Closed-Form Solutions in Low-Rank Subspace Recovery Models and Their Implications

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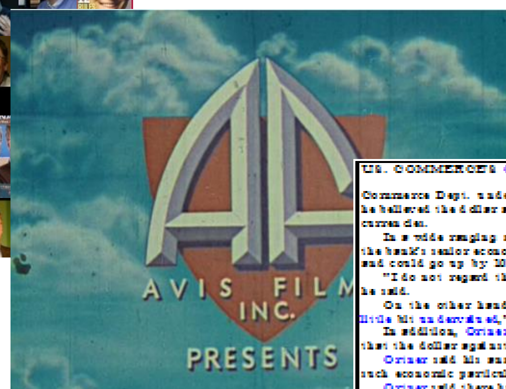
Outline

- Sparsity vs. Low-rankness
- Closed-Form Solutions of Low-rank Models
- Applications of Closed-Form Solutions
- Conclusions

Challenges of High-dim Data

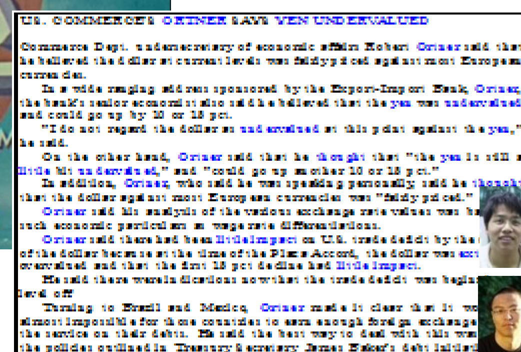
Images

> 1M dim



Videos

> 1B dim



U.S. COMMERCE'S ORTNER SAYS YEN UNDERVALUED

Commerce Dept. undersecretary of economic affairs Robert Ortner said that he believed the dollar at current levels was fairly priced against most European currencies.

In a wide ranging address sponsored by the Export-Import Bank, Ortner, the bank's senior economist also said he believed that the yen was undervalued and could go up by 10 or 15 pct.

"I do not regard the dollar as undervalued at this point against the yen," he said.

On the other hand, Ortner said that he thought that "the yen is still a little bit undervalued," and "could go up another 10 or 15 pct."

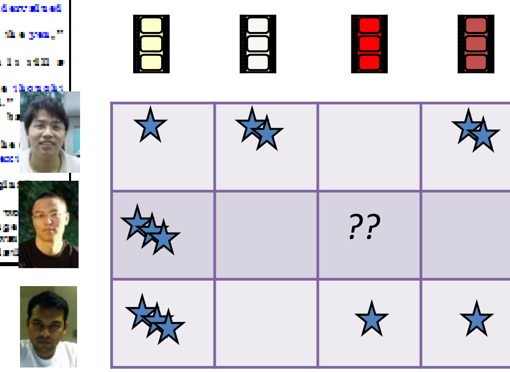
In addition, Ortner, who said he was speaking personally, said he thought that the dollar against most European currencies was "fairly priced."

Ortner said his analysis of the various exchange rate values was ba... such economic particulars as wage rate differentiations.

Ortner said there had been little impact on U.S. trade deficit by the ... of the dollar because at the time of the Plaza Accord, the dollar was ex... overvalued and that the first 15 pct decline had little impact.

He said there were indications now that the trade deficit was begin... level off.

Turning to Brazil and Mexico, Ortner made it clear that it wa... almost impossible for those countries to earn enough foreign exchange ... the service on their debts. He said the best way to deal with this wa... the policies outlined in Treasury Secretary James Baker's debt initiat...

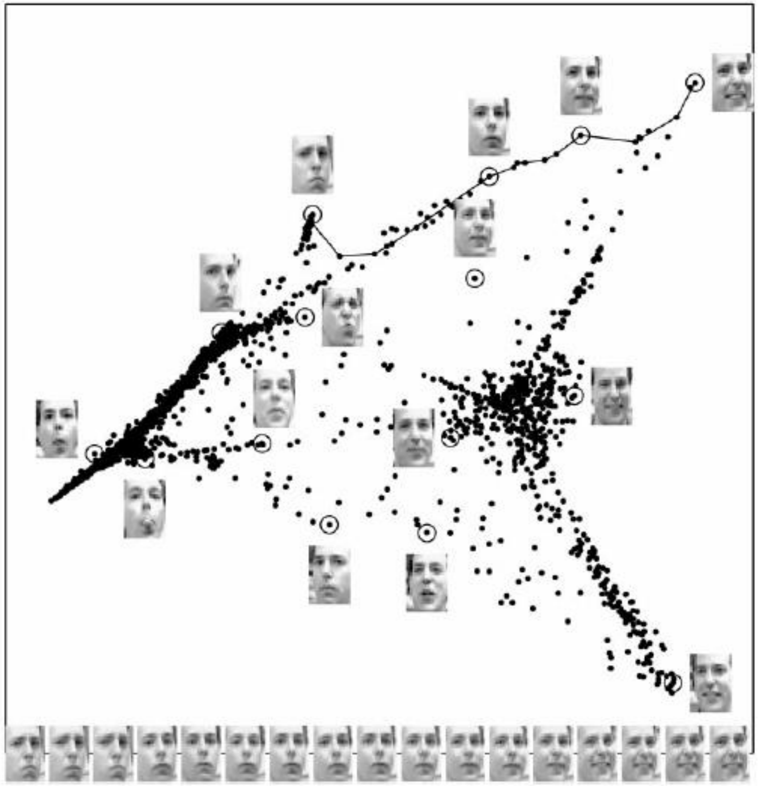


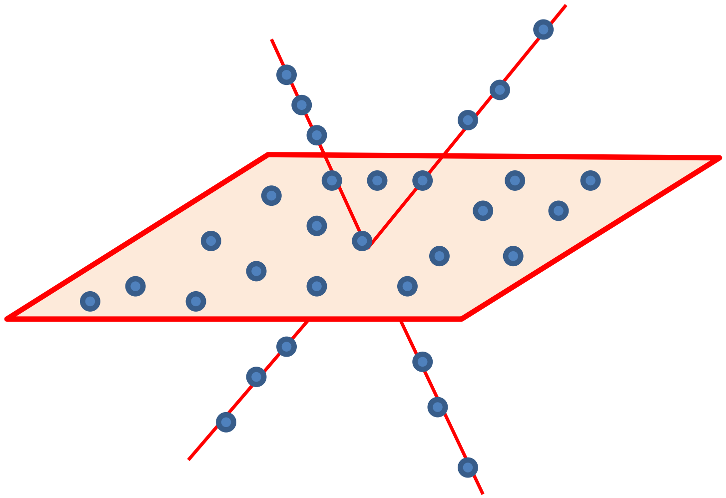
Web data

> 10B+ dim?

Courtesy of Y. Ma.

Sparsity vs. Low-rankness



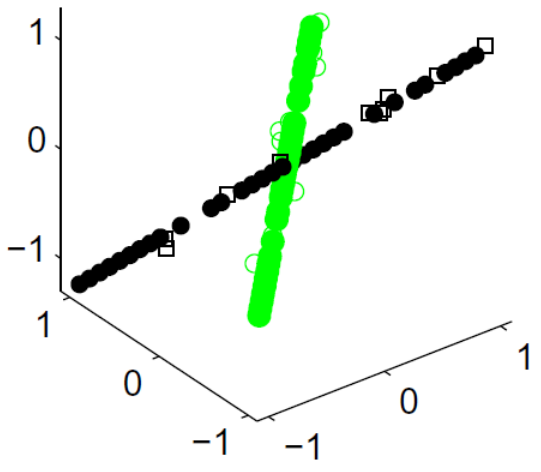


Sparse Models

- Sparse Representation

$$\min ||z||_0, \quad s.t.\ x = Az. \tag{1}$$

- Sparse Subspace Clustering



$$\min ||z_i||_0, \quad s.t.\ x_i = X_{\hat{i}} z_i, \quad \forall i. \tag{2}$$

where $X_{\hat{i}} = [x_1, \cdots, x_{i-1}, x_{i+1}, \cdots, x_n]$.

$$\min ||Z||_0, \quad s.t.\ X = XZ, diag(Z) = 0. \tag{3}$$

$$\min ||Z||_1, \quad s.t.\ X = XZ, diag(Z) = 0. \tag{4}$$

Elhamifar and Vidal. *Sparse Subspace Clustering*. CVPR2009.

Low-rank Models

- Matrix Completion (MC)

$$\min \operatorname{rank}(A), \quad s.t. \quad D = \pi \ (A).$$

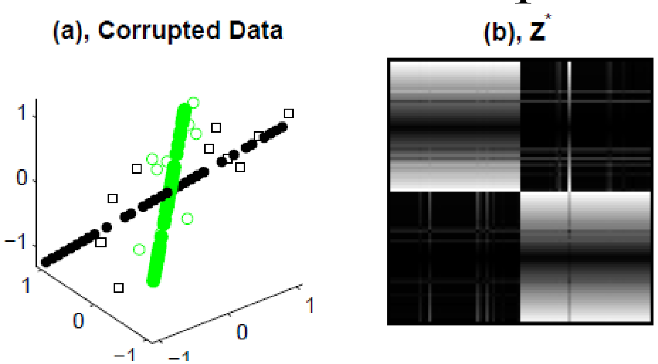
Filling in missing entries

- Robust PCA

$$\min \operatorname{rank}(A) + \lambda \|E\|_{l_0}, \quad s.t. \quad D = A + E.$$

Denoising

- Low-rank Representation (LRR)



$$\min \operatorname{rank}(Z) + \lambda ||E||_{2,0},$$
$$s.t. \ X = XZ + E.$$

Clustering

$$\|E\|_{l_0} = \#\{E_{ij} | E_{ij} \neq 0\} \qquad ||E||_{2,0} = \#\{i | ||E_{:,i}||_2 \neq 0\}.$$

NP Hard!

Convex Program Formulation

- Matrix Completion (MC)

$$\min \|A\|_*, \quad s.t. \quad D = \pi \ (A).$$

- Robust PCA

$$\min \|A\|_* + \lambda \|E\|_{l_1}, \quad s.t. \quad D = A + E.$$

- Low-rank Representation (LRR)

$$\min ||Z||_* + \lambda ||E||_{2,1},$$
$$s.t. \ X = XZ + E.$$

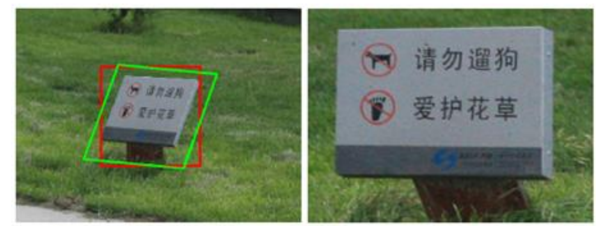
$$\|A\|_* = \sum_i \sigma_i(A),$$
$$\|E\|_{l_1} = \sum_{i,j} |E_{ij}|.$$
$$||E||_{2,1} = \sum_i ||E_{:,i}||_2.$$

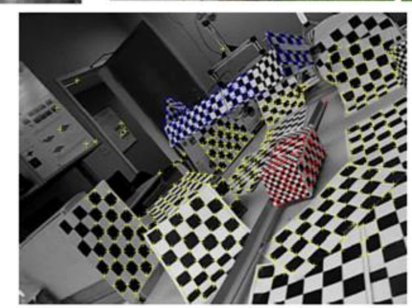
nuclear norm

Applications of Low-rank Models

- Background modeling
- Robust Alignment
- Image Rectification
- Motion Segmentation
- Image Segmentation
- Saliency Detection
- Image Tag Refinement
- Partial Duplicate Image Search
- ……








林宙辰、马毅，*信号与数据处理中的低秩模型*，中国计算机学会通讯，2015年第4期。

Closed-form Solution of LRR

- Closed form solution at noiseless case

$$\min_Z \|Z\|_*, \quad s.t. \quad X = XZ,$$

has a *unique closed-form* optimal solution: $Z^* = V_r V_r^T$, where $U_r \Sigma_r V_r^T$ is the skinny SVD of X.

  - Shape Interaction Matrix
  - when X is sampled from independent subspaces, Z^* is block diagonal, each block corresponding to a subspace

$$\min_{X=XZ} \|Z\|_* = \operatorname{rank}(X).$$

Wei and Lin. *Analysis and Improvement of Low Rank Representation for Subspace segmentation*, arXiv: 1107.1561, 2010.
Liu et al. *Robust Recovery of Subspace Structures by Low-Rank Representation*, TPAMI 2013.

Closed-form Solution of LRR

- Closed form solution at general case

$$\min_{Z} \|Z\|_*, \quad s.t. \quad X = AZ,$$

has a *unique closed-form* optimal solution: $Z^* = A^\dagger X$.

Valid for any unitary invariant norm!

$$\|X\|_{UI} = \|UXV^T\|_{UI}, \quad \forall\, U^TU = I,\ V^TV = I.$$

Liu et al., *Robust Recovery of Subspace Structures by Low-Rank Representation*, TPAMI 2013.
Yao-Liang Yu and Dale Schuurmans, *Rank/Norm Regularization with Closed-Form Solutions: Application to Subspace Clustering*, UAI2011.

Closed-form Solution of LRR

- Closed form solution of the original LRR

$$\min_{Z} \operatorname{rank}(Z), \quad \text{s.t.} \quad A = XZ. \tag{1}$$

Theorem: Suppose $U_X \Sigma_X V_X^T$ and $U_A \Sigma_A V_A^T$ are the skinny SVD of X and A, respectively. The complete solutions to feasible generalized LRR problem (1) are given by

$$Z^* = \boxed{X^\dagger A} + S V_A^T, \tag{2}$$

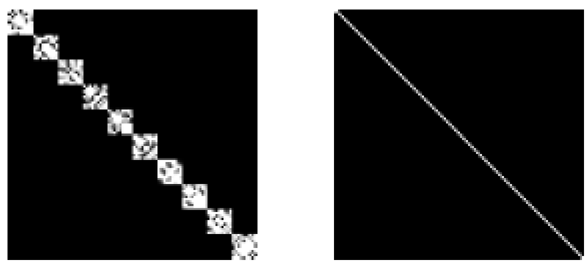
where S is any matrix such that $V_X^T S = 0$.

Hongyang Zhang, Zhouchen Lin, and Chao Zhang, *A Counterexample for the Validity of Using Nuclear Norm as a Convex Surrogate of Rank*, ECML/PKDD2013.

Latent LRR

- Small sample issue

$$\min \|Z\|_*, \\ s.t.\ X = XZ.$$



- Consider unobserved samples

$$\min \|Z\|_*, \\ s.t.\ X_O = [X_O, X_H]Z.$$

Liu and Yan. *Latent Low-Rank Representation for Subspace Segmentation and Feature Extraction*, ICCV 2011.

Latent LRR

$$\min \|Z\|_*,$$
$$s.t.\ X_O = [X_O, X_H]Z.$$

Theorem: Suppose $Z^*_{O,H} = [Z^*_{O|H}; Z^*_{H|O}]$. Then

$$Z^*_{O|H} = V_O V_O^T, \quad Z^*_{H|O} = V_H V_O^T,$$

where V_O and V_H are calculated as follows. Compute the skinny SVD: $[X_O, X_H] = U\Sigma V^T$ and partition V as $V = [V_O; V_H]$.

Proof: That $[X_O, X_H] = U\Sigma[V_O; V_H]^T$ implies

$$X_O = U\Sigma V_O^T, \quad X_H = U\Sigma V_H^T.$$

So $X_O = [X_O, X_H]Z$ reduces to:

$$V_O^T = V^T Z.$$

So $Z^*_{O,H} = VV_O^T = [V_O V_O^T; V_H V_O^T]$.

Liu and Yan. *Latent Low-Rank Representation for Subspace Segmentation and Feature Extraction*, ICCV 2011.

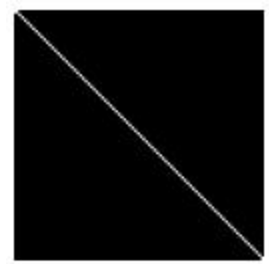
Latent LRR

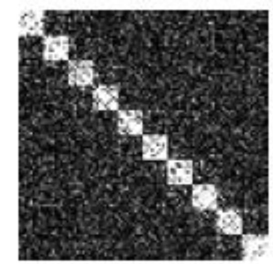
$$\min \|Z\|_*, \quad s.t.\ X_O = [X_O, X_H]Z.$$

$$Z^*_{O|H} = V_O V_O^T, \quad Z^*_{H|O} = V_H V_O^T.$$

$$\begin{aligned} X_O &= [X_O, X_H] Z^*_{O,H} \\ &= X_O Z^*_{O|H} + X_H Z^*_{H|O} \\ &= X_O Z^*_{O|H} + X_H V_H V_O^T \\ &= X_O Z^*_{O|H} + U\Sigma V_H^T V_H V_O^T \\ &= X_O Z^*_{O|H} + U\Sigma V_H^T V_H \Sigma^{-1} U^T X_O \\ &\equiv X_O Z^*_{O|H} + L^*_{H|O} X_O. \end{aligned}$$

low rank!





$$\min \operatorname{rank}(Z) + \operatorname{rank}(L), \quad s.t.\ X = XZ + LX.$$

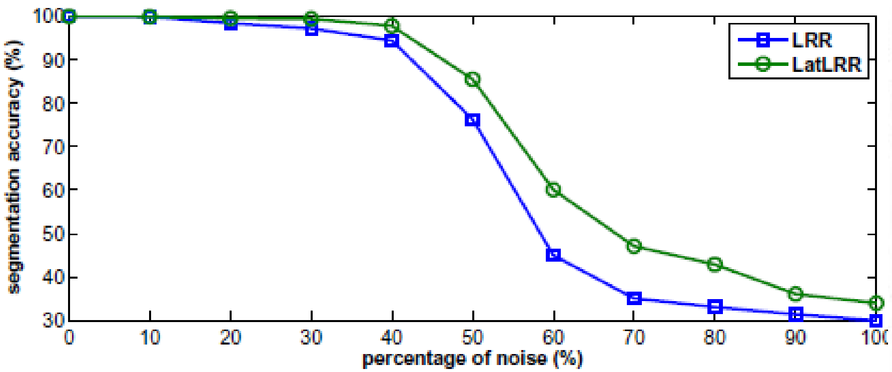
→

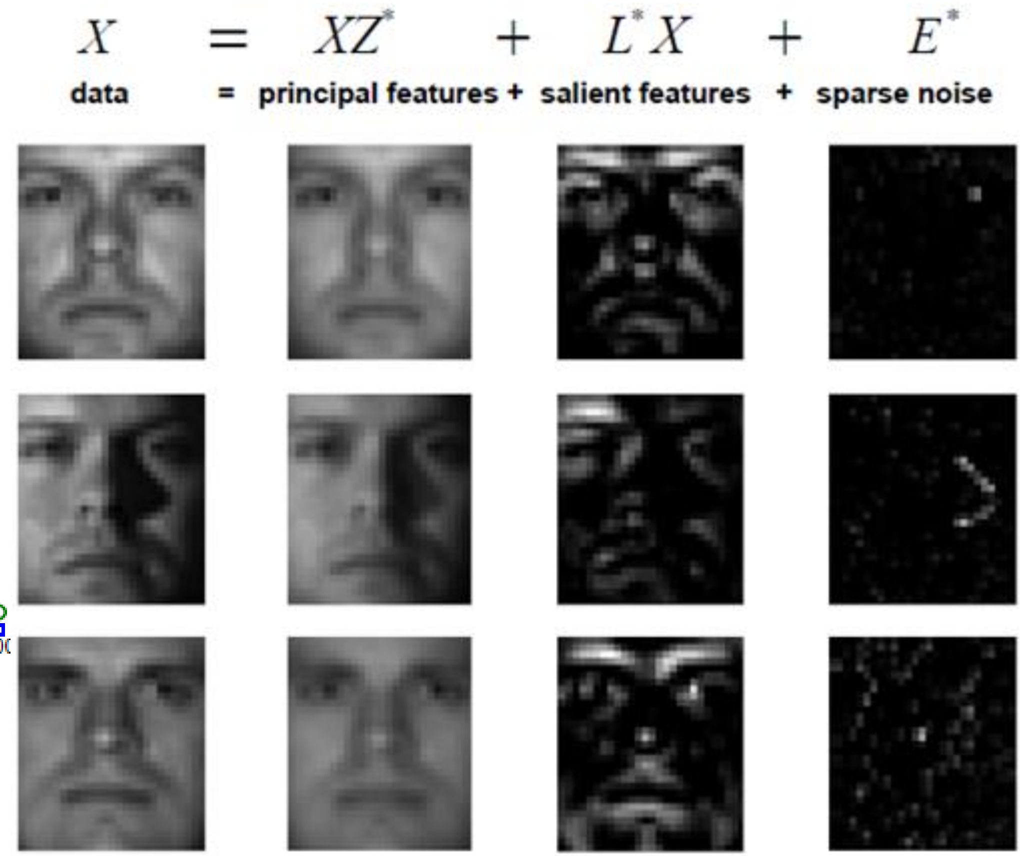
$$\min \|Z\|_* + \|L\|_*, \quad s.t.\ X = XZ + LX.$$

Liu and Yan. *Latent Low-Rank Representation for Subspace Segmentation and Feature Extraction*, ICCV 2011.

Latent LRR

$$\min \|Z\|_* + \|L\|_* + \lambda \|E\|_1,$$
$$s.t.\ X = XZ + LX + E.$$





Liu and Yan. *Latent Low-Rank Representation for Subspace Segmentation and Feature Extraction*, ICCV 2011.

Analysis on LatLRR

- Noiseless LatLRR has non-unique closed form solutions!

Theorem: The complete solutions to

$$\min_{Z,L} \operatorname{rank}(Z) + \operatorname{rank}(L), \quad s.t. \quad X = XZ + LX$$

$$\min_{Z} \operatorname{rank}(Z), \quad s.t. \quad \frac{1}{2}X = XZ.$$

$$\min_{L} \operatorname{rank}(L), \quad s.t. \quad \frac{1}{2}X = LX.$$

⇨

$$\min_{Z} \operatorname{rank}(Z), \quad s.t. \quad \alpha X = XZ.$$

$$\min_{L} \operatorname{rank}(L), \quad s.t. \quad \beta X = LX.$$

$\alpha + \beta = 1.$

are as follows

$$Z^* = V_X \tilde{W} V_X^T + S_1 \tilde{W} V_X^T \text{ and } L^* = U_X \Sigma_X (I - \tilde{W}) \Sigma_X^{-1} U_X^T + U_X \Sigma_X (I - \tilde{W}) S_2,$$

where $\tilde{W}$ is any idempotent matrix and S_1 and S_2 are any matrices satisfying:

1. $V_X^T S_1 = 0$ and $S_2 U_X = 0$; and
2. $\operatorname{rank}(S_1) \cdot \operatorname{rank}(\tilde{W})$ and $\operatorname{rank}(S_2) \cdot \operatorname{rank}(I - \tilde{W})$.

$A^2 = A$

Zhang et al, *A Counterexample for the Validity of Using Nuclear Norm as a Convex Surrogate of Rank*, ECML/PKDD 2013.

Analysis on LatLRR

Theorem: The complete solutions to

$$\min_{Z,L} \|Z\|_* + \|L\|_*, \quad s.t. \quad X = XZ + LX$$

are as follows

$$Z^* = V_X \widehat{W} V_X^T \text{ and } L^* = U_X(I - \widehat{W})U_X^T,$$

where $\widehat{W}$ is any block diagonal matrix satisfying:

1. its blocks are compatible with Σ_X, i.e., if $[\Sigma_X]_{ii} \neq [\Sigma_X]_{jj}$ then $[\widehat{W}]_{ij} = 0$; and
2. both $\widehat{W}$ and $I - \widehat{W}$ are positive semi-definite.

$$\Sigma_X = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix} \Rightarrow \widehat{W} = \begin{bmatrix} 0.8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0.1 & 0.3 & 0 & 0 \\ 0 & 0.1 & 0.6 & 0.2 & 0 & 0 \\ 0 & 0.3 & 0.2 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

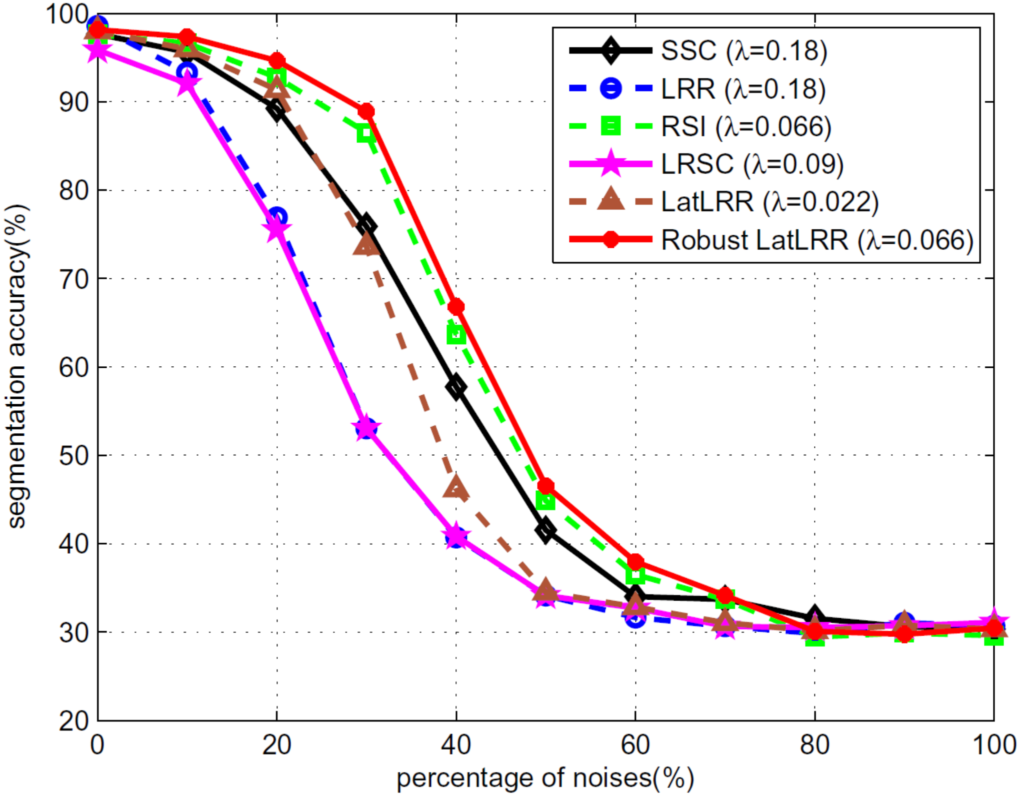
Positive Semi-definite

Zhang et al, *A Counterexample for the Validity of Using Nuclear Norm as a Convex Surrogate of Rank*, ECML/PKDD 2013.

Robust LatLRR

$$\min_{Z,W} \ ||Z||_1, \ \text{s.t.} \ \ Z = V_X W V_X^T, \ W \text{ is diagonal},$$

$$0 \leq \text{diag}(W) \leq 1, \ \text{tr}(W) = 1,$$



Comparison on the synthetic data as the percentage of corruptions increases.

Hongyang Zhang, Zhouchen Lin, Chao Zhang, and Junbin Gao, *Robust Latent Low Rank Representation for Subspace Clustering*, Neurocomputing, Vol. 145, pp. 369-373, December 2014.

Robust LatLRR

Table 1: Segmentation errors (%) on the Hopkins 155 data set. For robust LatLRR, the parameter λ is set as $0.806/\sqrt{n}$. The parameters of other methods are also tuned to be the best.

	SSC	LRR	RSI	LRSC	LatLRR	Robust LatLRR
MAX	46.75	49.88	47.06	40.55	42.03	**35.06**
MEAN	**2.72**	5.64	6.54	4.28	4.17	3.74
STD	8.20	10.35	9.84	8.55	9.14	**7.02**

Table 2: Segmentation accuracy (%) on the Extended Yale B data set, with different number of persons. For robust LatLRR, the parameter λ is set as 0.014, 0.013, and 0.0135, respectively. The parameters of other methods are also tuned to be the best.

Persons	SSC	LRR	RSI	LRSC	LatLRR	Robust LatLRR
5	90.31	91.25	88.44	75.94	69.06	**95.00**
7	87.05	72.77	89.73	65.63	42.86	**92.86**
9	71.35	60.42	87.67	51.04	36.11	**91.49**

Hongyang Zhang, Zhouchen Lin, Chao Zhang, and Junbin Gao, *Robust Latent Low Rank Representation for Subspace Clustering*, Neurocomputing, Vol. 145, pp. 369-373, December 2014.

Relationship Between LR Models

$$\min_{Z,E} \operatorname{rank}(Z) + \lambda||E||_\ell,$$
$$\text{s.t. } X = XZ + E.$$
$$\min_{Z,L,E} \operatorname{rank}(Z) + \operatorname{rank}(L) + \lambda||E||_\ell,$$
$$\text{s.t. } X = XZ + LX + E.$$

$$\text{(Original RPCA)} \min_{A,E} \operatorname{rank}(A) + \lambda||E||_\ell,$$
$$\text{s.t. } X = A + E.$$
$$\text{(Original Robust LRR)} \min_{Z,E} \operatorname{rank}(Z) + \lambda||E||_\ell,$$
$$\text{s.t. } X - E = (X - E)Z.$$
$$\text{(Original Robust LatLRR)} \min_{Z,L,E} \operatorname{rank}(Z) + \operatorname{rank}(L) + \lambda||E||_\ell,$$
$$\text{s.t. } X - E = (X - E)Z + L(X - E).$$

$$\min_{Z,D,E} \operatorname{rank}(Z) + \lambda||E||_\ell,$$
$$\text{s.t. } D = DZ, X = D + E.$$
$$\min_{Z,L,D,E} \operatorname{rank}(Z) + \operatorname{rank}(L) + \lambda||E||_\ell,$$
$$\text{s.t. } D = DZ + LD, X = D + E.$$

Hongyang Zhang, Zhouchen Lin, Chao Zhang, and Junbin Gao, *Relation among Some Low Rank Subspace Recovery Models*, Neural Computation, Vol. 27, No. 9, pp. 1915-1950, 2015.

Relationship Between LR Models

$$
\begin{aligned}
\text{(Heuristic RPCA)} \; & \min_{A,E} \|A\|_* + \lambda ||E||_\ell, \\
& \text{s.t.} \;\; X = A + E. \\
\text{(Heuristic Robust LRR)} \; & \min_{Z,E} \|Z\|_* + \lambda ||E||_\ell, \\
& \text{s.t.} \; X - E = (X - E)Z. \\
\text{(Heuristic Robust LatLRR)} \; & \min_{Z,L,E} \|Z\|_* + \|L\|_* + \lambda ||E||_\ell, \\
& \text{s.t.} \; X - E = (X - E)Z + L(X - E).
\end{aligned}
$$

Hongyang Zhang, Zhouchen Lin, Chao Zhang, and Junbin Gao, *Relation among Some Low Rank Subspace Recovery Models*, Neural Computation, Vol. 27, No. 9, pp. 1915-1950, 2015.

Relationship Between LR Models

$$\min_{Z,E} \operatorname{rank}(Z) + \lambda ||E||_\ell, \quad \text{s.t. } X - E = (X - E)Z.$$

Original Robust LRR

$$\min_{Z,E} \|Z\|_* + \lambda ||E||_\ell, \quad \text{s.t. } X - E = (X - E)Z.$$

Heuristic Robust LRR

Original RPCA

$$\min_{A,E} \operatorname{rank}(A) + \lambda ||E||_\ell, \quad \text{s.t. } X = A + E.$$

Original Robust LatLRR

$$\min_{Z,L,E} \operatorname{rank}(Z) + \operatorname{rank}(L) + \lambda ||E||_\ell, \quad \text{s.t. } X - E = (X - E)Z + L(X - E).$$

Heuristic Robust LatLRR

$$\min_{Z,L,E} \|Z\|_* + \|L\|_* + \lambda ||E||_\ell, \quad \text{s.t. } X - E = (X - E)Z + L(X - E).$$

Hongyang Zhang, Zhouchen Lin, Chao Zhang, and Junbin Gao, *Relation among Some Low Rank Subspace Recovery Models*, Neural Computation, Vol. 27, No. 9, pp. 1915-1950, 2015.

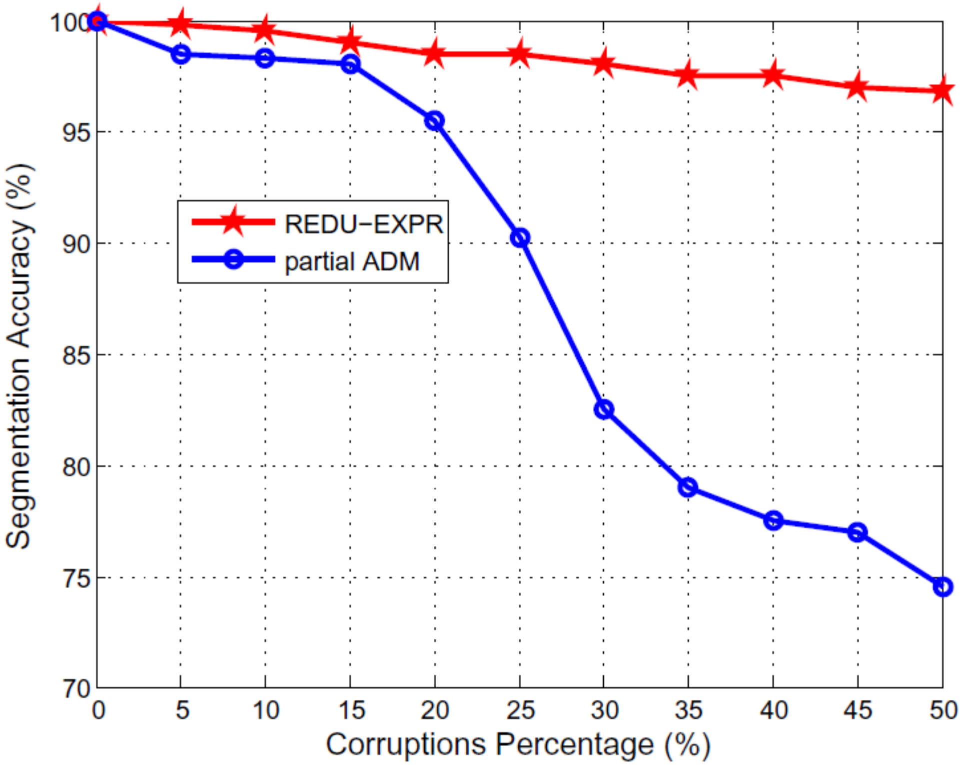
Implications

- We could obtain a *globally optimal* solution to other low rank models.
- We could have *much faster* algorithms for other low rank models.

Hongyang Zhang, Zhouchen Lin, Chao Zhang, and Junbin Gao, *Relation among Some Low Rank Subspace Recovery Models*, Neural Computation, Vol. 27, No. 9, pp. 1915-1950, 2015.

Implications

- Comparison of optimality



Comparison of accuracies of solutions to relaxed R-LRR computed by REDU-EXPR and partial ADM, where the parameter is adopted as 1/\sqrt(log n) and n is the input size. The program is run by 10 times and the average accuracies are reported.

Hongyang Zhang, Zhouchen Lin, Chao Zhang, and Junbin Gao, *Relation among Some Low Rank Subspace Recovery Models*, Neural Computation, Vol. 27, No. 9, pp. 1915-1950, 2015.

Implications

- Comparison of speed

Model	Method	Accuracy	CPU Time (h)
LRR	ADM	-	>10
R-LRR	ADM	-	did not converge
R-LRR	partial ADM	-	>10
R-LRR	REDU-EXPR	61.6365%	0.4603

Table 1: Unsupervised face image clustering results on the Extended YaleB database. REDU-EXPR means reducing to RPCA first and then express the solution as that of RPCA.

Hongyang Zhang, Zhouchen Lin, Chao Zhang, and Junbin Gao, *Relation among Some Low Rank Subspace Recovery Models*, Neural Computation, Vol. 27, No. 9, pp. 1915-1950, 2015.

Implications

- Comparison of optimality and speed

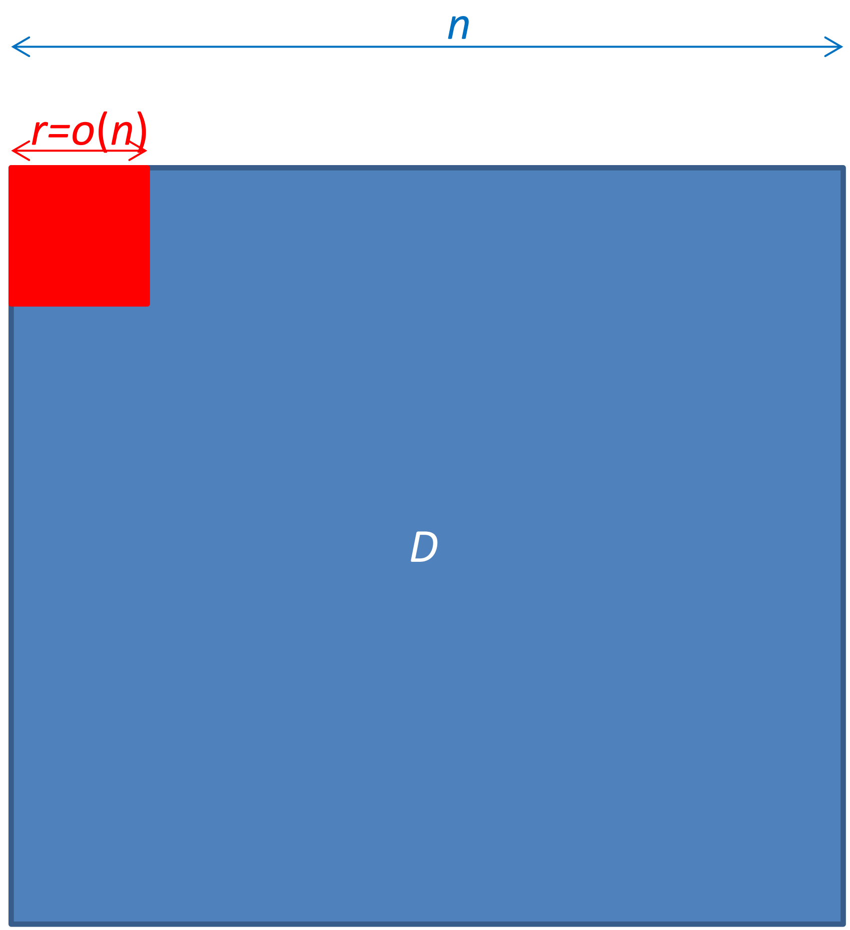
Table 4: Comparison of robustness and speed between partial ADM (LRSC) (Favaro et al., 2011) and REDU-EXPR (RSI) (Wei and Lin, 2010) methods for solving R-LRR when the percentage of corruptions increases. All the experiments are run ten times and the λ is set to be the same: $\lambda = 1/\sqrt{\log n}$, where n is the data size.

Noise Percentage (%)	0	10	20	30	40	50
Rank(Z) (partial ADM)	20	30	30	30	30	30
Rank(Z) (REDU-EXPR)	20	20	20	20	20	20
$\|\|E\|\|_{\ell_{2,0}}$ (partial ADM)	0	99	200	300	400	500
$\|\|E\|\|_{\ell_{2,0}}$ (REDU-EXPR)	0	100	200	300	400	500
Objective (partial ADM)	**20.00**	67.67	106.10	144.14	182.19	220.24
Objective (REDU-EXPR)	**20.00**	**58.05**	**96.10**	**134.14**	**172.19**	**210.24**
Time (s, partial ADM)	**4.89**	124.33	126.34	119.12	115.20	113.94
Time (s, REDU-EXPR)	10.67	**9.60**	**8.34**	**8.60**	**9.00**	**12.86**

Hongyang Zhang, Zhouchen Lin, Chao Zhang, and Junbin Gao, *Relation among Some Low Rank Subspace Recovery Models*, Neural Computation, Vol. 27, No. 9, pp. 1915-1950, 2015.

$O(n^1)$ RPCA by l_1-Filtering

- Assumption: $\text{rank}(A) = o(n)$.

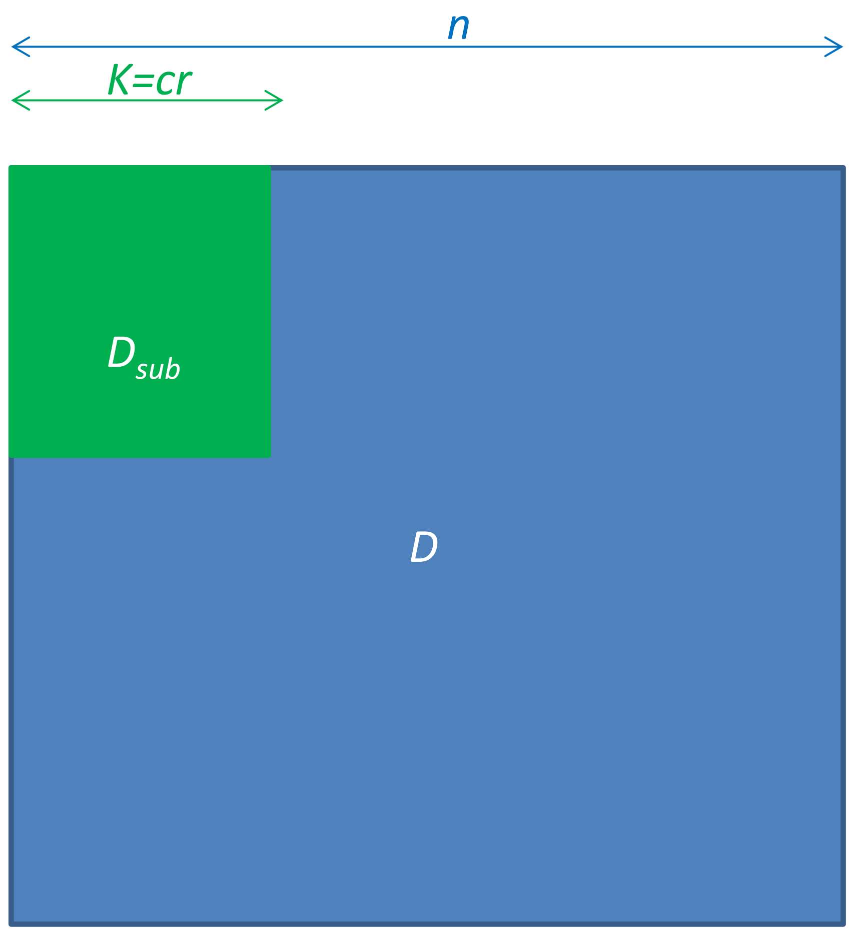


$$\min \|A\|_* + \lambda \|E\|_{l_1}, \quad s.t. \quad D = A + E.$$

Risheng Liu, Zhouchen Lin, Zhixun Su, and Junbin Gao, *Linear time Principal Component Pursuit and its extensions using l1 Filtering*, Neurocomputing, Vol. 142, pp. 529-541, 2014.

$O(n^1)$ RPCA by l_1-Filtering

- First, randomly sample D_{sub}.



$$\min \|A\|_* + \lambda \|E\|_{l_1},$$
$$s.t. \quad D = A + E.$$

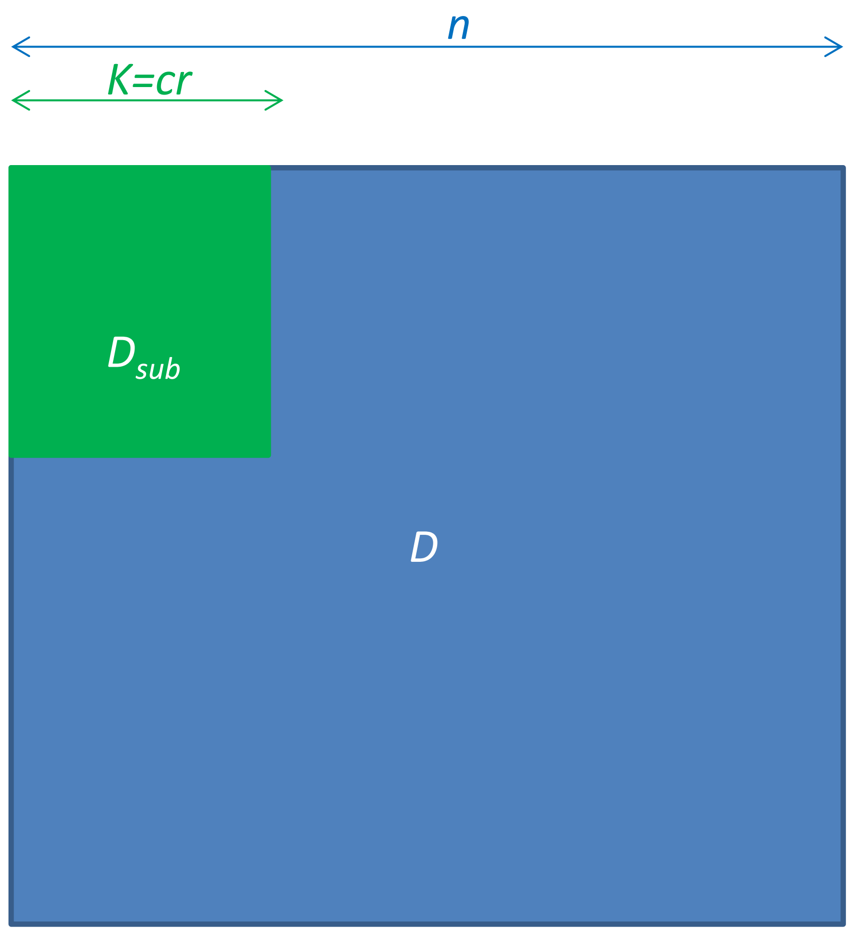
Risheng Liu, Zhouchen Lin, Zhixun Su, and Junbin Gao, *Linear time Principal Component Pursuit and its extensions using l1 Filtering*, Neurocomputing, Vol. 142, pp. 529-541, 2014.

$O(n^1)$ RPCA by l_1-Filtering

- Second, solve RPCA for D_{sub}: $D_{sub}=A_{sub}+E_{sub}$.

$$\min \|A_{sub}\|_* + \lambda_1 \|E_{sub}\|_1$$
$$\text{subj } D_{sub} = A_{sub} + E_{sub}.$$

$< O(n)$ complexity !

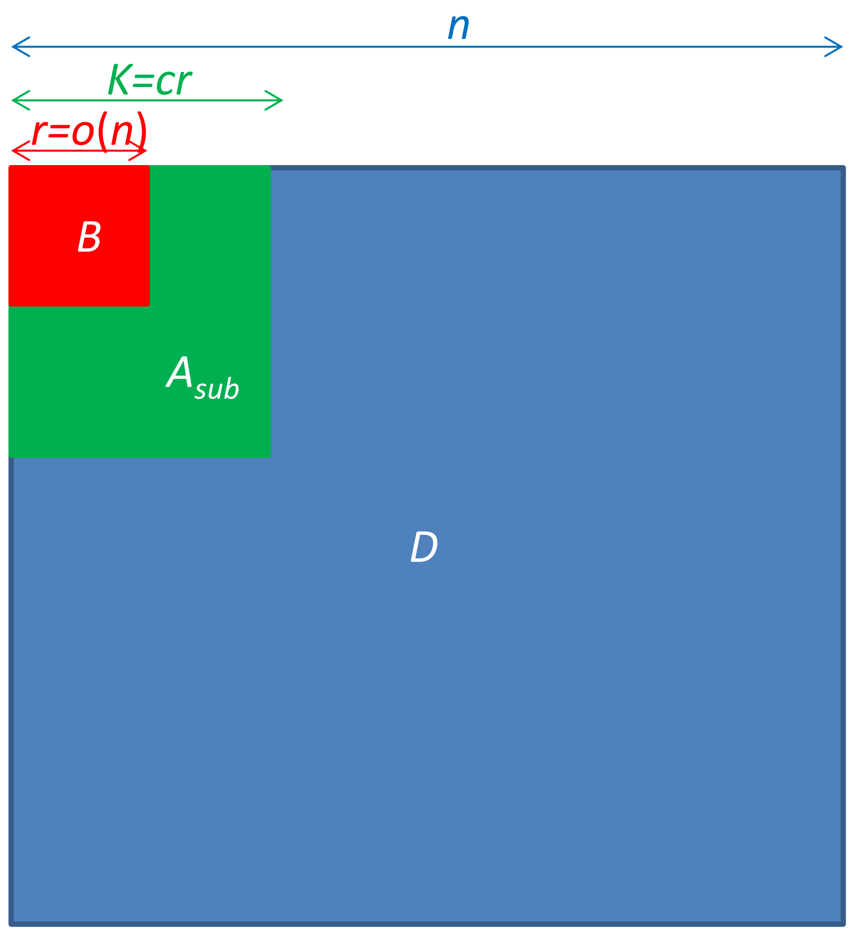


$$\min \|A\|_* + \lambda \|E\|_{l_1},$$
$$s.t. \quad D = A + E.$$

Risheng Liu, Zhouchen Lin, Zhixun Su, and Junbin Gao, *Linear time Principal Component Pursuit and its extensions using l1 Filtering*, Neurocomputing, Vol. 142, pp. 529-541, 2014.

$O(n^1)$ RPCA by l_1-Filtering

- Third, find the full rank submatrix B of A_{sub}.



$$\min \|A\|_* + \lambda \|E\|_{l_1}, \quad s.t. \quad D = A + E.$$

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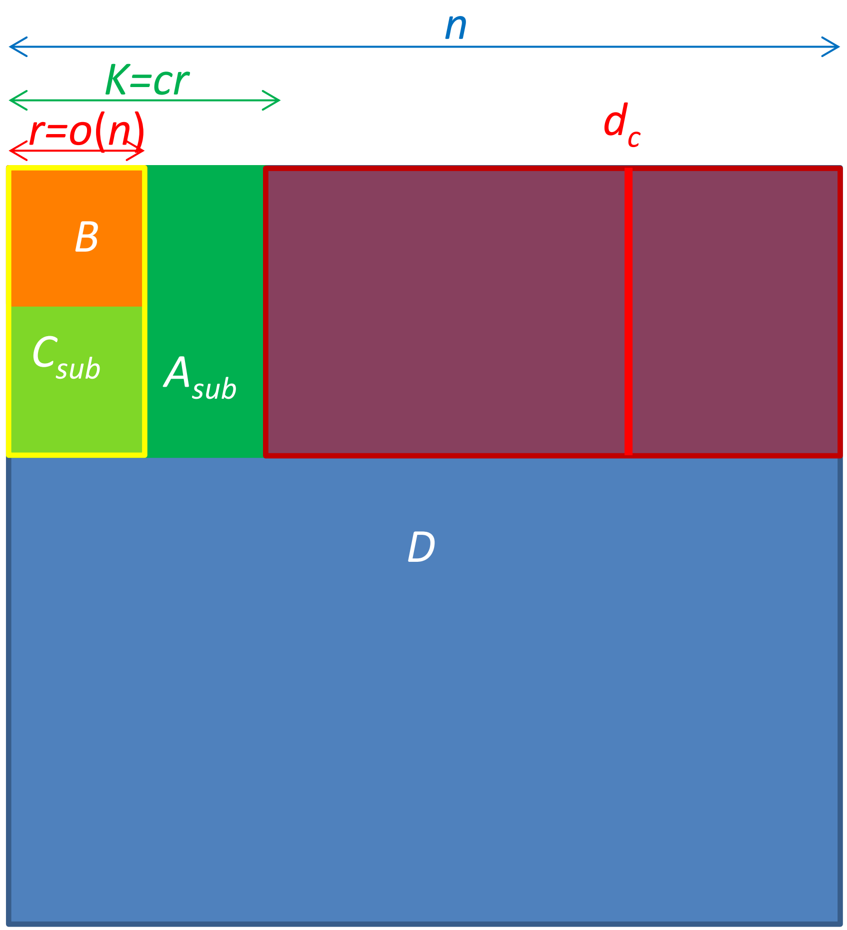
$O(n^1)$ RPCA by l_1-Filtering

- Fourth, correct the Kx(n-K) submatrix of D by C_{sub}.

$$\min_{p_c} \|d_c - C_{sub} p_c\|_1$$

$O(n)$ complexity! Can be done in parallel!!



$$\min \|A\|_* + \lambda \|E\|_{l_1}, \quad s.t. \quad D = A + E.$$

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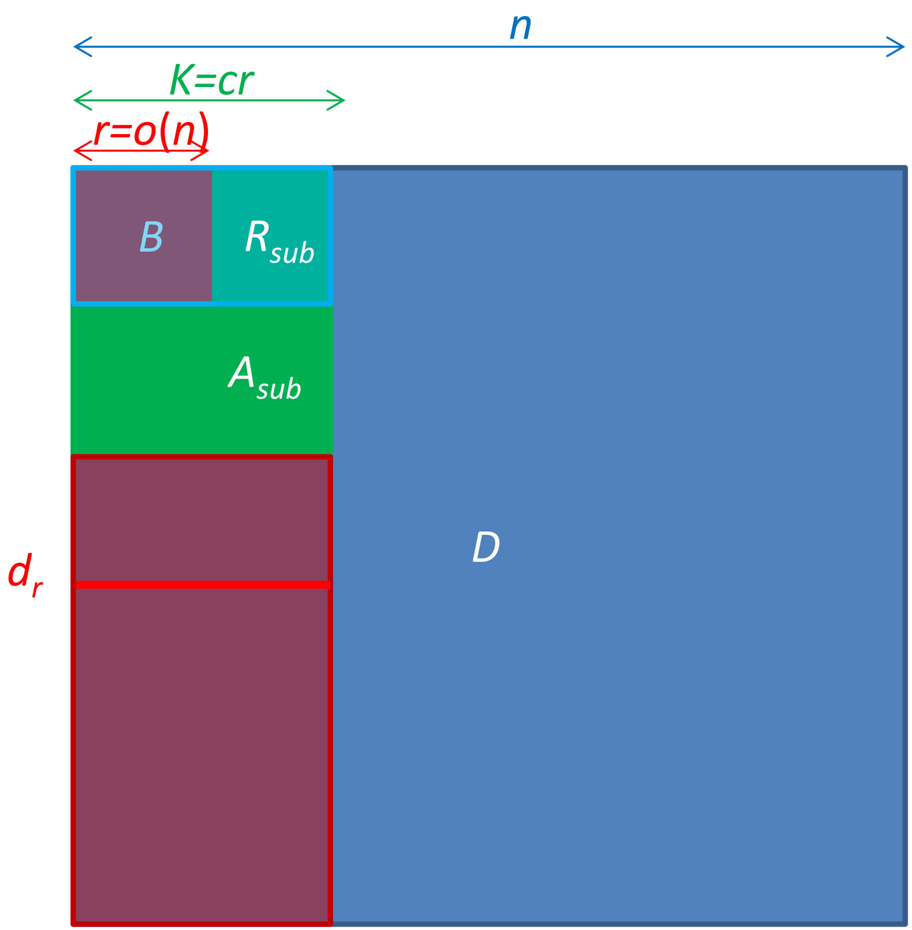
$O(n^1)$ RPCA by l_1-Filtering

- Fifth, correct the $(n\text{-}K)\text{x}K$ submatrix of D by R_{sub}.

$$\min_{q_r} \|d_r - q_r R_{sub}\|_1$$

$O(n)$ complexity! Can be done in parallel!!



Rows and columns can also be processed in parallel!!

$$\min \|A\|_* + \lambda \|E\|_{l_1}, \quad s.t. \quad D = A + E.$$

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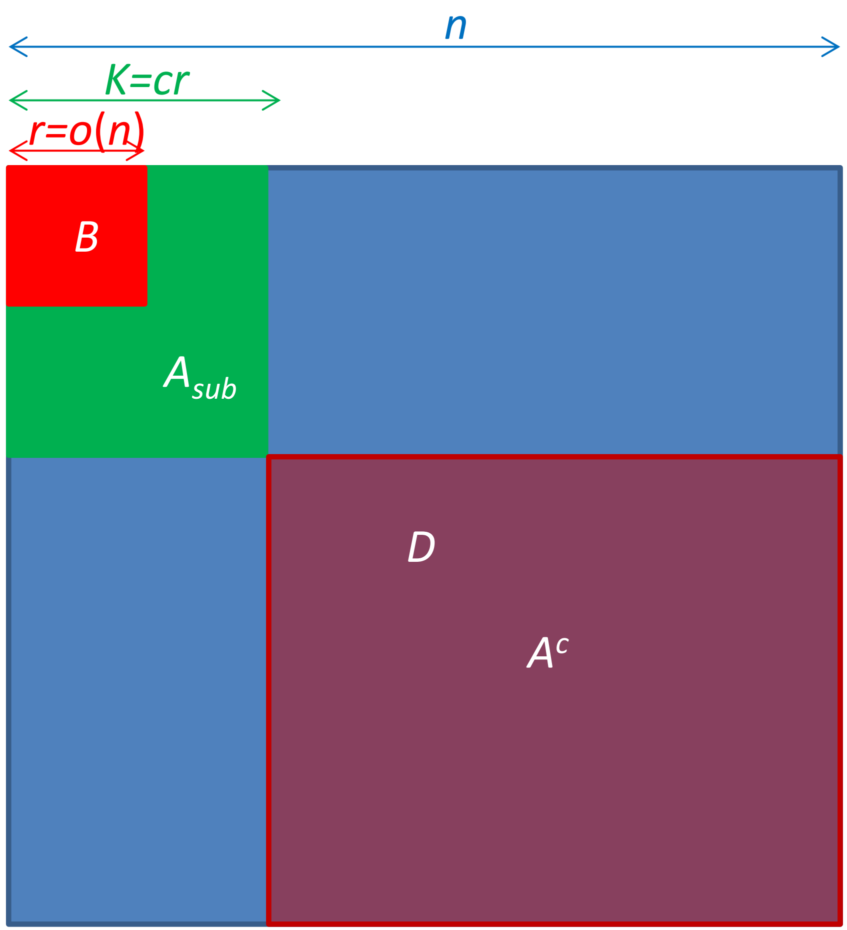
$O(n^1)$ RPCA by l_1-Filtering

- Finally, the rest part of A is $A^c = Q_r B P_c$.

A compact representation of A!

$$P_c = (p_1, p_2, \cdots, p_{n-K})$$

$$Q_r = \begin{pmatrix} q_1 \\ q_2 \\ \cdots \\ q_{n-K} \end{pmatrix}$$



$$\min \|A\|_* + \lambda \|E\|_{l_1},$$
$$s.t. \quad D = A + E.$$

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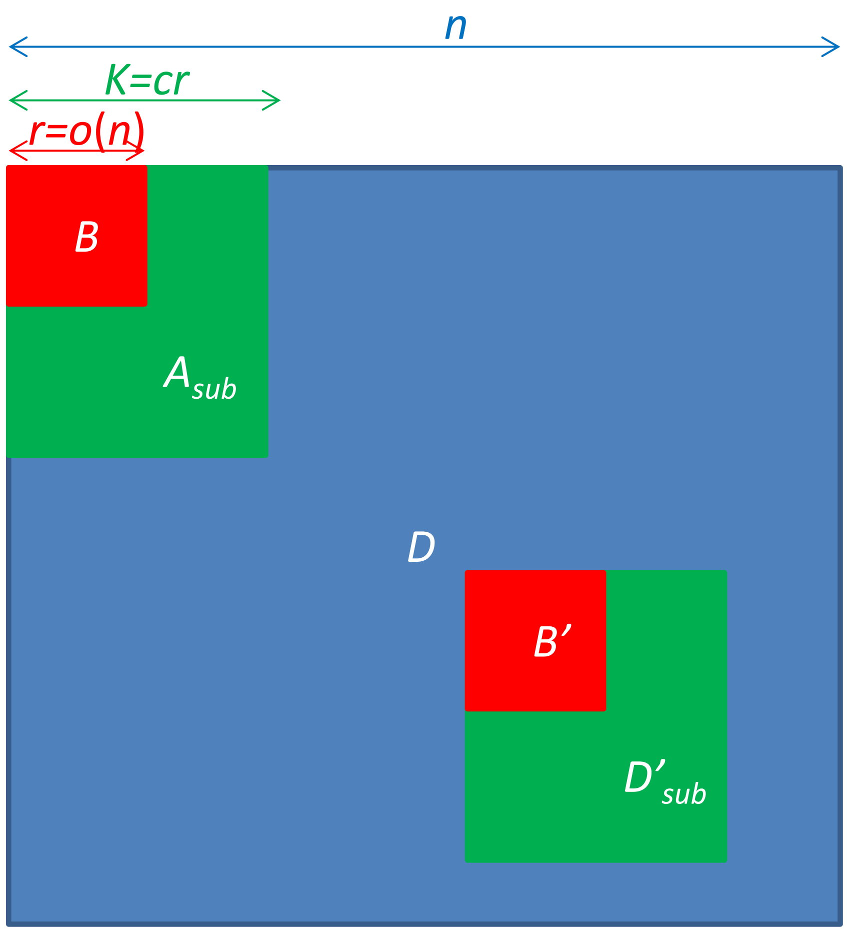
$O(n^1)$ RPCA by l_1-Filtering

- What if the rank is unknown?

1. If $\text{rank}(A_{sub}) > K/c$, then increase K to $c\,\text{rank}(A_{sub})$.
2. Otherwise, resample another D_{sub} for cross validation.



$$\min \|A\|_* + \lambda \|E\|_{l_1}, \quad s.t. \quad D = A + E.$$

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Experiments

Synthetic Data							
Size	Method	$\frac{\|\mathbf{L}_0-\mathbf{L}^*\|_F}{\|\mathbf{L}_0\|_F}$	rank($\mathbf{L}^*$)	$\|\mathbf{L}^*\|_*$	$\|\mathbf{S}^*\|_{l_0}$	$\|\mathbf{S}^*\|_{l_1}$	Time(s)
2000	rank($\mathbf{L}_0$) = 20, $\|\mathbf{L}_0\|_* = 39546$, $\|\mathbf{S}_0\|_{l_0} = 40000$, $\|\mathbf{S}_0\|_{l_1} = 998105$						
	S-ADM	1.46×10^{-8}	20	39546	39998	998105	84.73
	L-ADM	4.72×10^{-7}	20	39546	40229	998105	27.41
	l_1	1.66×10^{-8}	20	39546	40000	998105	**5.56 = 2.24 + 3.32**
5000	rank($\mathbf{L}_0$) = 50, $\|\mathbf{L}_0\|_* = 249432$, $\|\mathbf{S}_0\|_{l_0} = 250000$, $\|\mathbf{S}_0\|_{l_1} = 6246093$						
	S-ADM	7.13×10^{-9}	50	249432	249995	6246093	1093.96
	L-ADM	4.28×10^{-7}	50	249432	250636	6246158	195.79
	l_1	5.07×10^{-9}	50	249432	250000	6246093	**42.34=19.66 + 22.68**
10000	rank($\mathbf{L}_0$) = 100, $\|\mathbf{L}_0\|_* = 997153$, $\|\mathbf{S}_0\|_{l_0} = 1000000$, $\|\mathbf{S}_0\|_{l_1} = 25004070$						
	S-ADM	1.23×10^{-8}	100	997153	1000146	25004071	11258.51
	L-ADM	4.26×10^{-7}	100	997153	1000744	25005109	1301.83
	l_1	2.90×10^{-10}	100	997153	1000023	25004071	**276.54 = 144.38 + 132.16**
Structure form Motion Data							
4002 × 2251	rank($\mathbf{L}_0$) = 4, $\|\mathbf{L}_0\|_* = 31160$, $\|\mathbf{S}_0\|_{l_0} = 900850$, $\|\mathbf{S}_0\|_{l_1} = 3603146$						
	S-ADM	5.38×10^{-8}	4	31160	900726	3603146	925.28
	L-ADM	3.25×10^{-7}	4	31160	1698387	3603193	62.08
	l_1	1.20×10^{-8}	4	31160	900906	3603146	**5.29 = 3.51 + 1.78**

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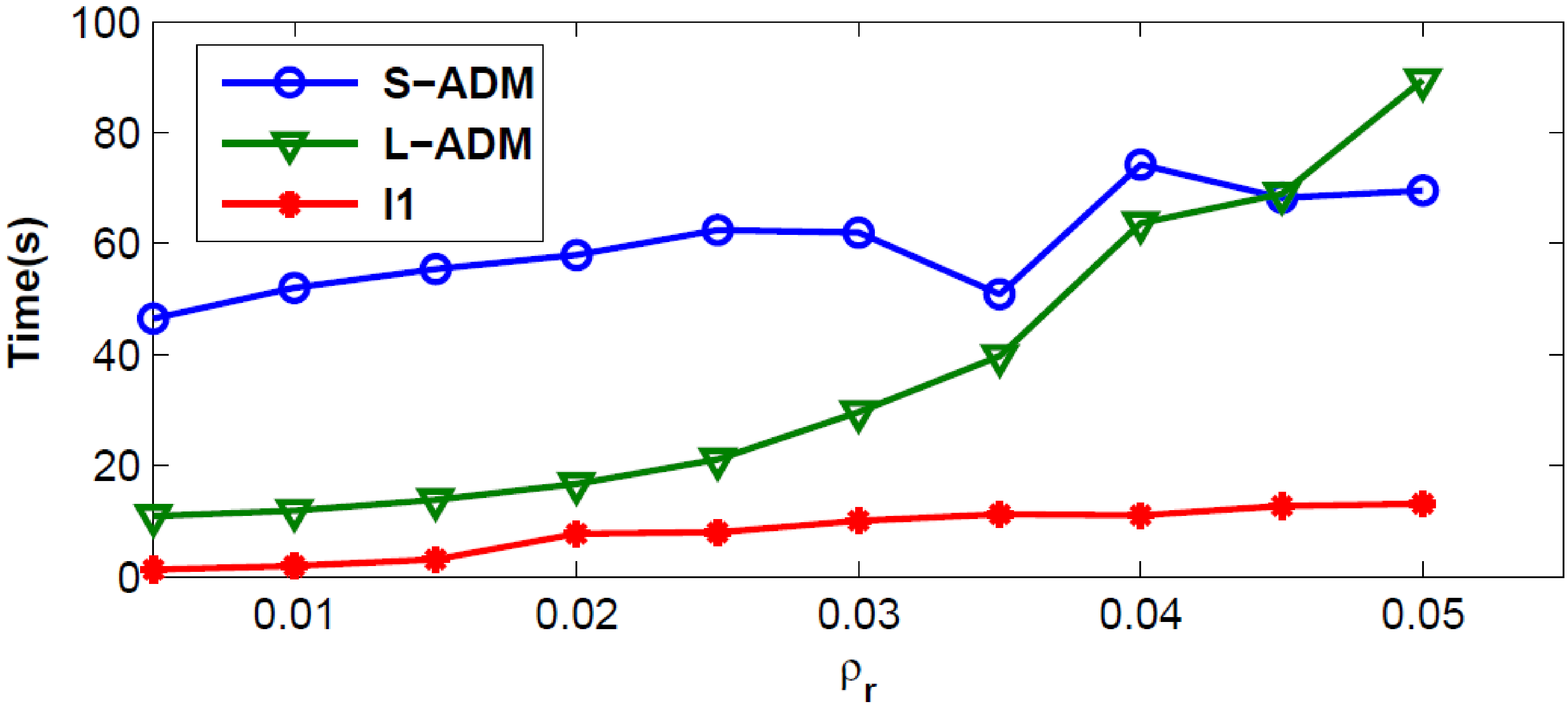
Experiments



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Conclusions

- Low-rank models have much richer mathematical properties than sparse models.
- Closed-form solutions to low-rank models are useful in both theory and applications.

Thanks!

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- http://www.cis.pku.edu.cn/faculty/vision/zlin/zlin.htm