Efficient Rectification of Neuro-Symbolic Reasoning Inconsistencies by Abductive Reflection

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NeSy AI

Neuro-Symbolic AI models human dual-process cognition,

<-> System 1 (Intuitive) Neural Networks Fast, yet often incorrect

Symbolic Reasoning <-> System 2 (Algorithmic) Slow, leverage domain knowledge

A Humans can easily use System 2 to rectify System 1 errors, but this is challenging for AI systems.

Inspiration from **Human Cognition**

Abductive Learning

A framework that preserves full expressive power in both neural networks and symbolic reasoning.

Learning And Mining from DatA

IAAI-25





EAAI-25

Key in human cognition: **Cognitive Reflection**,

promptly forms upon System 1 response, detects which part may contain errors and invokes System 2 to focus on.



- directly maps x into output \hat{y} ;
- \mathcal{KB} : holds constraints between symbols in y, and can perform reasoning (e.g., abduction) with a symbolic solver.

 Λ Involves consistency optimization before abduction, requiring large number of queries to \mathcal{KB} that increases exponentially with data scale, largely hindering efficiency.

Our Method: Abductive Learning with Reflection Mechanism (ABL-Refl)

Replaces the time-consuming consistency optimization in ABL with an efficient reflection mechanism:

Incorporating a reflection layer R after the body block f_1 , generating a reflection vector r, each element, r_i , acts as a classifier to indicate whether the corresponding intuitive output \hat{y}_i is an error leading to inconsistencies with \mathcal{KB} (flagged as 1) or not (0).



Key features

- Reflection vector is generated **simultaneously** with intuitive output;
- During each time of inference, \mathcal{KB} is only invoked only **once**, regardless of data scale;

Training Paradigm

For the reflection, we want to achieve two goals:

1. Leverage training information directly from \mathcal{KB} :

Reflection mechanism is trained using information **directly from** \mathcal{KB} .

The reflection can be seen as an efficient attention mechanism, generated from neural network, and abduced from \mathcal{KB} , promptly directing the focus for symbolic search on a reduced space.

Experiments

Method	Training Time (min)	Inference Time (s)	Inference Accuracy
RRN	$114.8 {\pm} 7.8$	$0.19{\pm}0.01$	73.1±1.2
CL-STE	173.6±9.9	$0.19{\scriptstyle\pm0.02}$	76.5 ± 1.8
SATNet	$140.3 {\pm} 6.8$	$0.11{\scriptstyle \pm 0.01}$	74.1 ± 0.4
ABL-Refl	$109.8{\scriptstyle\pm10.8}$	$0.22{\pm}0.02$	97.4±0.3

Achieves far better **reasoning accuracy** in only **a few epochs** (<10);

KB Form	Solver	Method	Inference Accuracy	Inference Time (s)		
				NN Time	Abduction Time	Overall Time
Propositional logic	MiniSAT	Solver only	100 ± 0	-	0.227 ± 0.024	$0.227{\scriptstyle\pm0.024}$
		ABL-Refl	97.4 ± 0.3	$0.021 {\pm} 0.004$	0.196±0.015	0.217±0.019

indicates how well r identifies errors in \hat{y} , use REINFORCE to maximize it $\rightarrow L_{con}(x)$.

2. Constrain the size of reasoning, invoking it judiciously: $L_{size}(\boldsymbol{x}) = \max(0, \ C - \frac{1}{n} \sum_{i=1}^{n} (1 - R(f_1(\boldsymbol{x}))_i))^2$

Overall loss function:

$$\mathcal{L} = \frac{1}{|D_l|} \sum_{(\boldsymbol{x}, \boldsymbol{y}) \in D_l} L_{label}(\boldsymbol{x}, \boldsymbol{y}) \\ + \frac{1}{|D_l \cup D_u|} \sum_{\boldsymbol{x} \in D_l \cup D_u} (\alpha L_{con}(\boldsymbol{x}) + \beta L_{size}(\boldsymbol{x}))$$

Reflection, do not need information from data label

First-order logic	Prolog with CLP(FD)	Solver only	100 ± 0	-	105.81 ± 5.62	105.81 ± 5.62
		ABL-Refl	97.4 ± 0.3	$0.021 {\pm} 0.004$	$31.86{\scriptstyle \pm 1.88}$	$31.88{\scriptstyle \pm 1.89}$

Enhances the **efficiency of symbolic solvers**, applicable for both propositional logic (MiniSAT) and first-order logic (Prolog).

Conclusion

ABL-Refl efficiently integrates neural networks with symbolic reasoning while preserving their integrity, enabling broad uses requiring rigorous reasoning.

