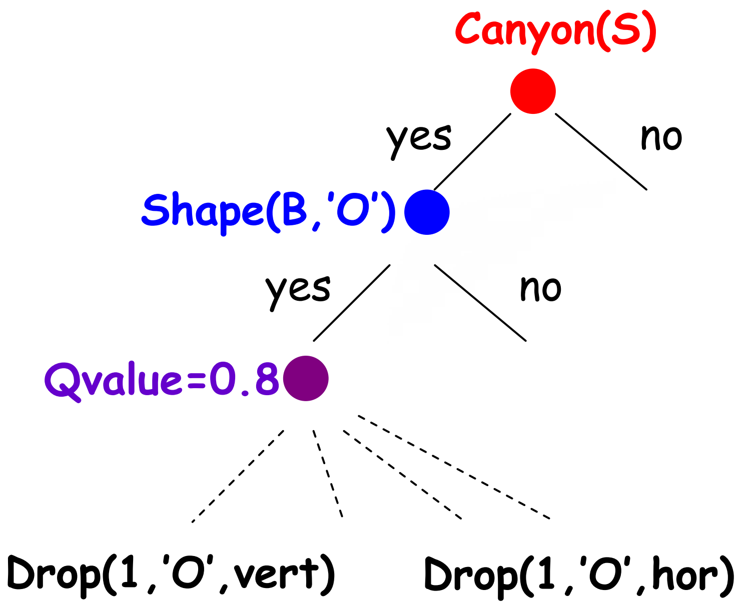
- Differences between LDT and DT
  - Example: a relational database
  - Test: query
- Example 1 : (s, a, Q)
  - Qvalue(1)
  - WidCanyon(b, 1)
  - HeightCanyon(b, 4)
  - Hole(3,2)
  - NumHoles(1)
  - Hight(3,3)
  - Shape(a, 'O')
  - Drop(1,'O',vert)

Q-value

state

action

For example: a LDT



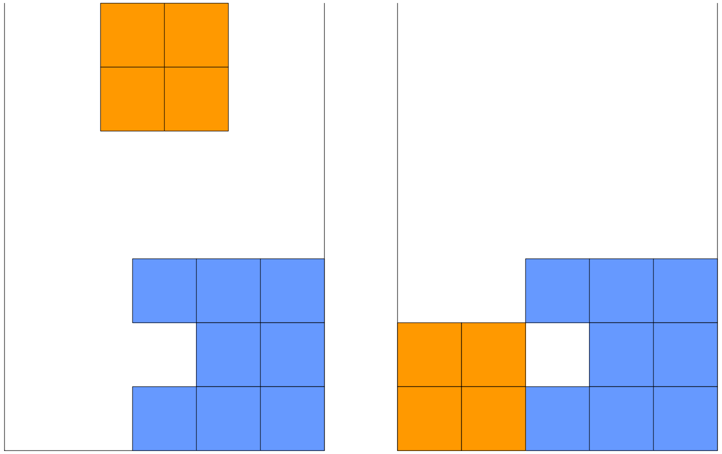
Relational RL algorithm

- RRL algorithm [Driessens et al, 2001]
  - 0. Represent the state and action with relational method, initialize the *Q*-values
  - 1. Run the first episode
    - Choose the action randomly
  - 2. Obtain examples *(s,a,Q)*
  - 3. Use TG algorithm to expand tree
  - 4. Run next episode
    - Choose the action according to the tree
    - Update the Q-value
  - 5. Return step 2

TG algorithm (1)

- Build first order logical tree
  - Create an empty leaf
  - While (examples available)
    - Sort example down to leaf
    - Update statistics in leaf
    - If (split needed)
      - Create two empty leafs
- The heuristical rule is same as in C4.5.

Example 1



State:

WidCanyon(1,2),---column 1, width 2

HeightCanyon(1,3),---column 1, height 3

Hole(3,2),

NumHoles(1),

Height(3,3),

Height(4,3),

Height(5,3),

Action:

Drop(1,'O',Vertical)---Put Shape 'O' on column 1 with direction Vertical

Qvalue:

Qvalue(1)---1 line is cleared

Example 2

State:

WidCanyon(1,2),---column 1, width 2

HeightCanyon(1,3),---column 1, height 3

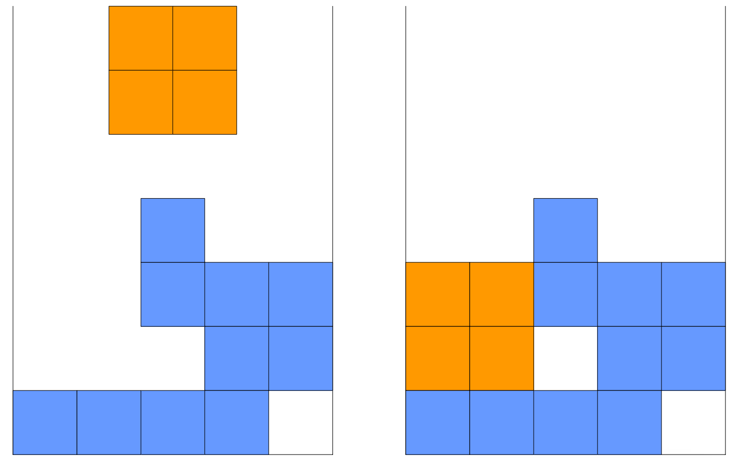
Hole(3,2), Hole(5,1)

NumHoles(2),

Height(1,1), Height(2,1),

Height(3,4), Height(4,3),

Height(5,3),



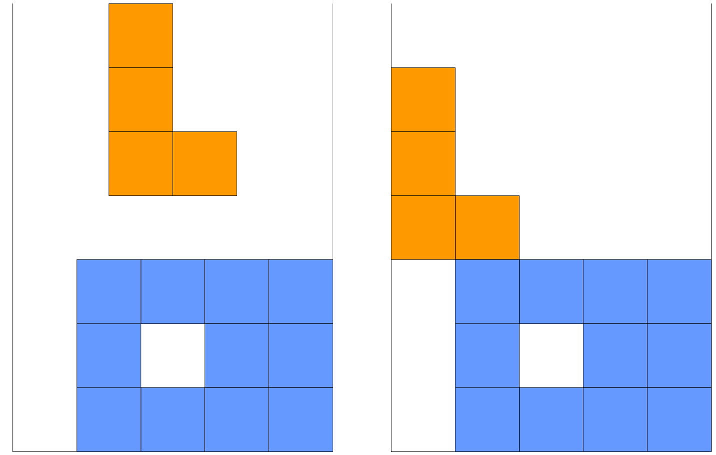
Action:

Drop(1,'O',Vertical)---Put Shape 'O' on column 1 with direction Vertical

Qvalue :

Qvalue(1)---1 line is cleared

Example 3



State:

WidCanyon(1,1),---column 1, width 1

HeightCanyon(1,3),---column 1, height 3

Hole(3,2),

NumHoles(1),

Height(2,3), Height(3,3),

Height(4,3), Height(5,3),

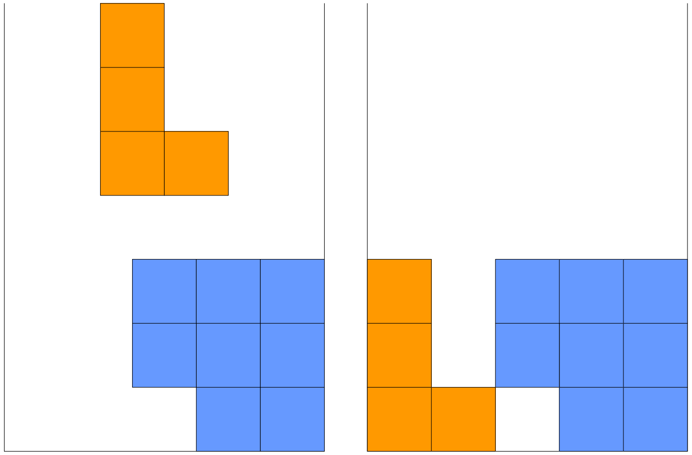
Action:

Drop(1,'L',Vertical)---Put Shape 'L' on column 1 with direction Vertical

Qvalue :

Qvalue(0)---No line is cleared

Example 4



State:

WidCanyon(1,2),---column 1, width 2

HeightCanyon(1,3),---column 1, height 4

Hole(3,1),

NumHoles(1),

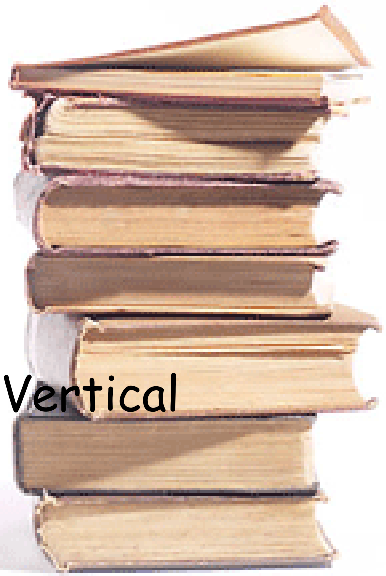
Height(3,3),

Height(4,3),

Height(5,3),

Action:

Drop(1,L,Vert)---Put Shape 'L' on column 1 with direction Vertical

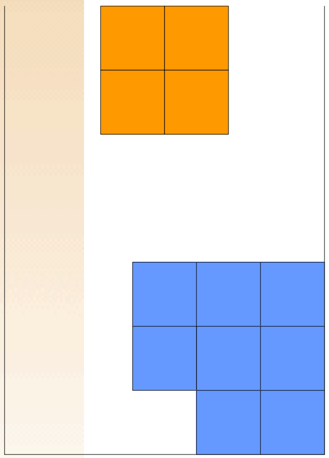


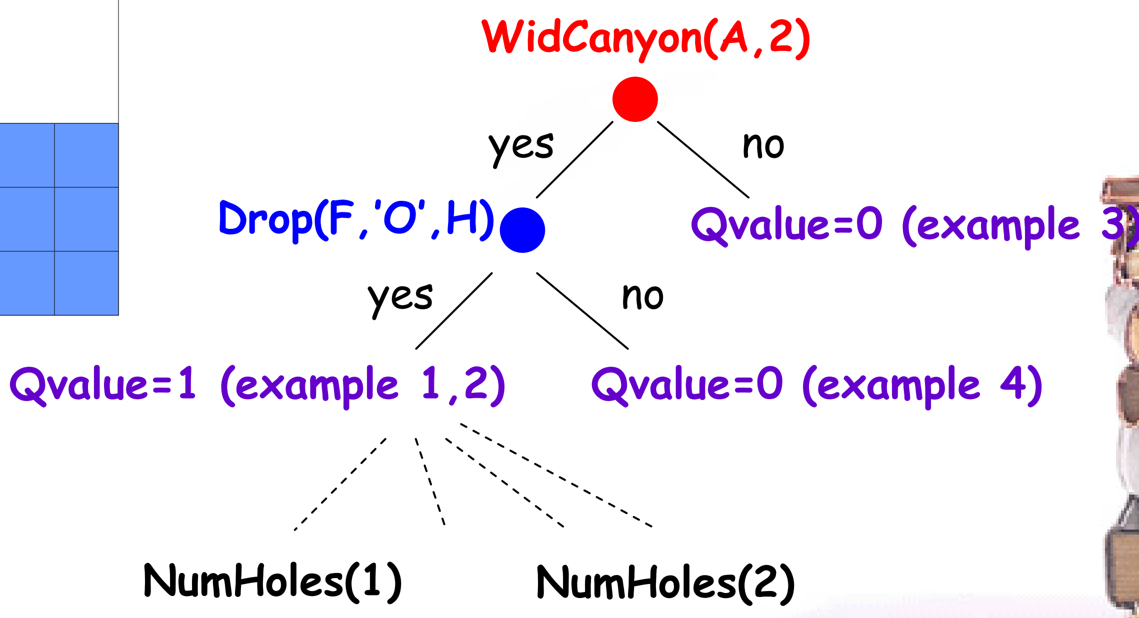
Qvalue :

Qvalue(0)---No line is cleared

How to build first order logical tree?

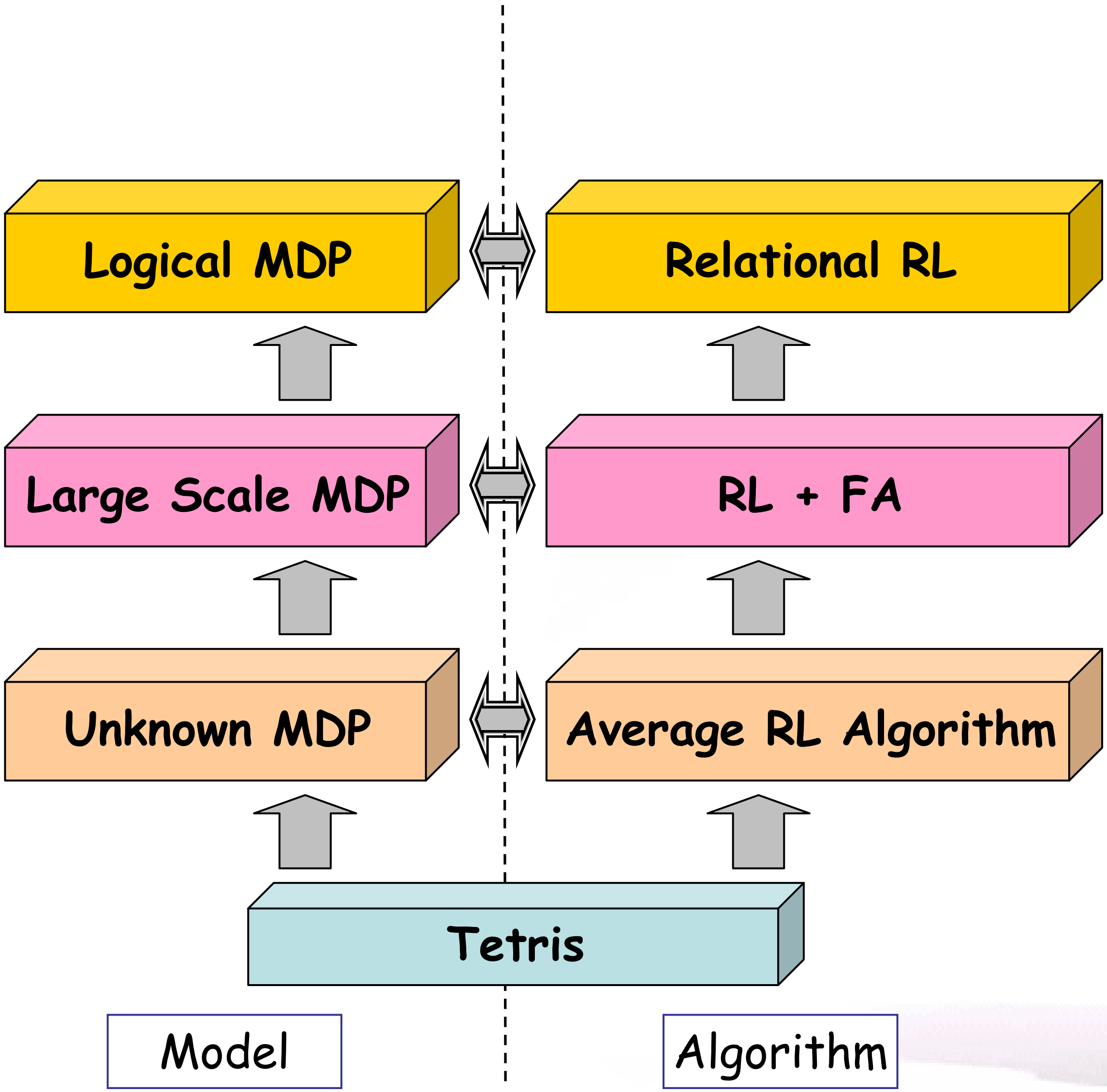
WidCanyon(A,B),HeightCanyon(C,D),NumHoles(E),Drop(F,G,H)





TG algorithm (2)

- TG algorithm
  - When stop to split the leaf node?
- Tree updated algorithm
  - New examples, update the tree incrementally
- Test to choose an action
  - Test all possible actions, combine any possible example, according to the tree to get their *Q*-values.


Logical MDP
Relational RL
Large Scale MDP
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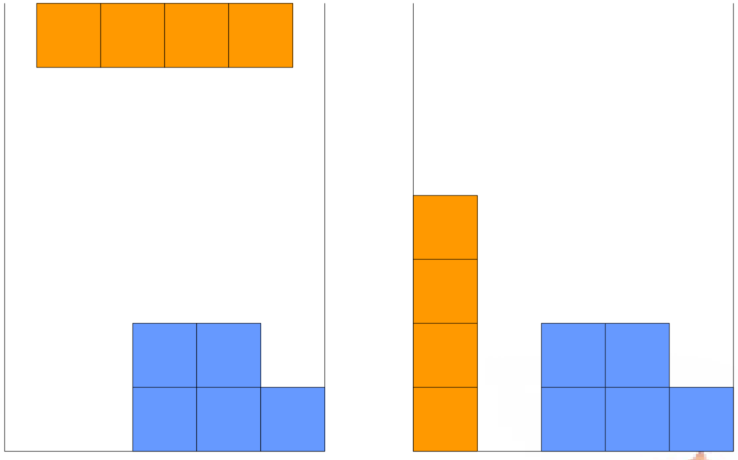
Prior Knowledge

- Formula 1
  - ‘If exist a canyon whose width is 2 and the shape of dropping block is I, put the block in the canyon, then the canyon’s width is 1.’



Markov logic networks

- What is MLN?
  - First order logic
    - Constants, variables, functions, predicates, formulas
  - Markov network

Explain

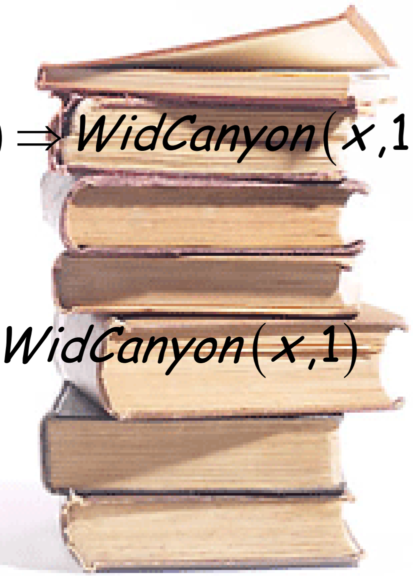
- Prior knowledge
  - 'If exist a canyon whose width is 2 and the shape of dropping block is I, put the block in the canyon, then the canyon's width is 1.'
- First-order logic

$$\exists x,\exists y \quad WidCanyon(x,2) \wedge BolckShape(y,O) \wedge Drop(y,x) \Rightarrow WidCanyon(x,1)$$

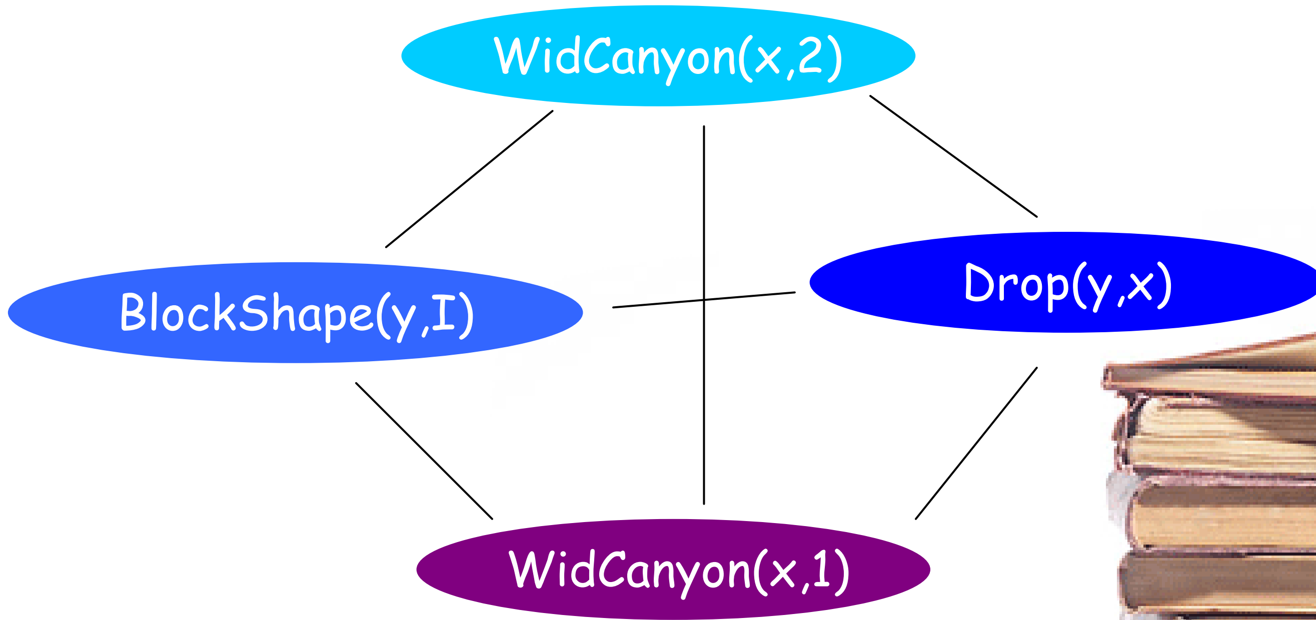
- Clausal form

$$\neg WidCanyon(x,2) \vee \neg BolckShape(y,I) \vee \neg Drop(y,x) \vee WidCanyon(x,1)$$

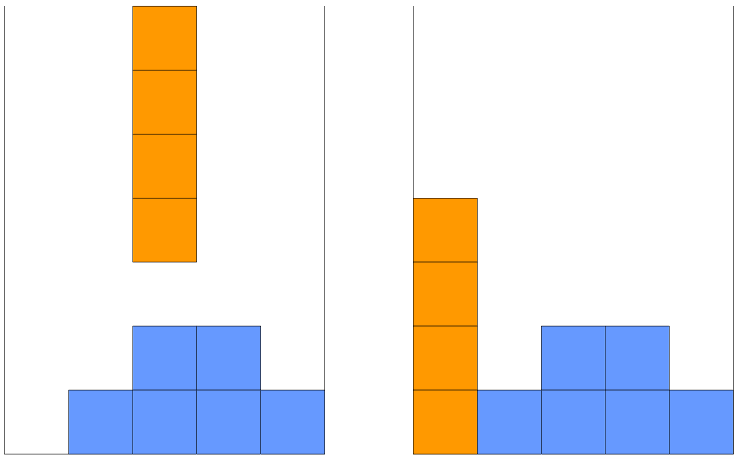
- Weight
  - 0.8



Markov logic network



Example



State:

WidCanyon(1,1),---column 1, width 1

HeightCanyon(1,2),---column 1, height 2

Height(2,1),

Height(3,2),

Height(4,2),

Height(5,1)

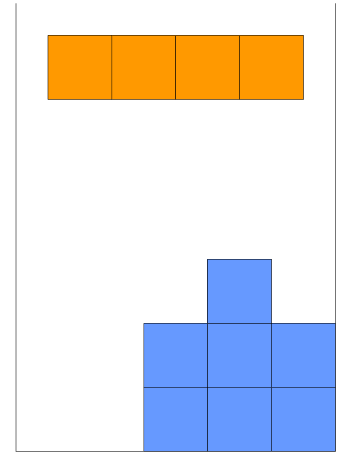
Action:

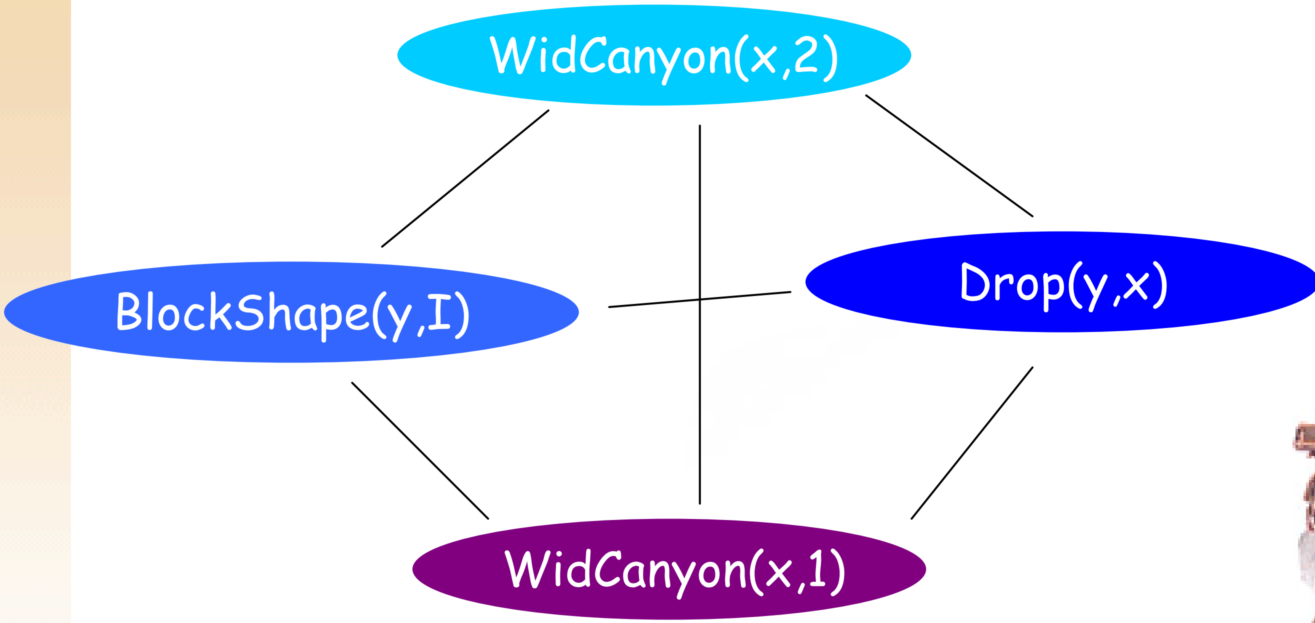
Drop(1,'I',Vertical)---Put Shape 'O' on column 1 with direction Vertical

Qvalue:

Qvalue(1)---1 line is cleared

How to predict?

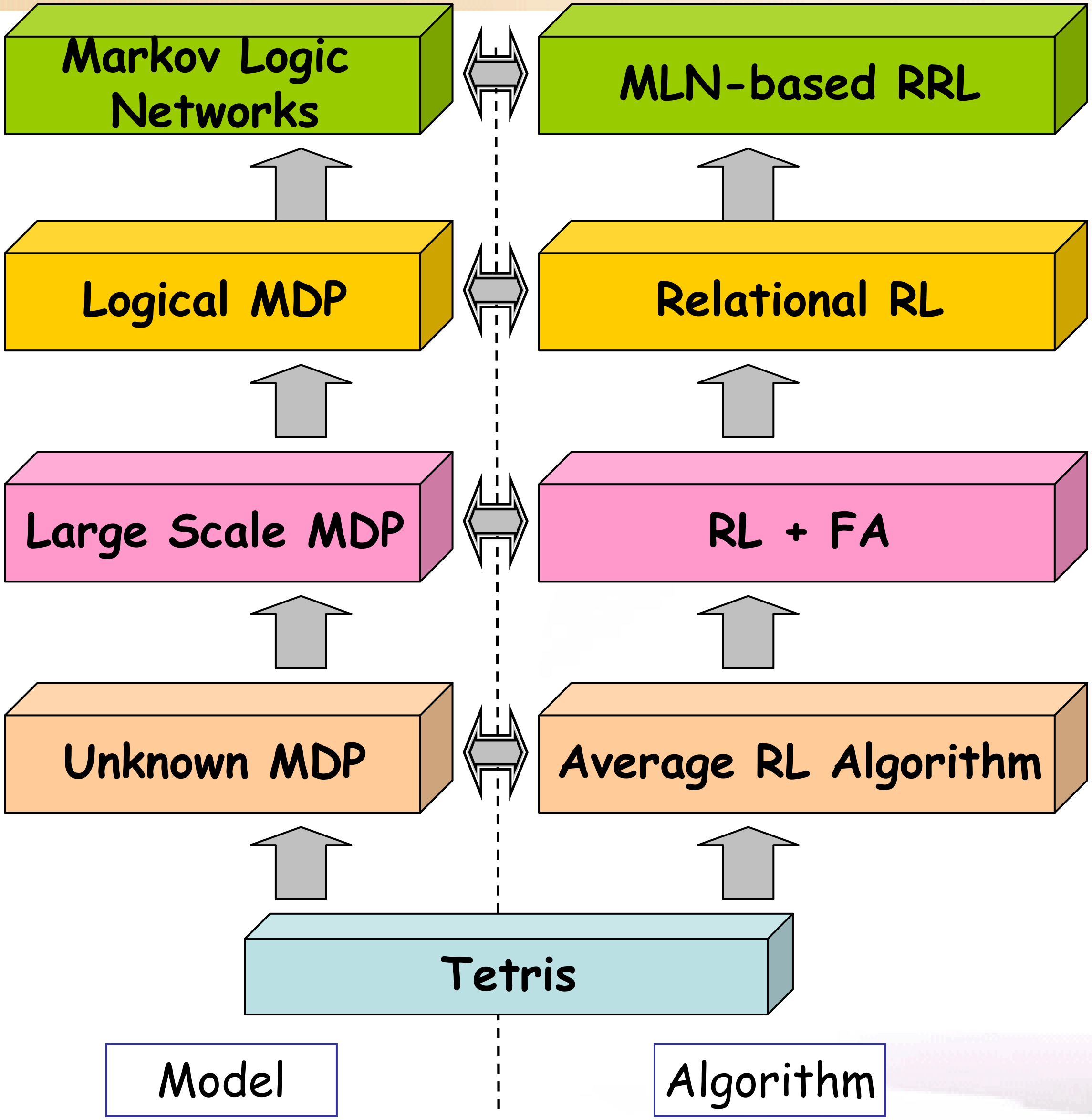




Compute the probability of WidCanyon(x,1) using MLN

Then compute the Q-value by LDT

Choose the predict (action) to maximize the Q-value


Markov Logic Networks
MLN-based RRL
Logical MDP
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Conclusion

- Traditional reinforcement learning
  - Too large state space, to re-visit it.
- RL + FA
  - Propagate the Q values to similar states.
- Features
  - Similar states have same features.
- Relational RL
  - Compute which feature is most important.
- Markov logic network

Doing the task is not difficult, Describing the task is difficult.

Thanks …

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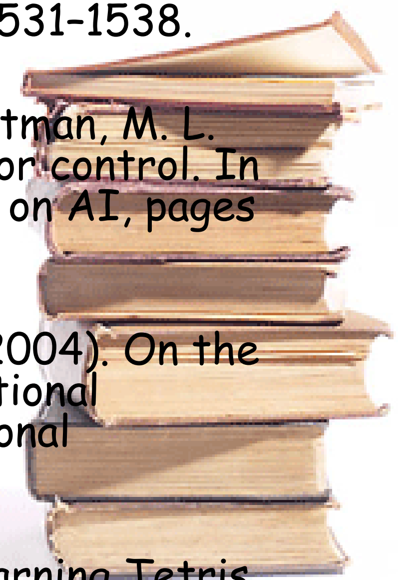

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