南京大学 人工智能学院
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Lecture 10: Learning 1

Previously...

Search

- Path-based search
- Iterative improvement search

Knowledge

- Propositional Logic
- First Order Logic (FOL)

Uncertainty

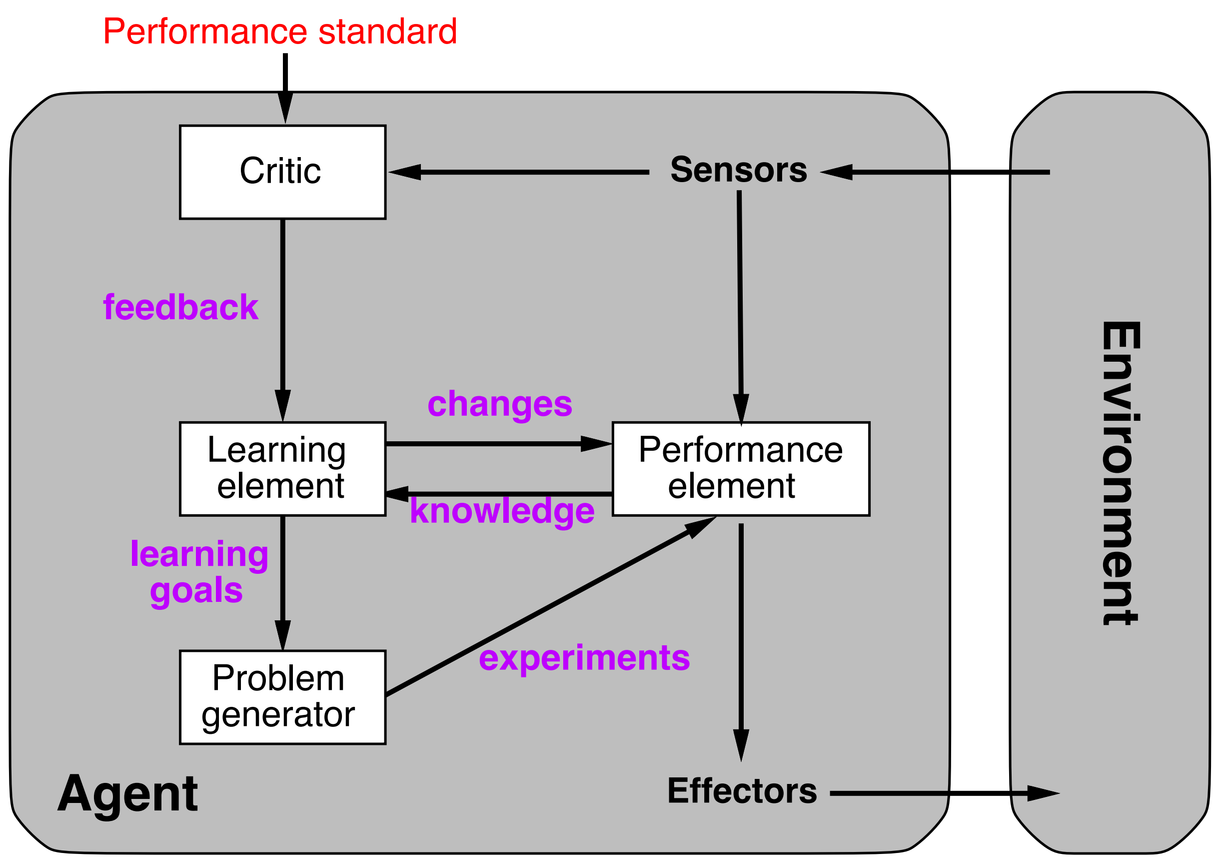
- Bayesian network

Learning

Learning is essential for unknown environments,
i.e., when designer lacks omniscience

Learning is useful as a system construction method,
i.e., expose the agent to reality rather than trying to write it down

Learning modifies the agent's decision mechanisms to improve performance



Inductive Learning

Simplest form: learn a function from examples (**tabula rasa**)

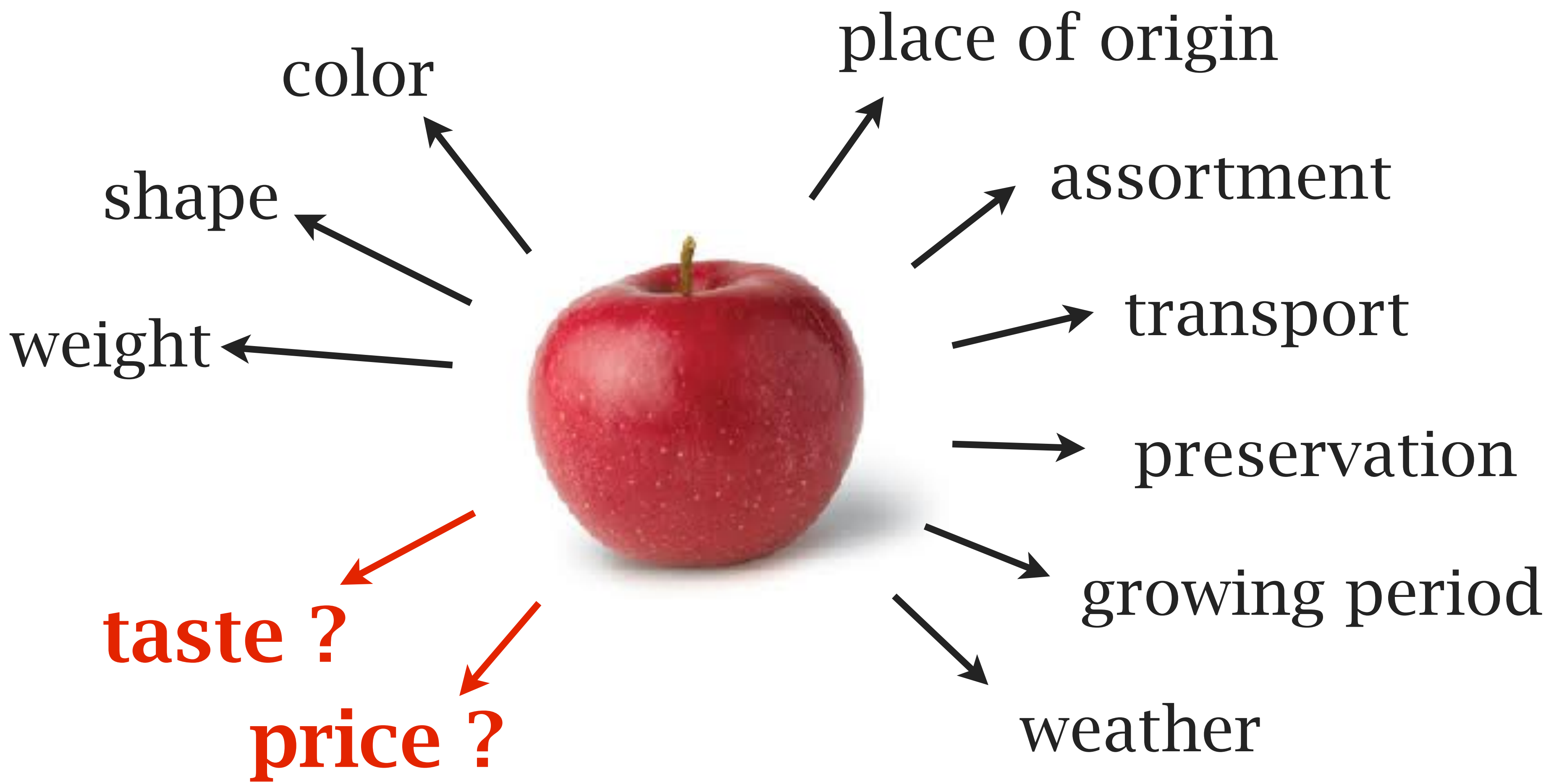
f is the target function

An example is a pair x, $f(x)$, e.g., $\begin{array}{c|c|c} O & O & X \\ \hline & X & \\ \hline X & & \end{array}\ ,\quad +1$

Problem: find a(n) hypothesis h
such that $h \approx f$
given a training set of examples

(**This is a highly simplified model of real learning:**
- **Ignores prior knowledge**
- **Assumes a deterministic, observable "environment"**
- **Assumes examples are given**
- **Assumes that the agent wants to learn f—why?**)

Attribute-based representations



Attribute-based representations

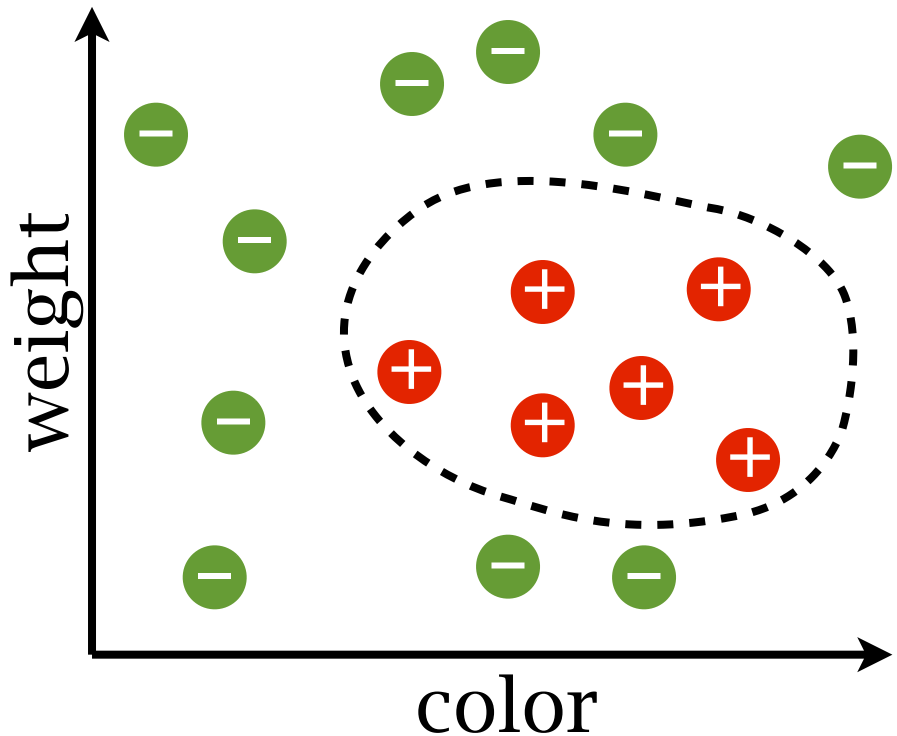
Examples described by attribute values (Boolean, discrete, continuous, etc.)
E.g., situations where I will/won't wait for a table:

Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	*T*	*F*	*F*	*T*	*Some*	*$$$*	*F*	*T*	*French*	*0–10*	*T*
X_2	*T*	*F*	*F*	*T*	*Full*	*$*	*F*	*F*	*Thai*	*30–60*	*F*
X_3	*F*	*T*	*F*	*F*	*Some*	*$*	*F*	*F*	*Burger*	*0–10*	*T*
X_4	*T*	*F*	*T*	*T*	*Full*	*$*	*F*	*F*	*Thai*	*10–30*	*T*
X_5	*T*	*F*	*T*	*F*	*Full*	*$$$*	*F*	*T*	*French*	*>60*	*F*
X_6	*F*	*T*	*F*	*T*	*Some*	*$$*	*T*	*T*	*Italian*	*0–10*	*T*
X_7	*F*	*T*	*F*	*F*	*None*	*$*	*T*	*F*	*Burger*	*0–10*	*F*
X_8	*F*	*F*	*F*	*T*	*Some*	*$$*	*T*	*T*	*Thai*	*0–10*	*T*
X_9	*F*	*T*	*T*	*F*	*Full*	*$*	*T*	*F*	*Burger*	*>60*	*F*
X_{10}	*T*	*T*	*T*	*T*	*Full*	*$$$*	*F*	*T*	*Italian*	*10–30*	*F*
X_{11}	*F*	*F*	*F*	*F*	*None*	*$*	*F*	*F*	*Thai*	*0–10*	*F*
X_{12}	*T*	*T*	*T*	*T*	*Full*	*$*	*F*	*F*	*Burger*	*30–60*	*T*

Classification of examples is positive (T) or negative (F)

Learning task: Classification

Features: color, weight
Label: taste is sweet (positive/+) or not (negative/-)



(color, weight) → sweet ?

$\mathcal{X} \rightarrow \{-1, +1\}$

ground-truth function f

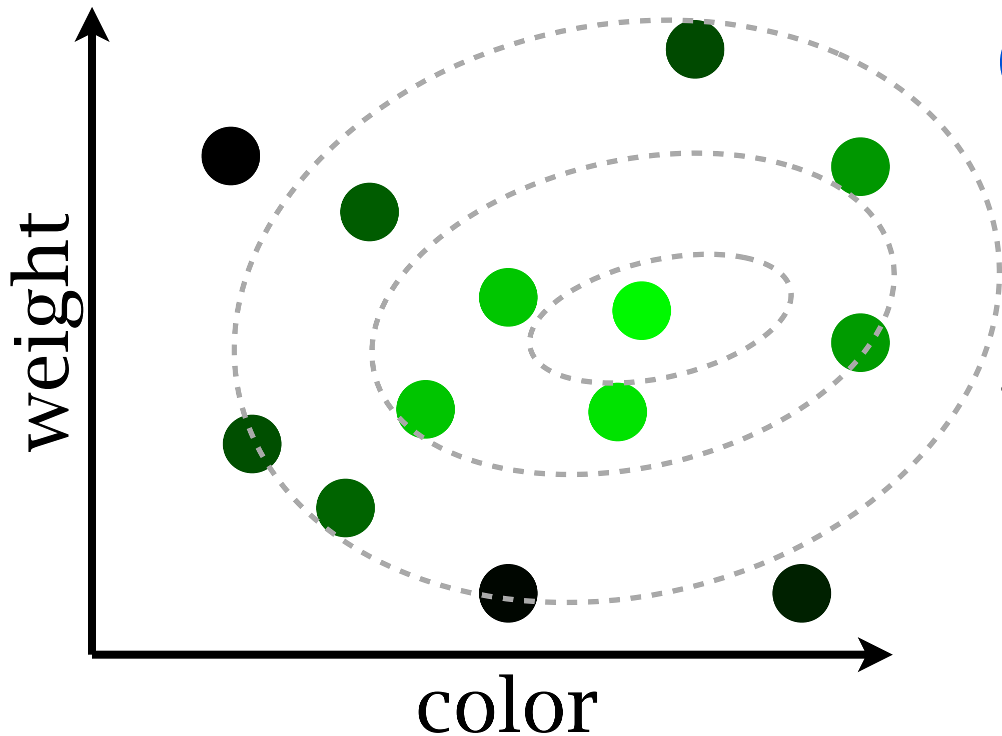
examples/training data:
$\{(\boldsymbol{x}_1, y_1), \ldots, (\boldsymbol{x}_m, y_m)\}$

$y_i = f(\boldsymbol{x}_i)$

learning: find an f' that is close to f

Learning task: Regression

Features: color, weight
Label: price [0,1]



(color, weight) → price

$\mathcal{X} \rightarrow [0, +1]$

ground-truth function f

examples/training data:
$\{(\boldsymbol{x}_1, y_1), \ldots, (\boldsymbol{x}_m, y_m)\}$

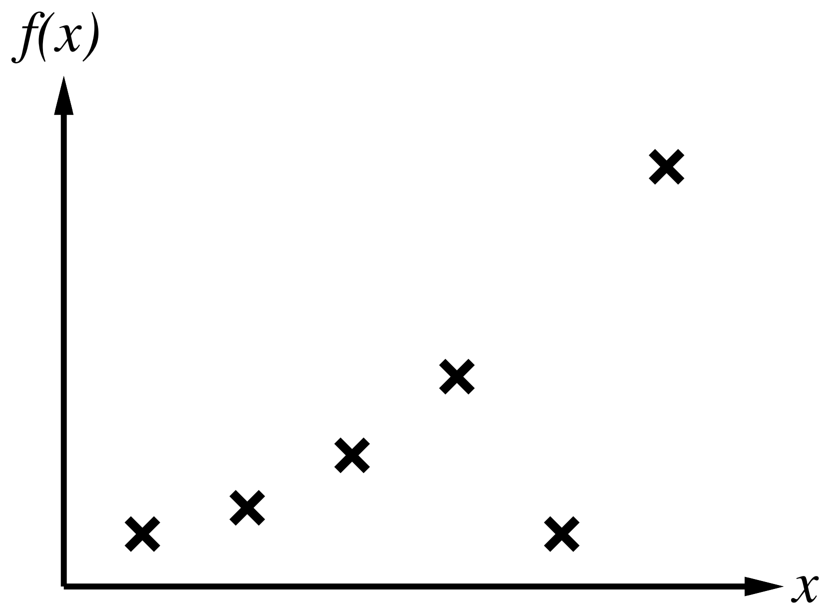
$y_i = f(\boldsymbol{x}_i)$

learning: find an f' that is close to f

Learning task: Regression

Construct/adjust h to agree with f on training set
(h is consistent if it agrees with f on all examples)

E.g., curve fitting:



Learning task: Regression

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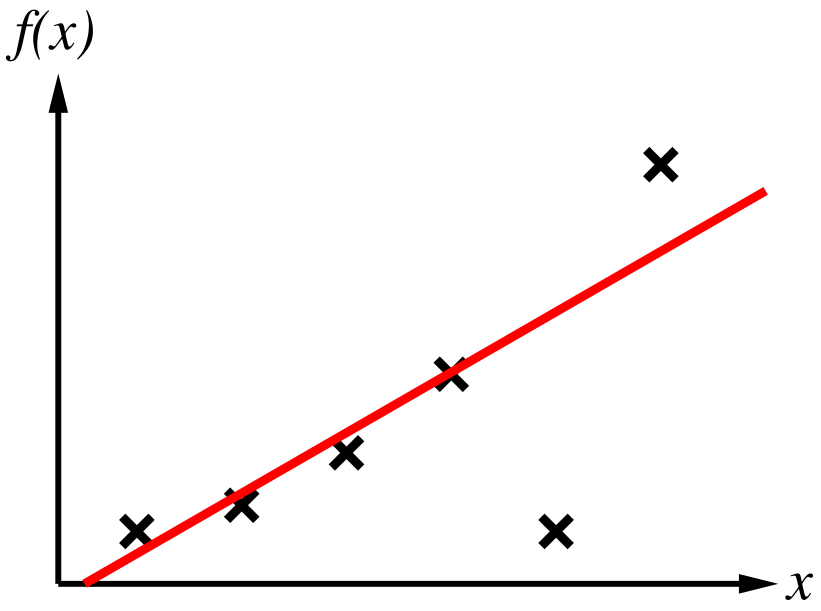

Learning task: Regression

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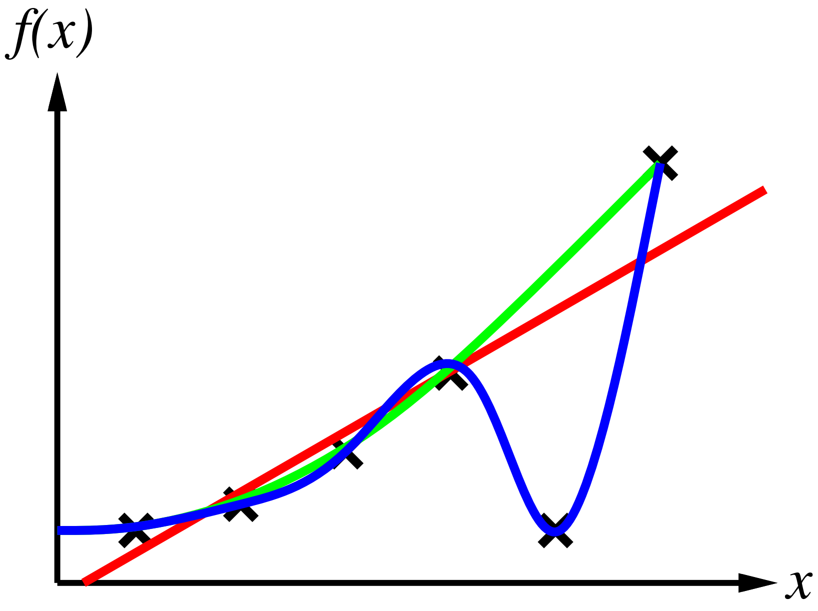

Learning task: Regression

Construct/adjust h to agree with f on training set
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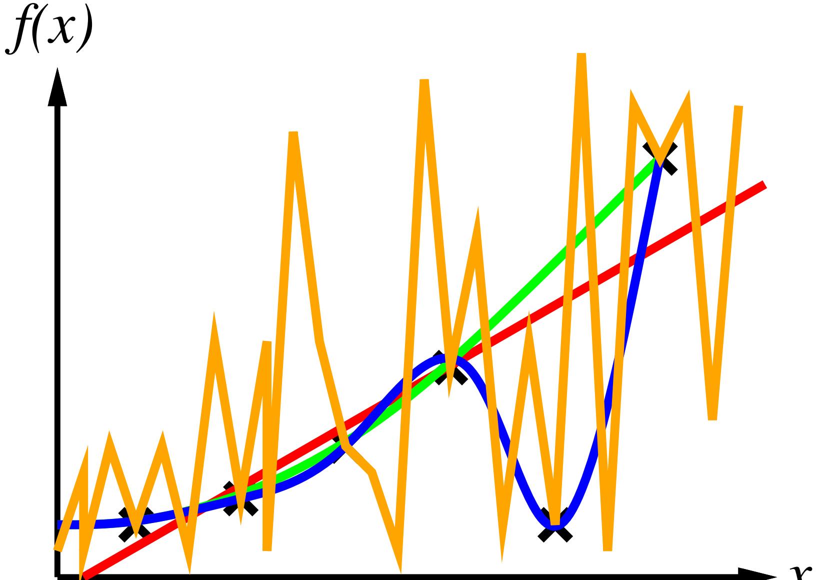
E.g., curve fitting:



how to learn? why it can learn?

Learning algorithms

Decision tree

Neural networks

Linear classifiers

Bayesian classifiers

Lazy classifiers

...

Why different classifiers?

heuristics

viewpoint

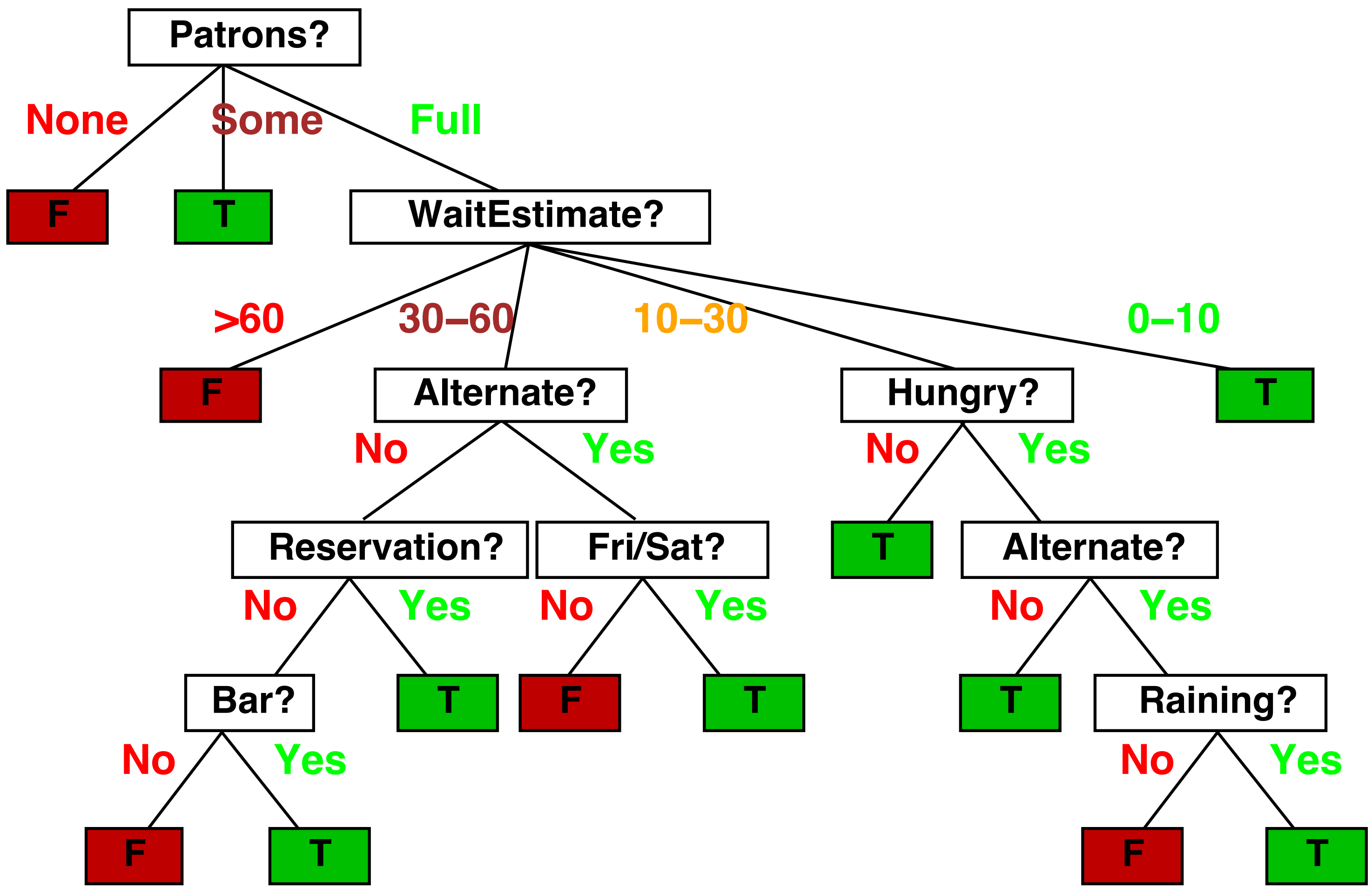
performance

Decision tree learning

what is a decision tree

One possible representation for hypotheses

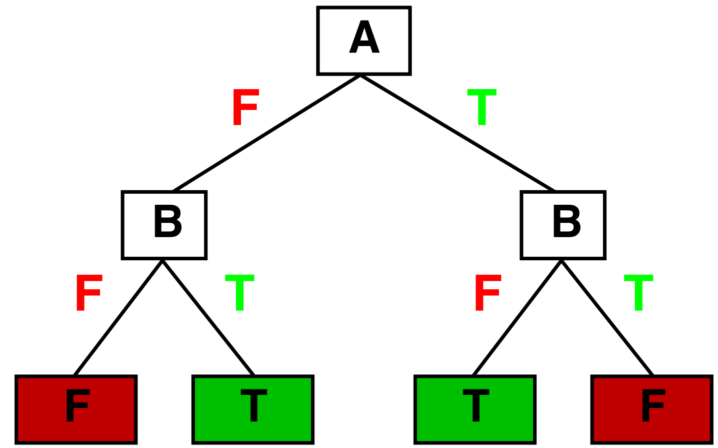
E.g., here is the "true" tree for deciding whether to wait:



Expressiveness

Decision trees can express any function of the input attributes.
E.g., for Boolean functions, truth table row $\longrightarrow$ path to leaf:

A	B	A xor B
F	F	F
F	T	T
T	F	T
T	T	F



Trivially, there is a consistent decision tree for any training set w/ one path to leaf for each example (unless f nondeterministic in x) but it probably won't generalize to new examples

Prefer to find more **compact** decision trees

Hypothesis spaces (all possible trees)

How many distinct decision trees with n Boolean attributes??

= number of Boolean functions
= number of distinct truth tables with 2^n rows $= 2^{2^n}$

E.g., with 6 Boolean attributes, there are 18,446,744,073,709,551,616 trees

How many purely conjunctive hypotheses (e.g., $Hungry \wedge \neg Rain$)??

Each attribute can be in (positive), in (negative), or out
$\Rightarrow$ 3^n distinct conjunctive hypotheses

More expressive hypothesis space
– increases chance that target function can be expressed
– increases number of hypotheses consistent w/ training set
$\Rightarrow$ may get worse predictions

Decision tree learning algorithm

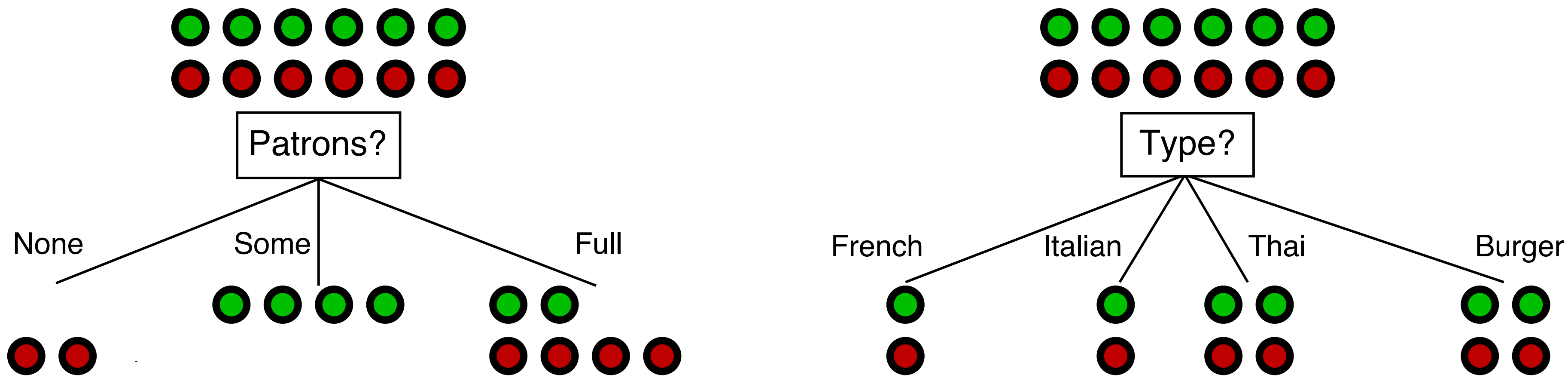
Aim: find a small tree consistent with the training examples

Idea: (recursively) choose "most significant" attribute as root of (sub)tree

```
function DTL(examples, attributes, default) returns a decision tree
    if examples is empty then return default
    else if all examples have the same classification then return the classification
    else if attributes is empty then return MODE(examples)
    else
        best ← CHOOSE-ATTRIBUTE(attributes, examples)
        tree ← a new decision tree with root test best
        for each value v_i of best do
            examples_i ← {elements of examples with best = v_i}
            subtree ← DTL(examples_i, attributes − best, MODE(examples))
            add a branch to tree with label v_i and subtree subtree
        return tree
```

Choosing an attribute

Idea: a good attribute splits the examples into subsets that are (ideally) "all positive" or "all negative"



Patrons? is a better choice—gives **information** about the classification

Information

Information answers questions

The more clueless I am about the answer initially, the more information is contained in the answer

Scale: 1 bit = answer to Boolean question with prior $\langle 0.5, 0.5\rangle$

Information in an answer when prior is $\langle P_1, \ldots, P_n\rangle$ is

$$H(\langle P_1, \ldots, P_n\rangle) = \sum_{i=1}^{n} - P_i \log_2 P_i$$

(also called entropy of the prior)

Information

Suppose we have p positive and n negative examples at the root

$\Rightarrow$ $H(\langle p/(p+n), n/(p+n)\rangle)$ bits needed to classify a new example

E.g., for 12 restaurant examples, $p = n = 6$ so we need 1 bit

An attribute splits the examples E into subsets E_i, each of which (we hope) needs less information to complete the classification

Let E_i have p_i positive and n_i negative examples

$\Rightarrow$ $H(\langle p_i/(p_i+n_i), n_i/(p_i+n_i)\rangle)$ bits needed to classify a new example

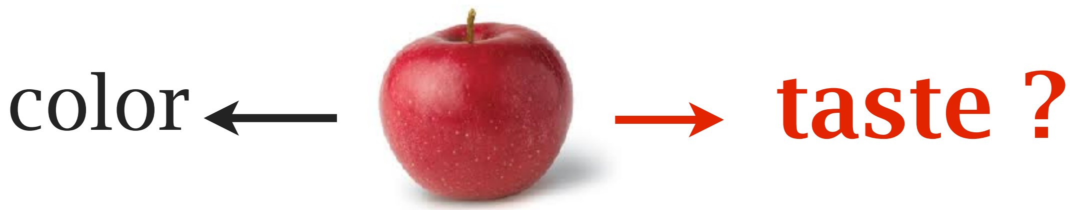
$\Rightarrow$ **expected** number of bits per example over all branches is

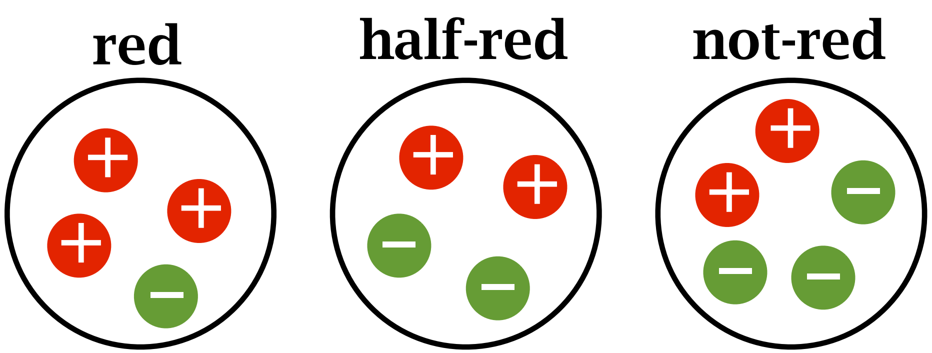
$$\Sigma_i \; \frac{p_i + n_i}{p + n} H(\langle p_i/(p_i + n_i), n_i/(p_i + n_i)\rangle)$$

For $Patrons?$, this is 0.459 bits, for $Type$ this is (still) 1 bit

$\Rightarrow$ choose the attribute that minimizes the remaining information needed

Example





id	color	taste
1	red	sweet
2	red	sweet
3	half-red	sweet
4	not-red	sweet
5	not-red	not-sweet
6	half-red	sweet
7	red	not-sweet
8	not-red	not-sweet
9	not-red	sweet
10	half-red	not-sweet
11	red	sweet
12	half-red	not-sweet
13	not-red	not-sweet

information gain:

entropy before split: $H(X) = -\sum_i ratio(class_i) \ln ratio(class_i) = 0.6902$

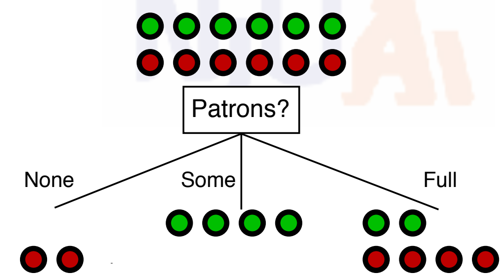
entropy after split: $I(X;\text{split}) = \sum_i ratio(split_i) H(split_i)$

$$= \frac{4}{13} 0.5623 + \frac{4}{13} 0.6931 + \frac{5}{13} 0.6730 = 0.6452$$

information gain:

$$Gain(X; split) = H(X) - I(X; \text{split}) = 0.045$$

Decision tree learning algorithm



Aim: find a small tree consistent with the training examples

Idea: (recursively) choose "most significant" attribute as root of (sub)tree

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            subtree ← DTL(examples_i, attributes − best, MODE(examples))
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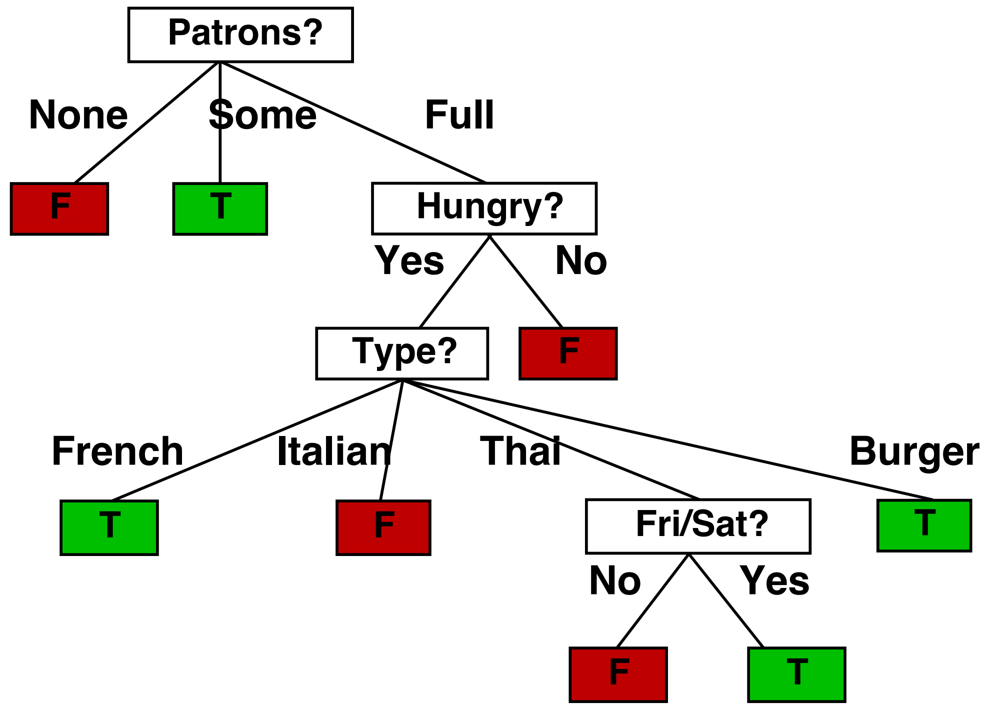
        return tree
```

Example of learned tree

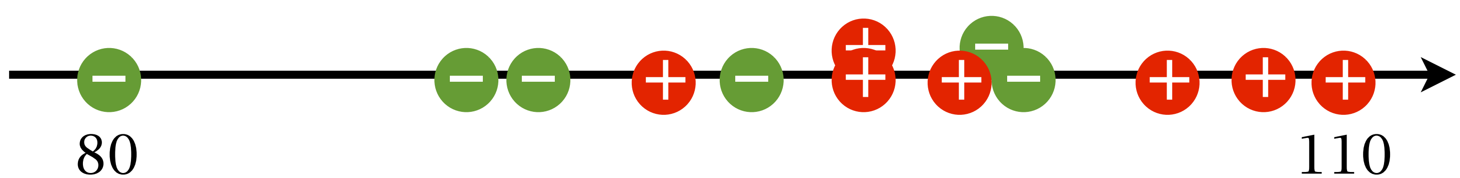
Decision tree learned from the 12 examples:



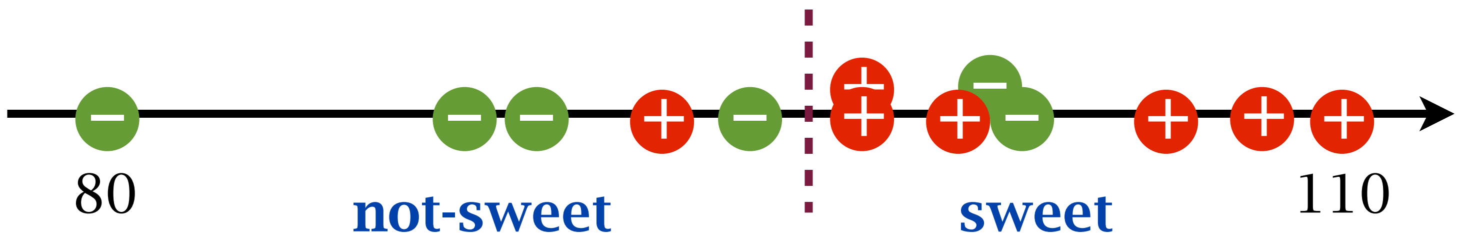
Substantially simpler than "true" tree—a more complex hypothesis isn't justified by small amount of data

Continuous attribute

id	weight	taste
1	110	sweet
2	105	sweet
3	100	sweet
4	93	sweet
5	80	not-sweet
6	98	sweet
7	95	not-sweet
8	102	not-sweet
9	98	sweet
10	90	not-sweet
11	108	sweet
12	101	not-sweet
13	89	not-sweet



Continuous attribute



for every split point

information gain:

entropy before split: $H(X) = -\sum_i ratio(class_i) \ln ratio(class_i) = 0.6902$

entropy after split: $I(X;\text{split}) = \sum_i ratio(split_i) H(split_i)$

$$= \frac{5}{13} 0.5004 + \frac{8}{13} 0.5623 = 0.5385$$

information gain:

$$Gain(X; split) = H(X) - I(X;\text{split}) = 0.1517$$

Non-generalizable feature

id	color	weight	taste
1	red	110	sweet
2	red	105	sweet
3	half-red	100	sweet
4	not-red	93	sweet
5	not-red	80	not-sweet
6	half-red	98	sweet
7	red	95	not-sweet
8	not-red	102	not-sweet
9	not-red	98	sweet
10	half-red	90	not-sweet
11	red	108	sweet
12	half-red	101	not-sweet
13	not-red	89	not-sweet

the system may not know non-generalizable features

$$\mathrm{IG} = H(X) - 0$$

Gain ratio as a correction:

$$\text{Gain ratio}(X) = \frac{H(X) - I(X;\text{split})}{IV(\text{split})}$$

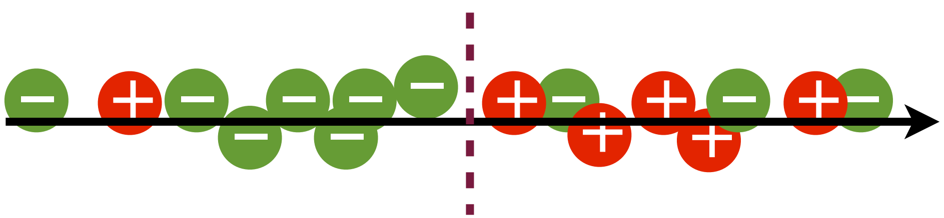
$$IV(\text{split}) = H(\text{split})$$

Alternative to information: Gini index

Gini index (CART):

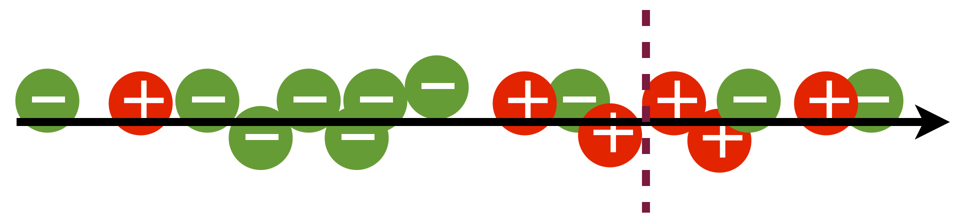
Gini: $Gini(X) = 1 - \sum_i p_i^2$

Gini after split: $\frac{\#\text{left}}{\#\text{all}} Gini(\text{left}) + \frac{\#\text{right}}{\#\text{all}} Gini(\text{right})$



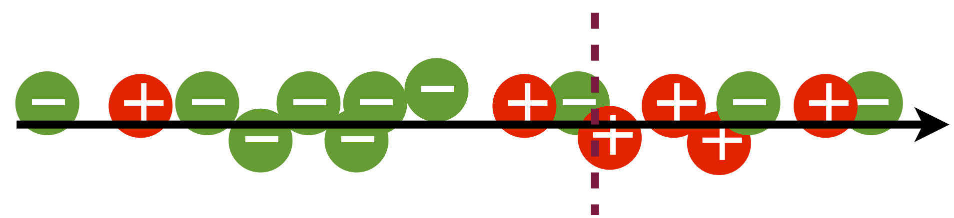
$\text{IG} = H(X) - 0.5192$

$Gini = 0.3438$



$\text{IG} = H(X) - 0.6132$

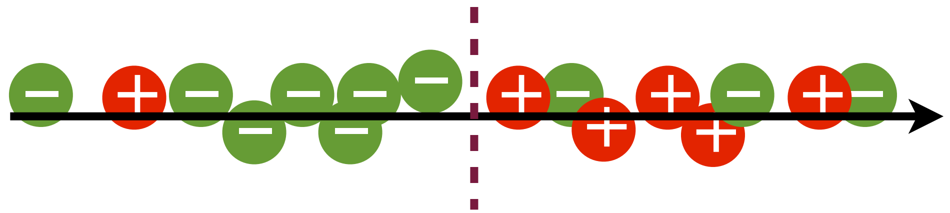
$Gini = 0.4427$



$\text{IG} = H(X) - 0.5514$

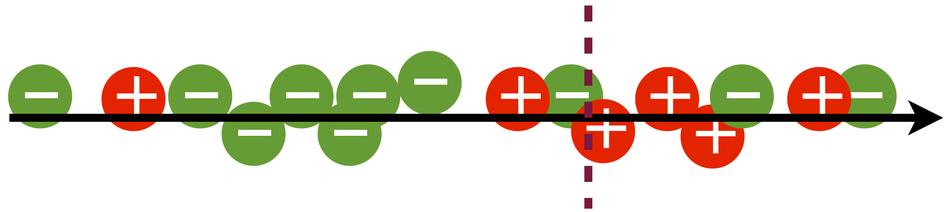
$Gini = 0.3667$

Training error v.s. Information gain



training error: 4

information gain: $\mathrm{IG} = H(X) - 0.5192$



training error: 4

information gain: $\mathrm{IG} = H(X) - 0.5514$

training error is less smooth

Decision tree learning algorithms

ID3: information gain

Ross Quinlan

C4.5: gain ratio, handling missing values

CART: gini index

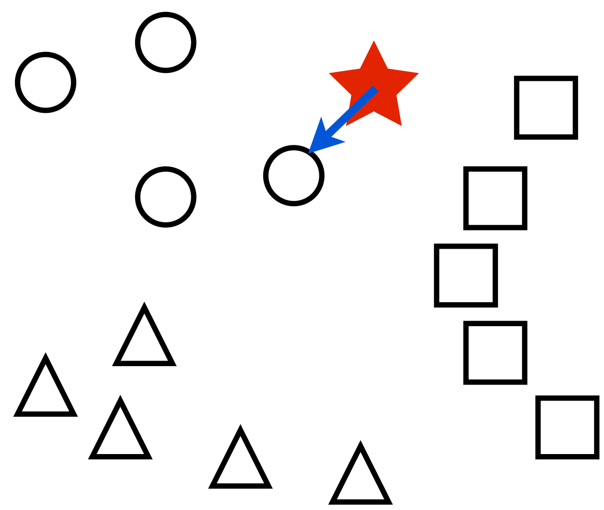
Leo Breiman 1928-2005

Jerome H. Friedman

Nearest Neighbor Classifier

Nearest neighbor

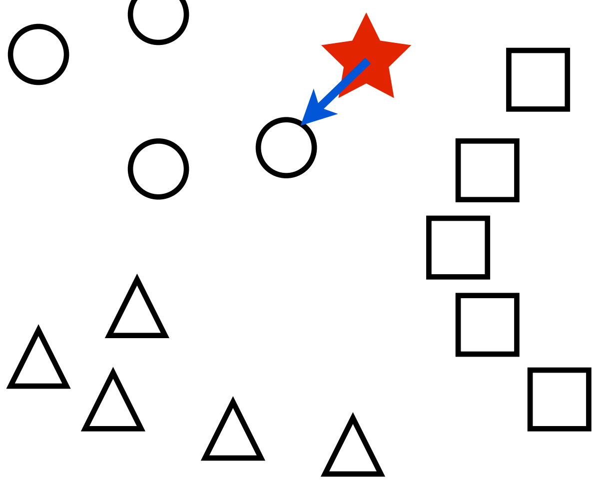
what looks similar are similar



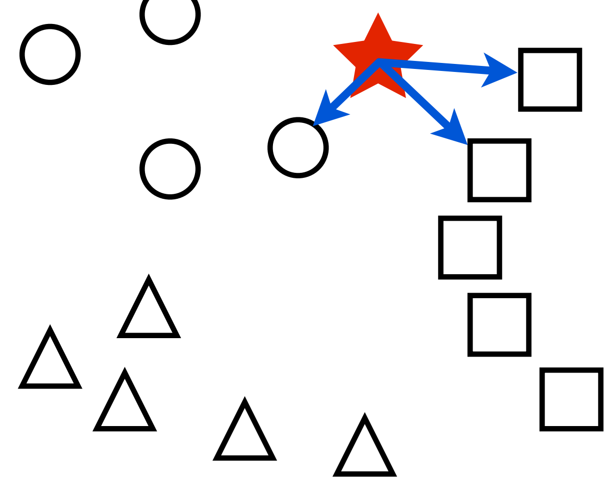
Nearest neighbor

for classification:

1-nearest neighbor:



k-nearest neighbor:

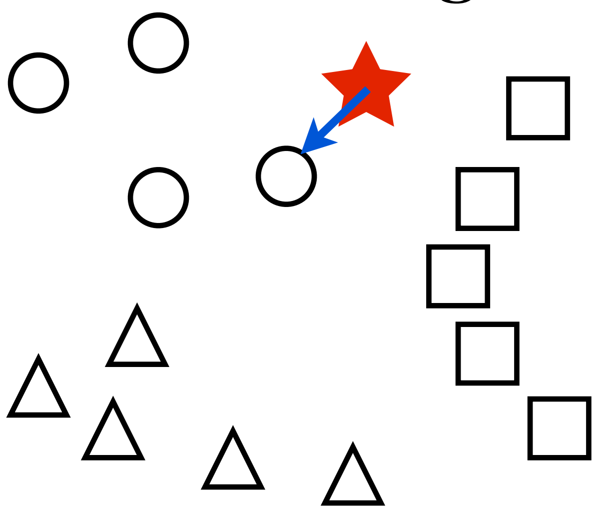


Predict the label as that of the NN
or the (weighted) majority of the k-NN

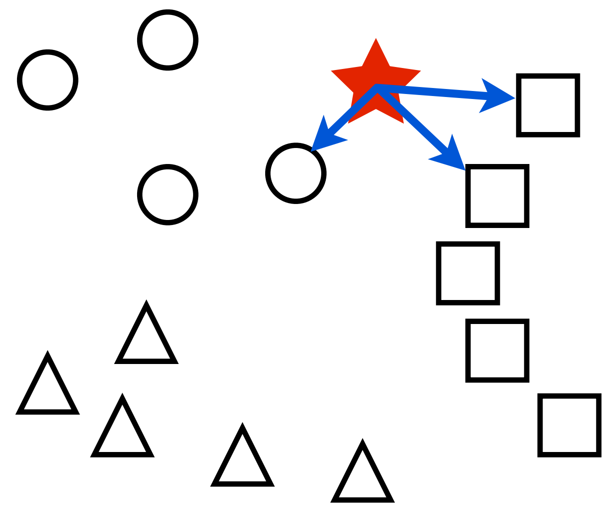
Nearest neighbor

for regression:

1-nearest neighbor:



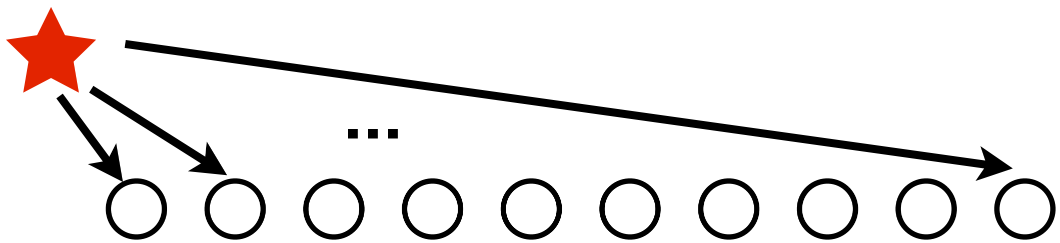
k-nearest neighbor:



Predict the label as that of the NN
or the (weighted) *average* of the k-NN

Search for the nearest neighbor

Linear search



n times of distance calculations

$O(dn \ln k)$

d is the dimension, *n* is the number of samples

Nearest neighbor classifier

- as classifier, asymptotically less than 2 times of the optimal Bayes error
- naturally handle multi-class
- no training time
- nonlinear decision boundary

- slow testing speed for a large training data set
- have to store the training data
- sensitive to similarity function

nonparametric method

Naive Bayes Classifier

Bayes rule

classification using posterior probability

for binary classification

$$f(x) = \begin{cases} +1, & P(y=+1 \mid \boldsymbol{x}) > P(y=-1 \mid \boldsymbol{x}) \\ -1, & P(y=+1 \mid \boldsymbol{x}) < P(y=-1 \mid \boldsymbol{x}) \\ \text{random}, & otherwise \end{cases}$$

in general

$$\begin{aligned} f(x) &= \arg\max_y P(y \mid \boldsymbol{x}) \\ &= \arg\max_y P(\boldsymbol{x} \mid y)P(y)/P(\boldsymbol{x}) \\ &= \arg\max_y P(\boldsymbol{x} \mid y)P(y) \end{aligned}$$

how the probabilities be estimated

Naive Bayes

$$f(x) = \arg\max_y P(\boldsymbol{x} \mid y) P(y)$$

estimation the a priori by frequency:

$$P(y) \leftarrow \tilde{P}(y) = \frac{1}{m} \sum_i I(y_i = y)$$

Consider a very simple case

id	color	taste
1	red	sweet
2	red	sweet
3	half-red	not-sweet
4	not-red	not-sweet
5	not-red	not-sweet
6	half-red	not-sweet
7	red	sweet
8	not-red	not-sweet
9	not-red	not-sweet
10	half-red	not-sweet
11	red	sweet
12	half-red	not-sweet
13	not-red	not-sweet

$$P(\text{red} \mid \text{sweet}) = 1$$

$$P(\text{half-red} \mid \text{sweet}) = 0$$

$$P(\text{not-red} \mid \text{sweet}) = 0$$

$$P(\text{sweet}) = 4/13$$

$$P(\text{red} \mid \text{not-sweet}) = 0$$

$$P(\text{half-red} \mid \text{not-sweet}) = 4/9$$

$$P(\text{not-red} \mid \text{not-sweet}) = 5/9$$

$$P(\text{not-sweet}) = 9/13$$

Consider a very simple case

id	color	taste
1	red	sweet
2	red	sweet
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4	not-red	not-sweet
5	not-red	not-sweet
6	half-red	not-sweet
7	red	sweet
8	not-red	not-sweet
9	not-red	not-sweet
10	half-red	not-sweet
11	red	sweet
12	half-red	not-sweet
13	not-red	not-sweet

what the f' would be?

$$f(x) = \arg\max_y P(\boldsymbol{x} \mid y) P(y)$$

$$P(\text{red} \mid \text{sweet}) P(\text{sweet}) = 4/13$$

$$P(\text{red} \mid \text{not-sweet}) P(\text{not-sweet}) = 0$$

$$P(\text{half-red} \mid \text{sweet}) P(\text{sweet}) = 0$$

$$P(\text{half-red} \mid \text{not-sweet}) P(\text{not-sweet}) = \frac{4}{9} \times \frac{9}{13} = \frac{4}{13}$$

perfect
but not realistic

Naive Bayes

$$f(x) = \arg\max_y P(\boldsymbol{x} \mid y) P(y)$$

estimation the a priori by frequency:

$$P(y) \leftarrow \tilde{P}(y) = \frac{1}{m} \sum_i I(y_i = y)$$

assume features are conditional independence given the class (naive assumption):

$$\begin{aligned} P(\boldsymbol{x} \mid y) &= P(x_1, x_2, \ldots, x_n \mid y) \\ &= P(x_1 \mid y) \cdot P(x_2 \mid y) \cdot \ldots P(x_n \mid y) \end{aligned}$$

decision function:

$$f(x) = \arg\max_y \tilde{P}(y) \prod_i \tilde{P}(x_i \mid y)$$

Naive Bayes

color={0,1,2,3} weight={0,1,2,3,4}

color	weight	sweet?
3	4	yes
2	3	yes
0	3	no
3	2	no
1	4	no

$$P(y = yes) = 2/5$$

$$P(y = no) = 3/5$$

$$P(color = 3 \mid y = yes) = 1/2$$

...

$$f(y \mid color = 3, weight = 3) \to$$

$$P(color = 3 \mid y = yes)P(weight = 3 \mid y = yes)P(y = yes) = 0.5 \times 0.5 \times 0.4 = 0.1$$

$$P(color = 3 \mid y = no)P(weight = 3 \mid y = no)P(y = no) = 0.33 \times 0.33 \times 0.6 = 0.06$$

$$f(y \mid color = 0, weight = 1) \to$$

$$P(color = 0 \mid y = yes)P(weight = 1 \mid y = yes)P(y = yes) = 0$$

$$P(color = 0 \mid y = no)P(weight = 1 \mid y = no)P(y = no) = 0$$

Naive Bayes

color={0,1,2,3} weight={0,1,2,3,4}

color	weight	sweet?
3	4	yes
2	3	yes
0	3	no
3	2	no
1	4	no

+

color	sweet?
0	yes
1	yes
2	yes
3	yes

smoothed (Laplacian correction) probabilities:

$$P(color = 0 \mid y = yes) = (0+1)/(2+4)$$

$$P(y = yes) = (2+1)/(5+2)$$

for counting frequency, assume every event has happened once.

$$f(y \mid color = 0, weight = 1) \to$$

$$P(color = 0 \mid y = yes)P(weight = 1 \mid y = yes)P(y = yes) = \frac{1}{6} \times \frac{1}{7} \times \frac{3}{7} = 0.01$$

$$P(color = 0 \mid y = no)P(weight = 1 \mid y = no)P(y = no) = \frac{2}{7} \times \frac{1}{8} \times \frac{4}{7} = 0.02$$

Naive Bayes

advantages:

- very fast:
  - scan the data once, just count: $O(mn)$
  - store class-conditional probabilities: $O(n)$
  - test an instance: $O(cn)$ (c the number of classes)
- good accuracy in many cases
- parameter free
- output a probability
- naturally handle multi-class

disadvantages:

- the strong assumption may harm the accuracy
- does not handle numerical features naturally