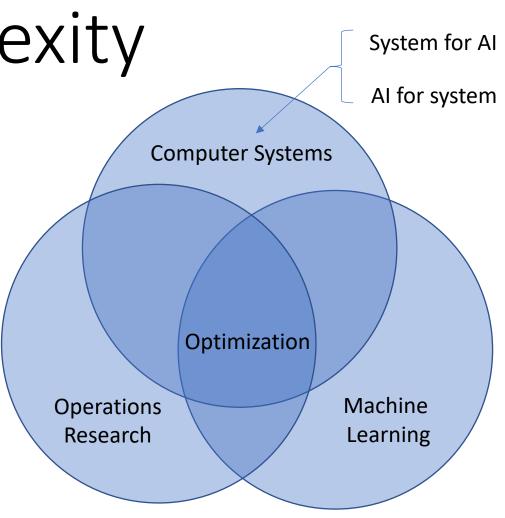


# Optimization in Alibaba: Beyond Convexity System for Al Al for system

Jian Tan Machine Intelligent Technology 阿里巴巴|达摩院 机器智能技术部 | 智能决策





## Agenda

Theories on non-convex optimization:
 Part 1. Parallel restarted SGD: it finds first-order stationary points

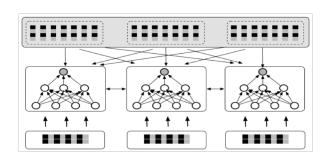
 (why model averaging works for Deep Learning?)

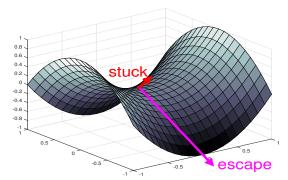
 Part 2. Escaping saddle points in non-convex optimization

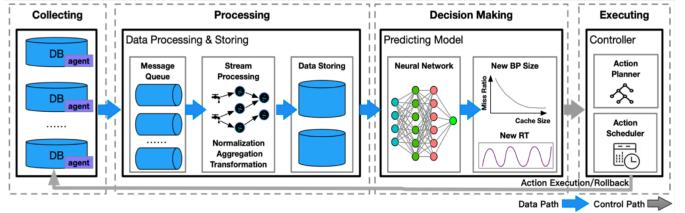
 (first-order stochastic algorithms to find second-order stationary points)

System optimization: BPTune for an intelligent database (from OR/ML perspectives)
 A real complex system deployment
 Combine pairwise DNN, active learning, heavy-tailed randomness ...

Part 3. Stochastic (large deviation) analysis for LRU caching



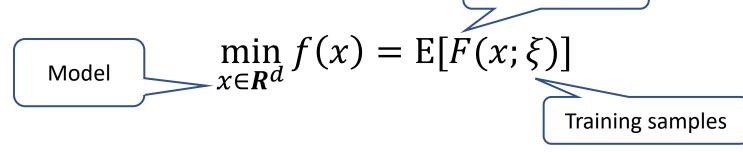






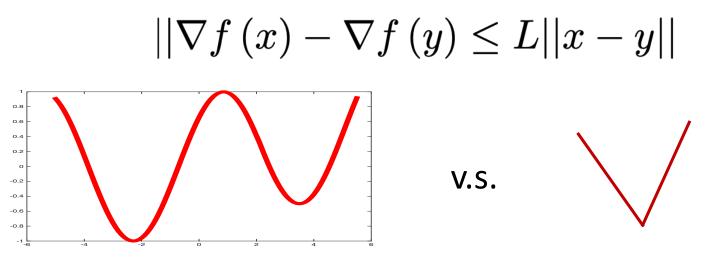
# Learning as Optimization

Stochastic (non-convex) optimization



Loss function

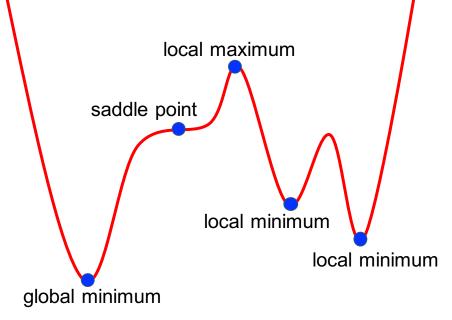
- $\xi$ : random training sample
- f(x): has Lipschitz continuous Gradient





# Non-Convex Optimization is Challenging

#### Many local minima & saddle points



For stationary points  $\nabla f(x)=0$  (first-order stationary)  $\nabla^2 f(x) > 0 \implies$  Local minimum  $\nabla^2 f(x) < 0 \implies$  Local maximum  $\nabla^2 f(x)$  has both +/- eigenvalues  $\implies$  saddle points  $\nabla^2 f(x)$  has 0/+ eigenvalues  $\implies$  Degenerate case: could be either local minimum or saddle points

#### In general, finding global minimum of non-convex optimization is NP-hard



## Instead ...

• For some applications, e.g., matrix completion, tensor decomposition, dictionary learning, and certain neural networks,

Good news: local minima

- Either all local minima are all global minima
- Or all local minima are close to global minima

Bad news: saddle points

- Poor function value compared with global/local minima
- Possibly many saddle points (even exponential number)



# Finding First-order Stationary Points (FSP)

• Stochastic Gradient Descent (SGD):

 $x_{t+1} = x_t - \eta \nabla F(x_t; \xi_t)$ 

- Complexity of SGD (Ghadimi & Lan, 2013, 2016; Ghadimi et al., 2016; Yang et al., 2016) :
  - $\epsilon$ -FSP,  $\mathbb{E}[\|\nabla f(x)\|_2^2] \le \epsilon^2$ : Iteration complexity  $O(1/\epsilon^4)$
- Improved Iteration complexity based on Variance Reduction:
  - SCSG (Lei et al.,2017):  $O(1/\epsilon^{10/3})$
- Workhorse of deep learning

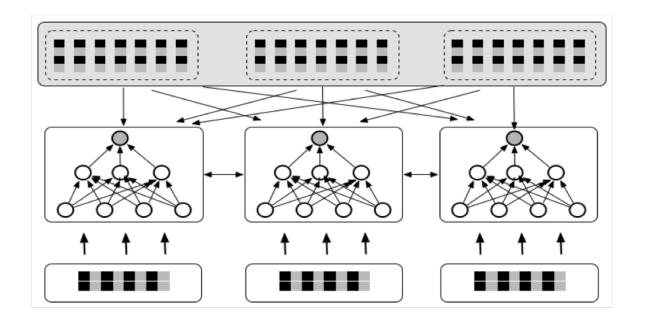




#### Part 1:

#### Parallel Restarted SGD with Faster Convergence and Less Communication: Demystifying Why Model Averaging works for Deep Learning

Hao Yu, Sen Yang, Shenghuo Zhu (AAAI 2019)

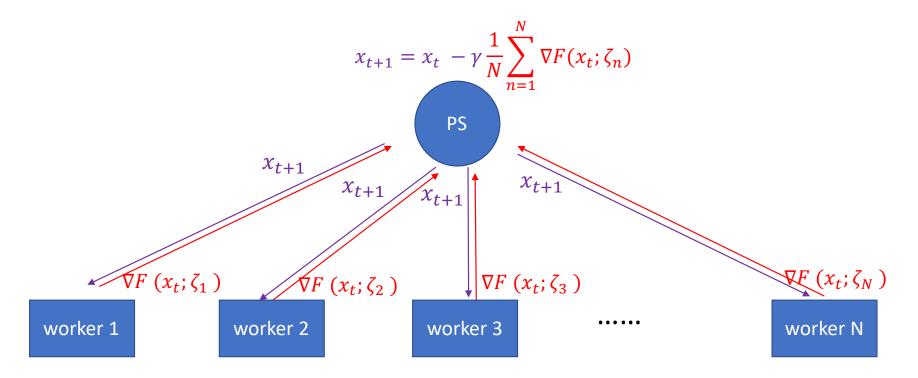


- One server is not enough:
  - too many parameters, e.g., deep neural networks
  - huge number of training samples
  - training time is too long
- Parallel on N servers:
  - With N machines, can we be N times faster? If yes, we have the linear speed-up (w.r.t. # of workers)



# Classical Parallel mini-batch SGD

• The classical Parallel mini-batch SGD (PSGD) achieves  $O(\frac{1}{\sqrt{NT}})$  convergence with N workers [Dekel et al. 12]. PSGD can attain a linear speed-up.



- Each iteration aggregates gradients from every workers. Communication too high!
- Can we reduce the communication cost? Yes, model averaging.



#### Model Averaging (Parallel Restarted SGD)

Algorithm 1 Parallel Restarted SGD

- 1: Input: Initialize  $\mathbf{x}_i^0 = \overline{\mathbf{y}} \in \mathbb{R}^m$ . Set learning rate  $\gamma > 0$  and node synchronization interval (integer) I > 0
- 2: for t = 1 to T do
- 3: Each node *i* observes an unbiased stochastic gradient  $\mathbf{G}_i^t$  of  $f_i(\cdot)$  at point  $\mathbf{x}_i^{t-1}$
- 4: **if** t is a multiple of I, i.e., t% I = 0, **then**

5: Calculate node average 
$$\overline{\mathbf{y}} \stackrel{\Delta}{=} \frac{1}{N} \sum_{i=1}^{N} \mathbf{x}_{i}^{t-1}$$

6: Each node i in parallel updates its local solution

$$\mathbf{x}_i^t = \overline{\mathbf{y}} - \gamma \mathbf{G}_i^t, \quad \forall i \tag{2}$$

#### 7: else

8: Each node i in parallel updates its local solution

$$\mathbf{x}_{i}^{t} = \mathbf{x}_{i}^{t-1} - \gamma \mathbf{G}_{i}^{t}, \quad \forall i$$
(3)

9: end if10: end for



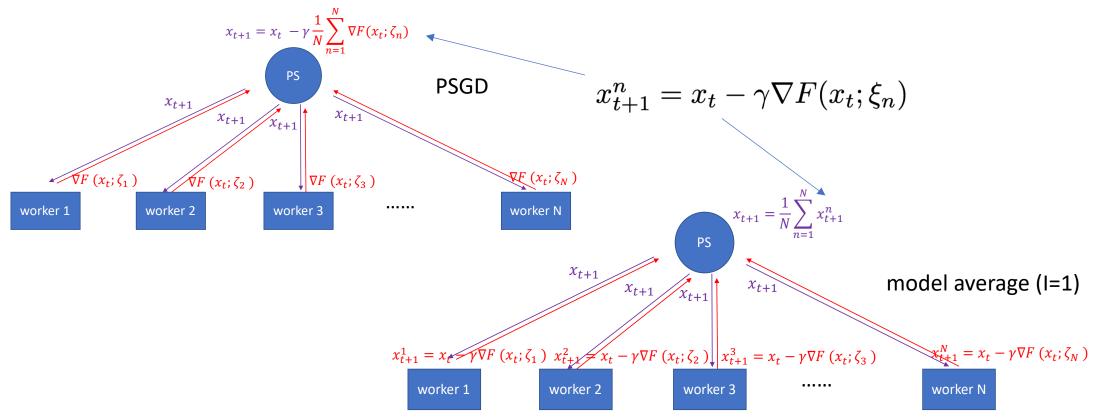
## Model Averaging

- Each worker train its local model + (periodically) average on all workers
  - **One-shot averaging**: [Zindevich et al. 2010, McDonalt et al. 2010] propose to average only once at the end.
  - [Zhang et al. 2016] shows averaging once can leads to poor solutions for nonconvex opt and suggest more frequent averaging.
- If averaging every | iterations, how large is | ?
  - One-shot averaging: I=T
  - PSGT: **I=1**



# Why I=1 works?

• If we average models each iteration (I=1), then it is equivalent to PSGD.



• What if we average after multiple iterations periodically (I>1)? Converge or not? Convergence rate? Linear speed-up or not?

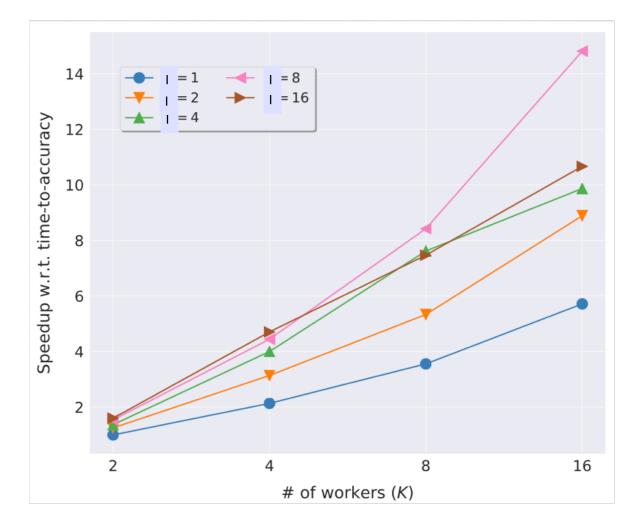


## Empirical work

- There has been a long line of empirical works ...
  - [Zhang et al. 2016]: CNN for MNIST
  - [Chen and Huo 2016] [Su, Chen, and Xu 2018] : DNN-GMM for speech recognition
  - [McMahan et al. 2017] :CNN for MNIST and Cifar10; LSTM for language modeling
  - [Kaamp et al. 2018] :CNN for MNIST
  - [Lin, Stich, and Jaggi 2018]: Res20 for Cifar10/100; Res50 for ImageNet
- These empirical works show that "model averaging" = PSGD with significantly less communication overhead!
- Recall PSGD = linear speed-up



### Model Averaging: almost linear speed-up in practice



- Good speed up (measured in wall time used to achieve target accuracy)
- I: averaging intervals (I=4 means "average every 4 iterations")
- Resnet20 over CIFAR10

• Figure 7(a) from "Tao Lin, Sebastian U. Stich, and Martin Jaggi 2018, Don't use large mini-batches, use local SGD"



## Related work

- For strongly convex opt, [Stich 2018] shows the convergence (with linear speed-up w.r.t. # of workers) is maintained as long as the averaging interval I <  $O(\sqrt{T}/\sqrt{N})$ .
- Why model averaging achieves almost linear speed-up for deep learning (non-convex) in practice for I>1?



## Main result

• Prove "model averaging" (communication reduction) has the same convergence rate as PSGD for non-convex opt under certain conditions

If the averaging interval  $I = O(T^{\frac{1}{4}}/N^{\frac{3}{4}})$ , then model averaging has the convergence rate  $O(\frac{1}{\sqrt{NT}})$ .

• "Model averaging" works for deep learning. It is as fast as PSGD with significantly less communication.



# Control bias-variance after I iterations

• Focus on

$$\bar{x}^t = \frac{1}{N} \sum_{i=1}^N x_i^t$$

average of local solution over all N workers

• Note...

$$\bar{x}^t = \bar{x}^{t-1} - \gamma \frac{1}{N} \sum_{i=1}^N G_i^t$$

 $G_i^t$ : independent gradients sampled at different points  $x_i^{t-1}$ 

• PSGD has i.i.d. gradients at  $\bar{x}^{t-1}$ , which are unavailable at local workers without communication



# Technical analysis

• Bound the difference between  $\bar{x}^t$  and  $x_i^t$ 

Our Algorithm ensures  $E[||\bar{x}^t - x_i^t||^2] \le 4\gamma^2 I^2 G^2, \forall i, \forall t$ 

• The rest part uses the smoothness and shows

$$\frac{1}{T}\sum_{t=1}^{T} \mathbb{E}\left[\|\nabla f(\overline{\mathbf{x}}^{t-1})\|^2\right] \le \frac{2}{\gamma T} \left(f(\overline{\mathbf{x}}^0) - f^*\right) + \frac{4\gamma^2 I^2 G^2 L}{N} + \frac{2}{N}\gamma \sigma^2$$

*Proof.* Fix  $t \ge 1$ . By the smoothness of f, we have

$$\mathbb{E}[f(\overline{\mathbf{x}}^{t})] \le \mathbb{E}[f(\overline{\mathbf{x}}^{t-1})] + \mathbb{E}[\langle \nabla f(\overline{\mathbf{x}}^{t-1}), \overline{\mathbf{x}}^{t} - \overline{\mathbf{x}}^{t-1} \rangle] + \frac{L}{2} \mathbb{E}[\|\overline{\mathbf{x}}^{t} - \overline{\mathbf{x}}^{t-1}\|^{2}]$$

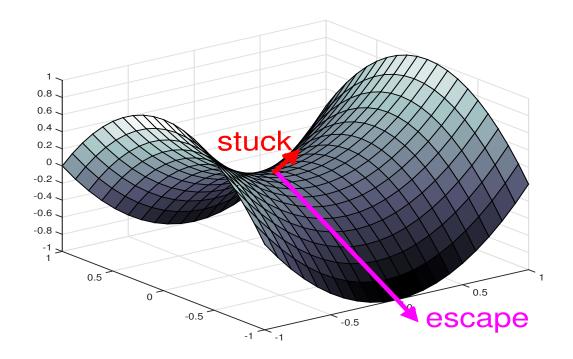
Note that

$$\mathbb{E}[\|\overline{\mathbf{x}}^t - \overline{\mathbf{x}}^{t-1}\|^2] \stackrel{(a)}{=} \gamma^2 \mathbb{E}[\|\frac{1}{N} \sum_{i=1}^N \mathbf{G}_i^t\|^2]$$

Assume:
$\mathbb{E}_{\zeta_i \sim \mathcal{D}_i} \ \nabla F_i(\mathbf{x}; \zeta_i) - \nabla f_i(\mathbf{x})\ ^2 \le \sigma^2$
$\mathbb{E}_{\zeta_i \sim \mathcal{D}_i} \  \nabla F_i(\mathbf{x}; \zeta_i) \ ^2 \le G^2$



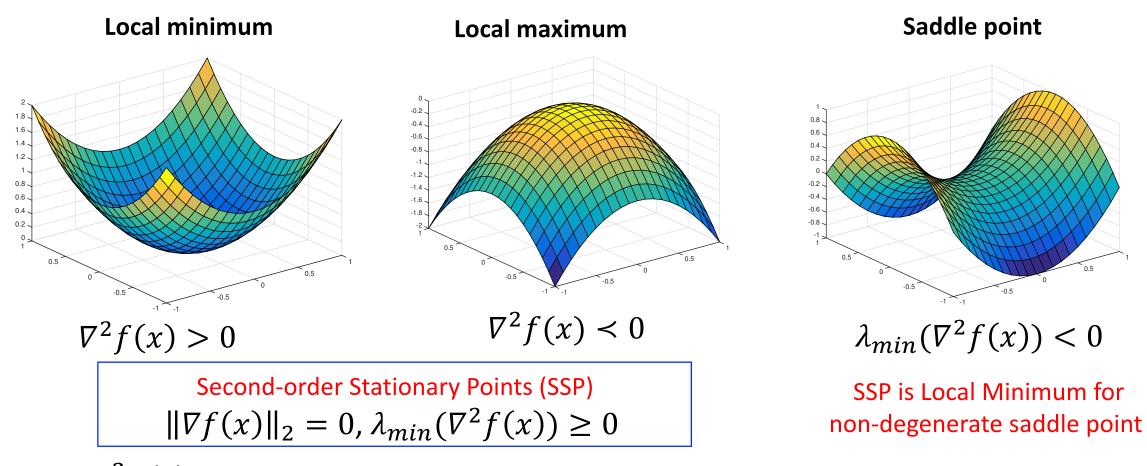
### Part 2: Escaping Saddle points in non-convex optimization Yi Xu\*, Rong Jin, Tianbao Yang\*



First-order Stochastic Algorithms for Escaping From Saddle Points in Almost Linear Time, NIPS 2018. \* Xu and Yang are with Iowa State University



# (First-order) Stationary Points (FSP) $||\nabla F(x)||_2 = 0$



 $\nabla^2 f(x)$  has both +/- eigenvalues  $\implies$  saddle points, which can be bad!  $\nabla^2 f(x)$  has both 0/+ eigenvalues  $\implies$  degenerate case: local minimum/saddle points



## The Problem

• Finding an approximate local minimum by using first-order methods

$$\epsilon$$
-SSP:  $\|\nabla f(x)\|_2 \leq \epsilon, \lambda_{min}(\nabla^2 f(x)) \geq -\gamma$ 

• Choice of  $\gamma$  : small enough, e.g.,  $\gamma = \sqrt{\epsilon}$  (Nesterov & Polyak 2006)

Nesterov, Yurii, and Polyak, Boris T. "Cubic regularization of Newton method and its global performance." *Mathematical Programming* 108.1 (2006): 177-205.



# Related Work

• Adding Isotropic Noise: Noisy SGD (Ge et al., 2015), SGLD (Zhang et al., 2017)

$$x_{t+1} = x_t - \eta(\nabla F(x_t; \xi_t) + n_t)$$

- n<sub>t</sub> is an isotropic noise vector (e.g., Gaussian)
- Iteration complexity:  $\tilde{O}(d^p/\epsilon^4)$ , where  $p \ge 4$ , d is dimension
- Noisy SGD is the first work on finding local minimum by first order methods
- For high-dimensional optimization problems, d is large
- Assume  $F(x; \xi)$  has Lipschitz continuous Gradient and Hessian



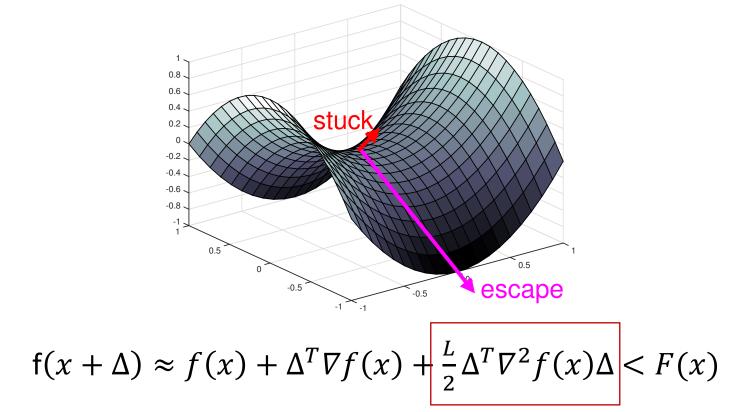
# More Related Work

- Using Full Gradient (FG) and Isotropic Noise: Perturbed GD (Jin et al., 2017)
  - Add Perturbation Around a Saddle Point  $\widetilde{x_t} = x_t + n_t$
  - Take Gradient Descent from  $\widetilde{x_t}$
  - Iteration Complexity:  $\tilde{O}(1/\epsilon^4)$ , which hides the team  $(\log d)^p$
- Using Hessian-vector product (HVP): (Allen-Zhu, 2017)[Natasha2]
  - Iteration Complexity:  $\tilde{O}$  (1/ $\epsilon^{3.5}$ )
  - The cost of computing HVP per-iteration could be as high as  $O(d^2)$
- Using both FG and HVP (Carmon et al., 2016; Agarwal et al., 2017)

Issue: FG and HVP could be more expensive than SG



## Motivation: How to Escape from Saddles?



- Saddle points have zero gradient, i.e.,  $\nabla f(x) = 0$
- Non-degenerate Hessian, i.e.  $\lambda_{min}(\nabla^2 f(x)) < 0$
- Negative eigenvector is a direction of escaping



## Negative Curvature

Suppose  $\lambda_{min}(\nabla^2 f(x)) \leq -\gamma$ , a direction  $v \in \mathbb{R}^d$  is called negative curvature (NC) direction if it satisfies (c > 0 is a constant)

$$v^T \nabla^2 f(x) v \leq -c\gamma$$
 and  $||v|| = 1$ 

- Find a NC direction v, update solution by  $x_{t+1} = x_t \eta v$
- Escape Saddles: we show  $f(x_t) f(x_{t+1}) \ge \Omega(\gamma^3)$



How to Find NC?

• Second-order Methods: Power Method and Lanczos method

$$v_0 = n //$$
 isotropic noise  
Iterate:  
 $v_{t+1} = (I - \eta \nabla^2 F(x)) v_t$ 

#### How to find NC without using HVP and Full Gradient?

Propose **NEON**: **NE**gative curvature **O**riginated from **N**oise



## NEON: A New Perspective of Noise Perturbation

- Adding Noise is for Extracting NC
  - x: around a saddle point
  - Inspired by Perturbed Gradient Descent (PGD):
    - $x_0 = x + e$ , noise *e* is from sphere of a Euclidean ball

• 
$$x_t = x_{t-1} - \eta \ \nabla F(x_{t-1}), t = 1, \cdots,$$

• An Equivalent Sequence: let  $u_t = x_t - x_t$ 

• 
$$u_t = u_{t-1} - \eta \nabla F(u_{t-1} + x)$$
  
 $\approx u_{t-1} - \eta \left[ \nabla F(u_{t-1} + x) - \nabla F(x) \right]$   
 $\approx u_{t-1} - \eta \nabla^2 F(x) u_{t-1} = \left[ I - \eta \nabla^2 F(x) \right] u_{t-1}$   
Lipschitz continuous Hessian when  $\|u_{t-1}\|$  is small: $\nabla F(u_{t-1} + x) - \nabla F(x) \ge \nabla^2 F(x) u_{t-1}$ 

 $\nabla F(x) \approx 0$ 

• Around Saddle Point: PGD  $\approx$  Power Method

NEON Update: Starting with a random noise  $u_0$ , the recurrence:  $u_{t+1} = u_t - \eta(\nabla F(x + u_t) - \nabla F(x))$  iteration complexity =  $\tilde{O}\left(\frac{1}{\gamma}\right)$ 



## NEON+: Another Perspective

- Recall the update of NEON:  $u_{t+1} = u_t \eta(\nabla F(x + u_t) \nabla F(x))$
- NEON is essentially an application of GD to decrease  $F_{\chi}(u)$ :

$$F_x(u) = F(x+u) - F(x) - \nabla F(x)^T u$$

Use Nesterov's Accelerated Gradient to decrease  $F_x(u)$ :  $y_{t+1} = u_t - \eta \nabla F_x(u_t), \ u_{t+1} = y_{t+1} + \zeta(y_{t+1} - y_t)$ 

For 
$$\zeta = 1 - \sqrt{\eta \gamma}$$
, # iteration can be reduced to  $t = \tilde{O}\left(\frac{1}{\sqrt{\gamma}}\right)$ 



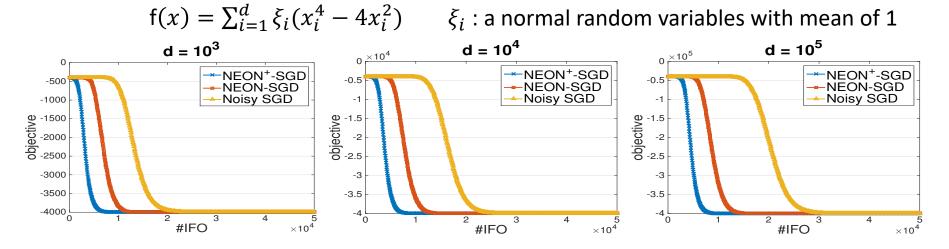
# Applications of NEON: Finding Local Minimum

#### Given a first-order alg. $\mathcal{A}$ (it can find a FSP)

- SGD, Stochastic Heavy-ball, Stochastic Nesterov's Accelerated Method
- Variance reduction methods, e.g., SCSG, SVRG

#### NEON + $A \rightarrow$ find a SSP point

• e.g., **NEON-SCSG** enjoy iteration complexity of  $\tilde{O}(1/\epsilon^{3.5})$  for finding  $(\epsilon, \sqrt{\epsilon})$ -SSP only using first-order information



#### Example: finding local minimum

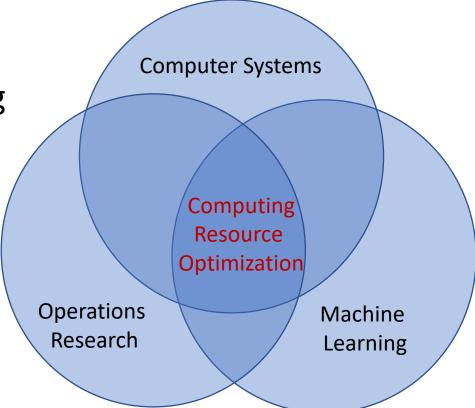


### Part 3: BPTune: Optimizing Buffer Pool Management for Large-Scale OLTP Database Clusters

J. Tan, T. Zhang, F. Li, J. Chen, Q. Zheng, P. Zhang, H. Qiao, Y. Shi, W. Cao, R. Zhang

A real system deployed for Alibaba database clusters Algorithm: large deviation, deep neural networks, active learning

> Large deviation on LRU: joint work with Quan, Ji and Shroff from The Ohio State University





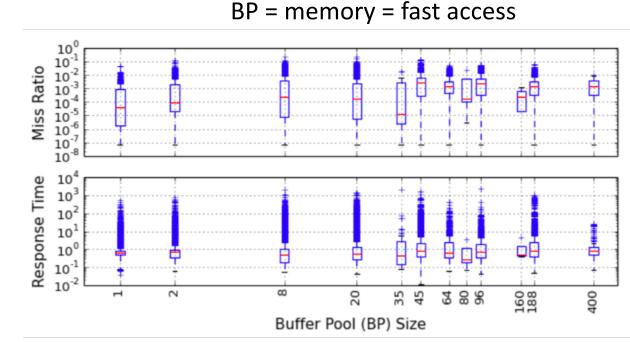
## "Personalization" for > 10,000 database instances

Measurements can NOT help much:

- 1. real BP usage pprox configured size
- 2. (miss ratio, response time)  $\leftrightarrow$  BP size

Current practice:

- 1. Overprovision (e.g., double BP size)
- 2. Use only a few BP sizes



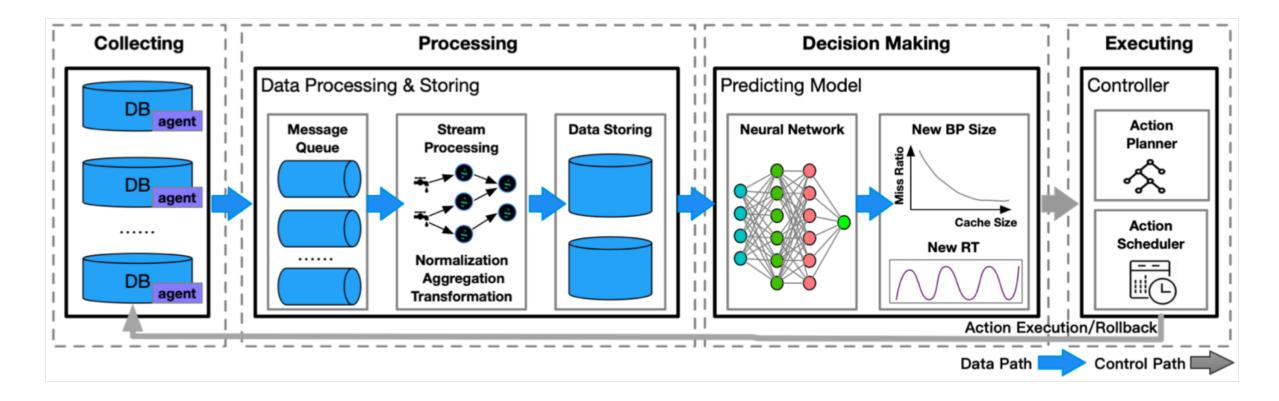
#### Challenges:

- 1. "Personalization" find the "best" BP size for each instance; manual optimization is not scalable.
- 2. Prediction estimate the response time for queries on each instance after changing its BP size?

Measurements on 10,000 database instances an instance = a database working unit Use only 11 different BP sizes by manual configurations



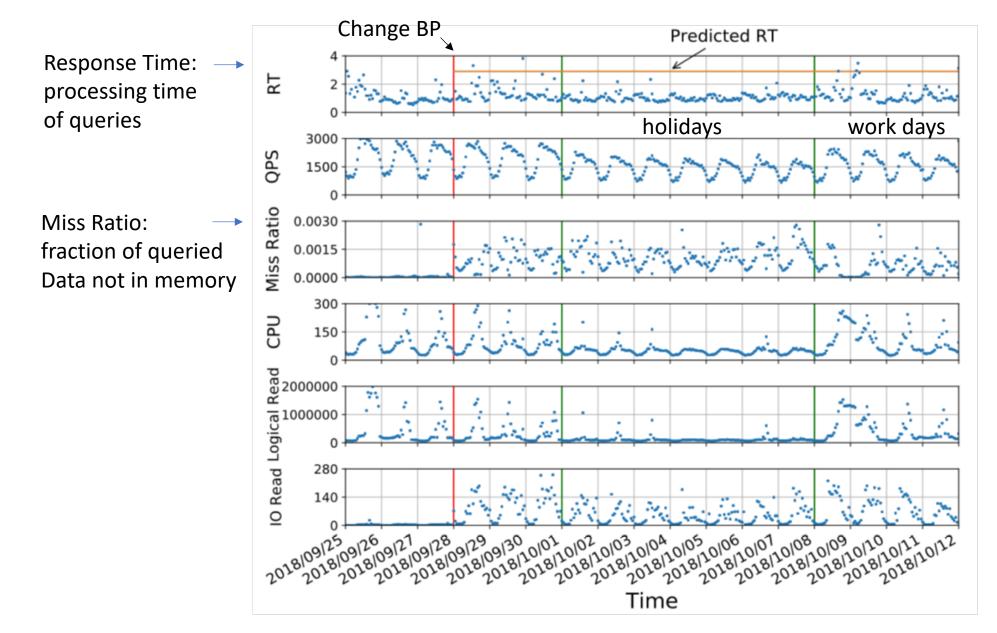
## BPTune architecture



Reduce > 20% BP memory, compared with manual configurations A bin-packing analysis shows BP is the bottleneck resource



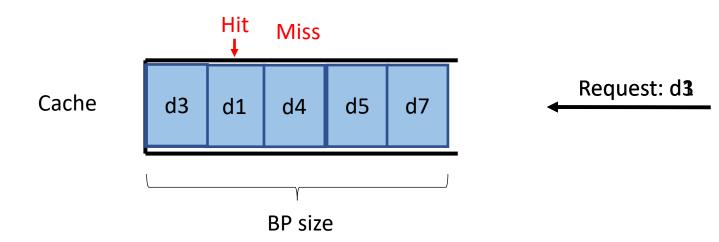
# Real experiment on an instance





# Today focus on LRU Caching algorithm

- Least recently used (LRU) algorithm (widely used: Memcached, Redis)
  - Store the most recently used data in the cache.
  - Easy to implement, adaptive to time-varying popularities
  - Q: What is the miss ratio of LRU?





## Goal & challenges

- Goal: characterize BP size = F (miss ratio)
  - Accurately and explicitly compute LRU miss ratio
  - A unified analysis solving all challenges below
- Challenges
  - Different data sizes
  - Time correlations
  - Multiple query flows on a single BP
  - Overlapped data across different flows
  - Long tailed data access probabilities e.g., Zipf's distribution, Weibull distribution

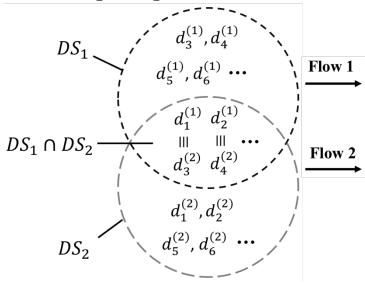


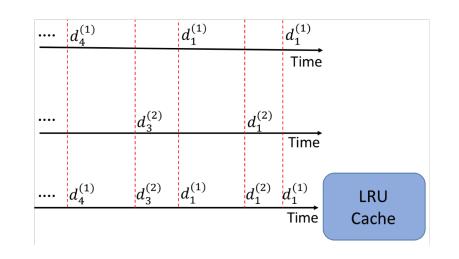
## Model

- *K* sets of data:  $DS_1, DS_2, ..., DS_K, DS_k = \{d_i^{(k)}, 1 \le i \le N_k\}$
- *K* data flows sharing a LRU cache:

Data flow k: a sequence of requests on the data set  $DS_k$ 

- Time correlation
  - $\{\Pi_t\}_{t\in\mathbb{R}}$ : a stationary and ergodic modulating process with finite states  $\{1, 2, ..., M\}$  and the stationary distribution  $(\pi_1, \pi_2, ..., \pi_M)$ .
  - Request rates, data popularities vary in different states.
- Goal:  $\mathbb{P}[Miss]$ .







### New functional representation

- Define the (conditional) popularities

  - $p_i^{(k)}$  and  $q_i^{(k)}$  can be very different.
- Functional relationship  $\Psi_k(\cdot)$  & finite support impacting  $\Theta_k(\cdot)$ :

For each flow k, for  $\forall \lambda > 1$ , let the size of the data set  $N_k \sim \lambda y$ . Find two eventually decreasing functions  $\Psi_k(\cdot)$  and  $\Theta_k(\cdot)$  that satisfy, as  $y \to \infty$ ,

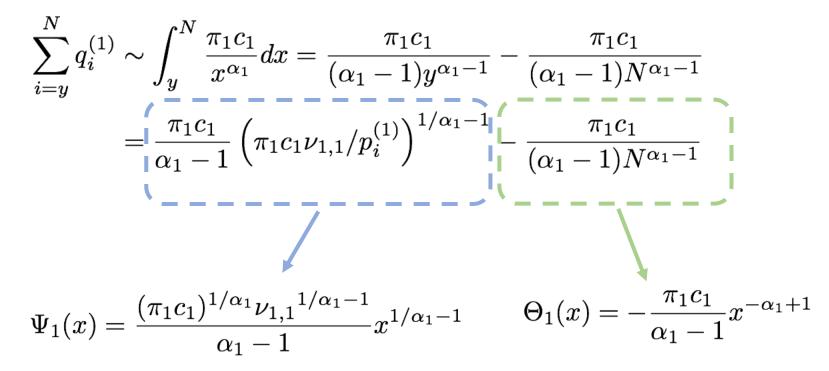
$$\sum_{i=y}^{N_k} q_i^{(k)} \sim \Psi_k \left( (p_y^{(k)})^{-1} \right) + \Theta_k(N_k)$$

where  $f(x) \sim g(x) \Leftrightarrow \lim_{x \to \infty} f(x)/g(x) = 1$ .



### New functional representation

• Example: If  $p_i^{(k)} = q_i^{(k)} = c_k/i^{\alpha_k}$ ,  $1 \le i \le N$ , k = 1, we have for flow 1:





### Main result

**Theorem** [Tan, Quan, Ji, Shroff]: Consider K flows without overlapped data that are modulated by the stationary and ergodic process  $\{\Pi_t\}_{t\in\mathbb{R}}$ . For flow k, if  $\Psi_k(x) \sim x^\beta l(x)$ , then under mild conditions, we have, as the cache size  $x \to \infty$ , for  $\forall \lambda > 0$ ,  $N_k = \lambda m^{\leftarrow}(x)$ ,

 $\mathbb{P}[\text{Miss}|\text{the request is from flow } k] \sim \beta \Gamma \left(\beta, m^{\leftarrow}(x) p_{N_k}^{(k)}\right) \Psi_k(m^{\leftarrow}(x)),$ 

where  $m^{\leftarrow}(x)$  is the inverse function of

$$m(x) = \sum_{k=1}^{K} \sum_{i=1}^{N_k} s_i^{(k)} \left( 1 - \exp\left(-\sum_{m=1}^{M} \pi_m \nu_{k,m} q_i^{(k,m)} x\right)\right).$$

Note:

- l(x) is any slowly varying function satisfying  $\lim_{x\to\infty} l(\lambda x)/l(x) = 1$  for any  $\lambda > 0$ . (e.g.,  $\log(x)$ , c, etc.)
- $\Gamma(\beta, s) = \int_{s}^{\infty} x^{\beta-1} e^{-x} dx$  is the incomplete gamma function.
- Quan, Ji and Shroff are with The Ohio State University



### Main result

**Corollary:** Consider one flow of unit-sized data. Assume  $q_i^{(1)} \sim c/i^{\alpha}$ ,  $1 \le i \le N$ . For  $\forall \lambda > 0$ ,  $N = \lambda m^{\leftarrow}(x)$ , we have, as the cache size  $x \to \infty$ ,

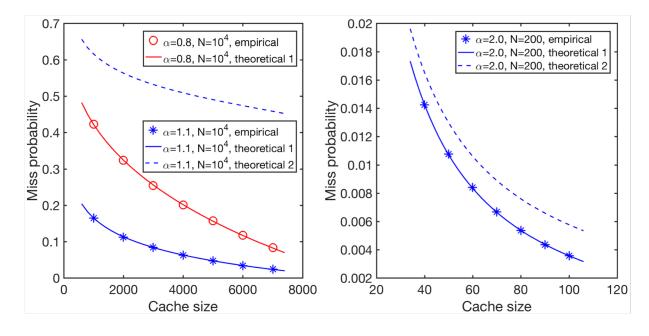
$$\mathbb{P}[\text{Miss}] \sim \frac{c^{1/\alpha}}{\alpha} \Gamma\left(1 - \frac{1}{\alpha}, \frac{cm^{\leftarrow}(x)}{N^{\alpha}}\right) m^{\leftarrow}(x)^{-1 + 1/\alpha},$$

where,  $m^{\leftarrow}(x)$  is the inverse function of

$$m(x) = \Gamma\left(1 - \frac{1}{\alpha}, \frac{cx}{N^{\alpha}}\right) (cx)^{1/\alpha} + N\left(1 - \exp\left(-\frac{cx}{N^{\alpha}}\right)\right).$$

Our result (labeled as 'theoretical 1')

Previous result (labeled as 'theoretical 2')





# Conclusion

#### > System for AI

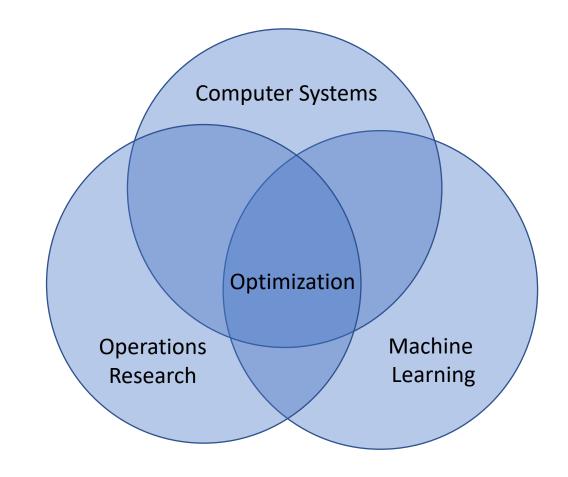
Part 1. Parallel restarted SGD (why model averaging works for Deep Learning?)

Part 2. Escaping saddle points in non-convex optimization (first-order stochastic algorithms to find second-order stationary points)

#### > AI for system

BPTune: intelligent database A real complex system deployment Combine OR/ML, e.g., pairwise DNN, active learning, heavy-tailed randomness ...

**Part 3**. Stochastic (large deviation) analysis for LRU caching





# Thank You! Questions?