





### Background

Federated Learning (FL) aims to collaborate these isolated data islands to complete the machine learning process.





Permutation Invariant Property of Neural Network (NN) : if we permute the parameters of NN properly, its output does not change. For example, for a 2 linear layer NN, we shuffle its intermediate neurons which is equivalent to multiply  $W_1$  by a permutation matrix  $\Pi$ . Then, the output of NN will be invariant by replacing  $W_2$  with  $W_2 \Pi^T$ .



Permutation Invariant Property may lead the neurons between models trained in clients to be *misplaced* and deteriorate coordinate-wise parameter averaging, especially under *non-i.i.d.* data across local clients.



- Training randomness may lead to neuron shuffling during local procedures
- Local non-i.i.d. data could exacerbate this phenomenon

# **Federated Learning with Position-Aware Neurons**

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# Methods





 $h_l = f_l (W_l h_{l-1} + b_l)$ Traditional Neurons:

Additive/Multiplicative PANs: adding or multiplying sinusoidal position encodings, i.e., *e*<sub>1</sub>. Neurons are related to their positions.

$$PAN_{+}: h_{l} = f_{l}(W_{l}h_{l-1} + b_{l} + \underline{e_{l}}), \qquad PAN_{+}: e$$

 $PAN_{\circ}: h_l = f_l((W_lh_{l-1} + b_l) \circ e_l),$  $\text{PAN}_{\circ}: e_{l,j} = 1 + A \sin(2\pi T j/J) \in [1 - A, 1 + A],$ 

Download

Turn off PANs



[Turn off PANs] Equal position encodings keep the permutation invariance property of NNs.



[Turn on PANs] Varying position encodings break the permutation invariance property of NNs.

Shuffle traditional Neurons and Additive/Multiplicative PANs:

| Traditional Neurons: |
|----------------------|
| Additive PANs:       |
| Multiplicative PANs: |

| $h_l = f_l (\Pi_l W_l \Pi_{l-1}^T h_{l-1})$           |
|-------------------------------------------------------|
| $h_{l,\mathrm{sf}} = f_l (\Pi_l W_l \Pi_{l-1}^T h_l)$ |
| $h_{l,\mathrm{sf}} = f_l ((\Pi_l W_l \Pi_{l-1}^T))$   |

PANs can remove Permutation Invariant Property!

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The reason of *Permutation Invariant Property* of

Traditional neurons does not consider their

Limit the *Permutation Invariant Property* of NNs:

Design of **Position-Aware Neurons** (PANs): fusing the position information into neuron's computation processes.

--NNs has no Permutation Invariant Property

 $e_{l,j} = A\sin\left(2\pi T j/J\right) \in [-A, A],$ 



Applying PANs to FL:

A straightforward implementation: replacing traditional neurons with PANs in FL.

Easy to implement.

Why does such a subtle improvement help? Pre-alignment!

$$(+\Pi_l b_l)$$
  
 $(l-1, \mathrm{sf} + \Pi_l b_l + e_l)$ 

$$h_{l-1,\mathrm{sf}} + \Pi_l b_l ) \odot e_l$$



Mnist, more datasets' results are in Supp.)



Figure 8. Preference vectors with PANs off/on, left vs. right. ( $\alpha =$ 1.0, VGG9 Conv6 on Cifar10, more results are shown in Supp.)



Figure 10. Comparisons under various levels of non-i.i.d. data on Cinic10. Smaller  $\alpha$  implies more non-i.i.d. data. (More datasets are shown in Supp.)



Figure 11. Comparisons under different FL scenes (K, E) based on FedAvg. (Scaffold results are shown in Supp.)

FL with PANs could improve several popular FL methods under various scenes, especially in the beginning of training or more non-i.i.d. scenes. Experimental details could be found in: <u>https://arxiv.org/abs/2203.14666</u>

Email: <u>lixc@lamda.nju.edu.cn</u> Homepage: <u>http://www.lamda.nju.edu.cn/lixc/</u> Code: https://github.com/lxcnju/FedRepo



### **Experiments**

OFF (A=0.0) [0.195]



Figure 7. Optimal assignment matrix with PANs off/on, left vs. right. ( $\alpha = 1.0, E = 20, VGG9$  Conv5 on Cifar10, more results are in Supp.)

We verify that turning on PANs could indeed align neurons better through some metrics including weight divergence, optimal assignment matrix, and preference vectors.

|                                       | FedAvg              | FedOpt                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   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| Lest Accuracy<br>0.40<br>0.35<br>0.30 | PANs OFF<br>PANs ON | 0.45 -<br>0.40 -<br>0.35 -<br>0.30 -<br>PANs OFF<br>PANs ON                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 0.50 -<br>0.45 -<br>0.40 -<br>PANs OFF<br>0.35 -<br>PANs ON<br>0.30 -                                             | 0.45 -<br>0.40 -<br>0.35 - PANs OFF<br>PANs ON<br>0.30 -                                                                                                       | 0.45 -<br>0.40 -<br>0.38 - PANs OFF<br>PANs ON<br>0.30 -                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| Lest Accuracy<br>0.70<br>0.65<br>0.60 | PANs OFF<br>PANs ON | 0.80 -<br>0.75 -<br>0.70 -<br>0.65 -<br>0.60 -<br>PANS OFF<br>PANS ON                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    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| Contract Accuracy 0.60                | PANs OFF<br>PANs ON | 0.60 - PANs OFF<br>PANs ON                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | 0.60 -<br>0.60 -<br>0.55 -<br>0.50 -<br>PANs OFF<br>PANS ON                                                       | 0.65 -<br>0.60 -<br>0.55 -<br>0.50 -<br>PANs OFF<br>PANS ON                                                                                                    | 0.65 - PANs OFF<br>0.55 - PANs OFF<br>0.55 - PANs ON                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| Lest Accuracy<br>0.50<br>0.50         | PANS OFF<br>PANS ON | 0.70 -<br>0.65 -<br>PANs OFF<br>PANs OFF<br>PANs OF<br>0.66 -<br>0.55 -<br>0 | 0.70 -<br>0.65 -<br>0.60 -<br>0.55 -<br>PANs OFF<br>PANs ON<br>0.50 -<br>PANs ON<br>0.50 -<br>PANs OFF<br>PANS ON | 0.65 -<br>0.60 -<br>0.55 -<br>PANs OFF<br>PANs OF<br>PANs ON<br>0.50 -<br><sup>5</sup> $\phi$ , | 0.70<br>0.65<br>0.65<br>0.65<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55<br>0.55 |

Figure 9. Comparison results on non-i.i.d. data ( $\alpha$ =0.1). Rows show datasets and columns show FL algorithms. PANs could universally improve these algorithms. (More datasets are shown in Supp.)



Figure 24. Comparison results on non-i.i.d. data ( $\alpha$ =0.1). Rows show datasets and columns show FL algorithms. PANs could universally improve these algorithms.

## **Contributions**

• We proposes PANs to control permutation invariance property of NNs. • We apply PANs to FL, which binds neurons in positions and pre-aligns parameters for better coordinate-wise parameter averaging.



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