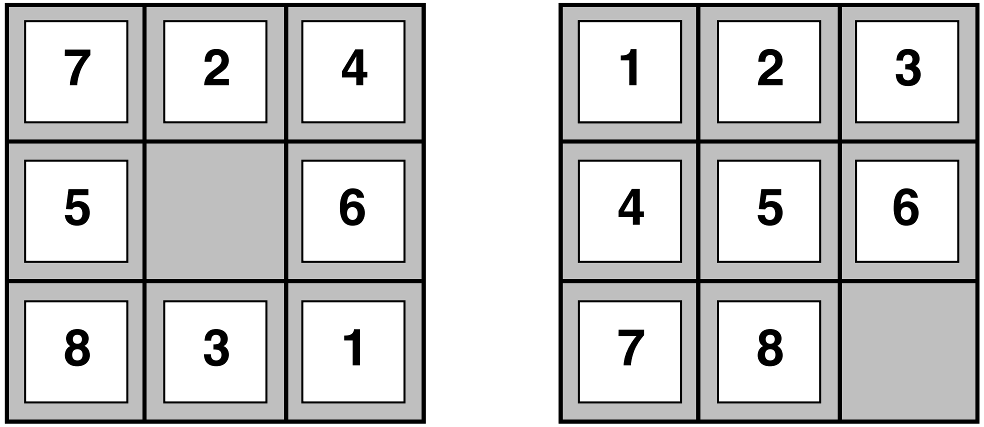
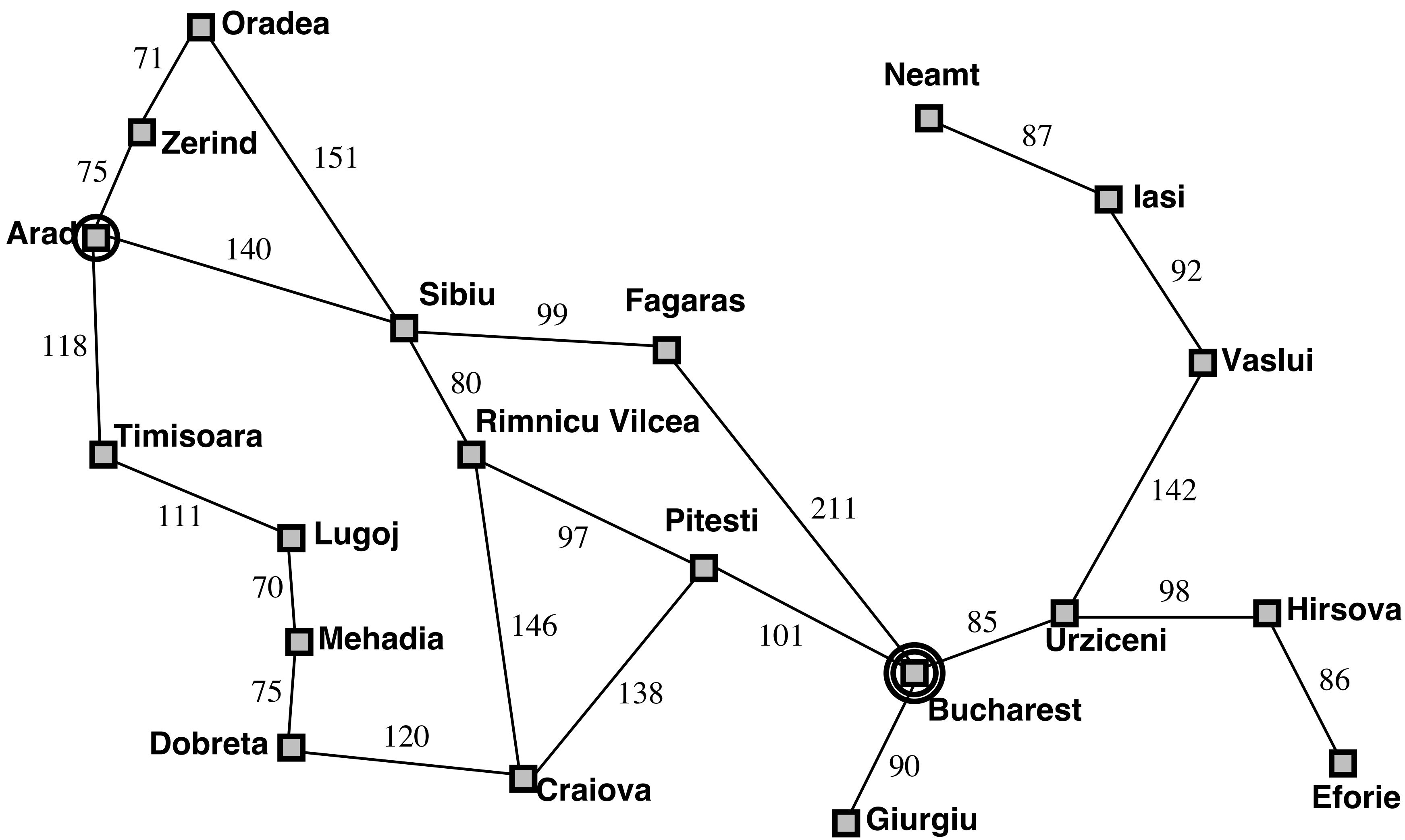
Lecture 2: Search 1

Search problem: example 1



Start State   Goal State

Search problem: example 2

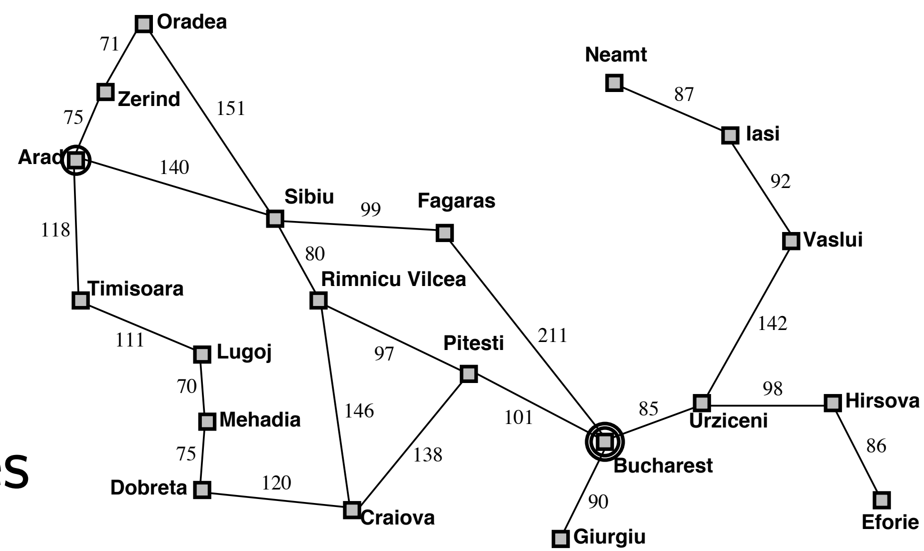


Key parts

On holiday in Romania; currently in Arad.
Flight leaves tomorrow from Bucharest

Formulate goal:
  be in Bucharest

Formulate problem:
  states: various cities
  actions: drive between cities



Find solution:
  sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

Search problems

A search problem is defined by 5 components:

initial state

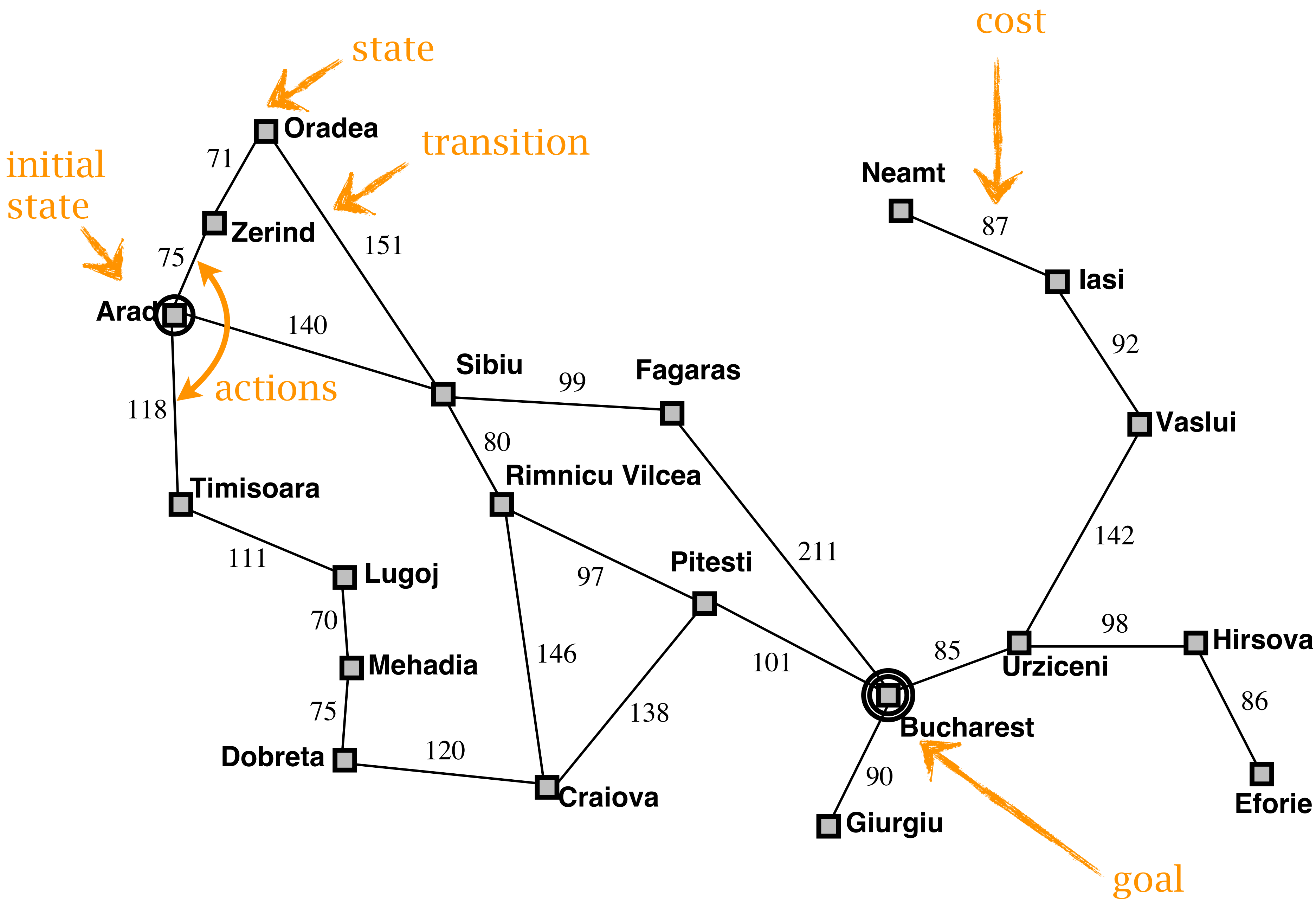
possible actions (and state associated actions)

transition model
taking an action will cause a state change

goal test
judge if the goal state is found

path cost
constitute the cost of a solution

Problems



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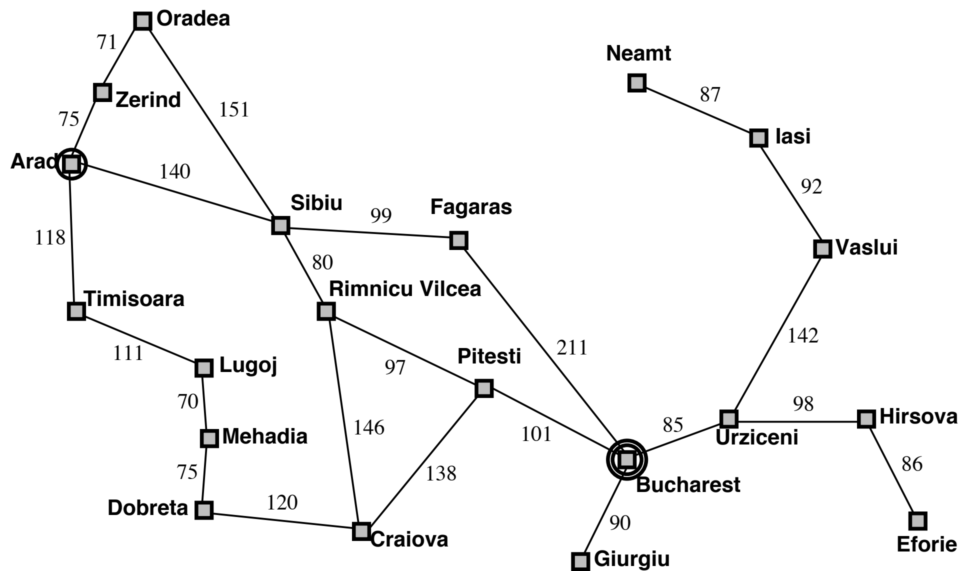
goal

Problems



initial state e.g., "at Arad"

successor function $S(x)$ = set of action–state pairs

e.g., $S(Arad) = \{\langle Arad \rightarrow Zerind, Zerind \rangle, \ldots\}$

<-- transition

goal test, can be

explicit, e.g., x = "at Bucharest"

implicit, e.g., $NoDirt(x)$

path cost (additive)

e.g., sum of distances, number of actions executed, etc.

$c(x, a, y)$ is the step cost, assumed to be ≥ 0

A solution is a sequence of actions

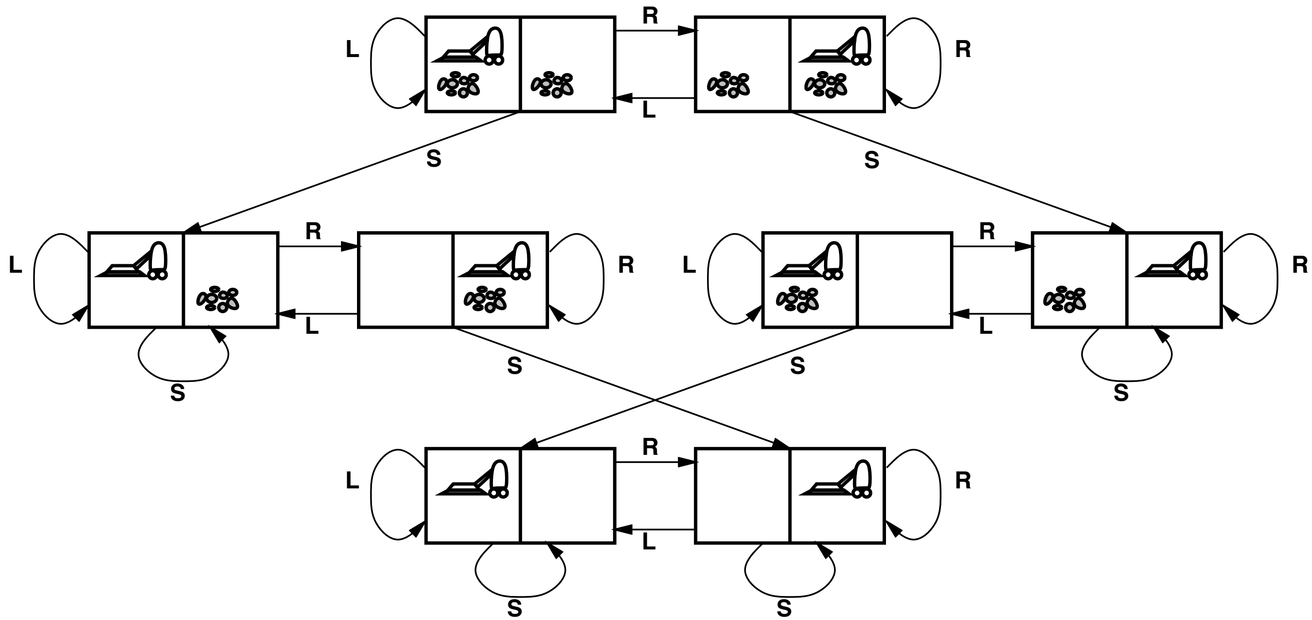
leading from the initial state to a goal state

Problems

we assume

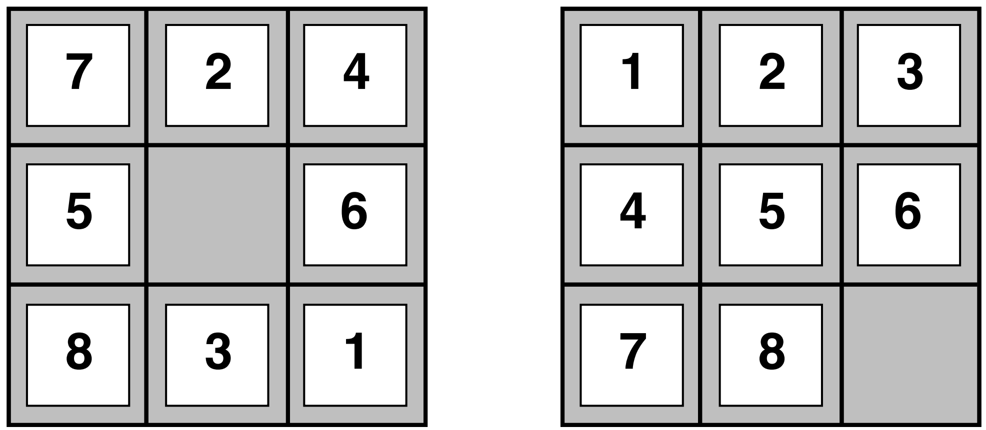
- observable states (a seen state is accurate)
  - in partial observable case, states are not accurate
- discrete states
  - there are also continuous state spaces
- deterministic transition
  - there could be stochastic transitions

Example: vacuum world



states??: integer dirt and robot locations (ignore dirt amounts etc.)
actions??: $Left$, $Right$, $Suck$, $NoOp$
goal test??: no dirt
path cost??: 1 per action (0 for $NoOp$)

Example: 8-puzzle



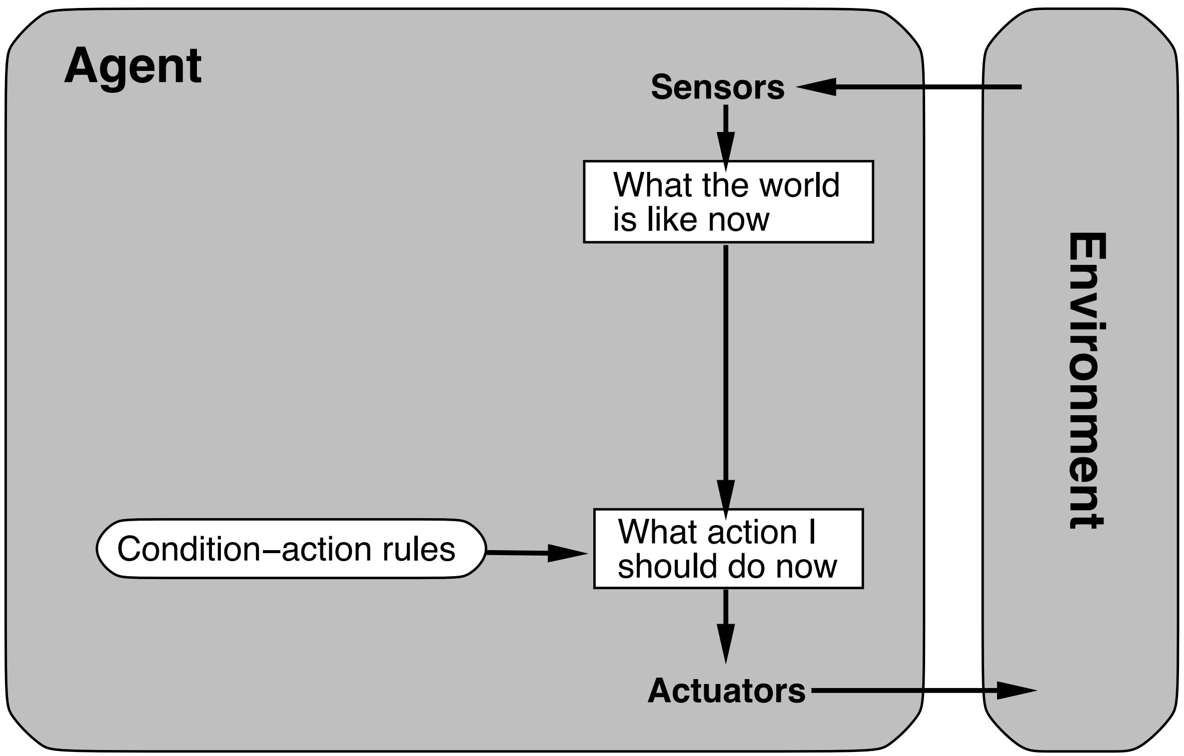
Start State **Goal State**

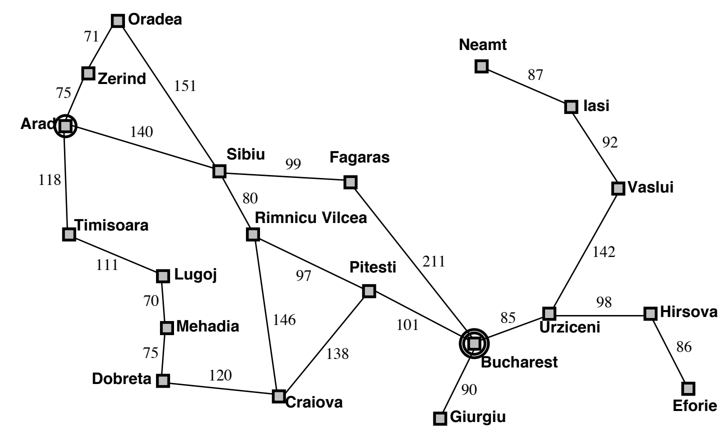
states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down (ignore unjamming etc.)
goal test??: = goal state (given)
path cost??: 1 per move

[Note: optimal solution of n-Puzzle family is NP-hard]

Agent that searches

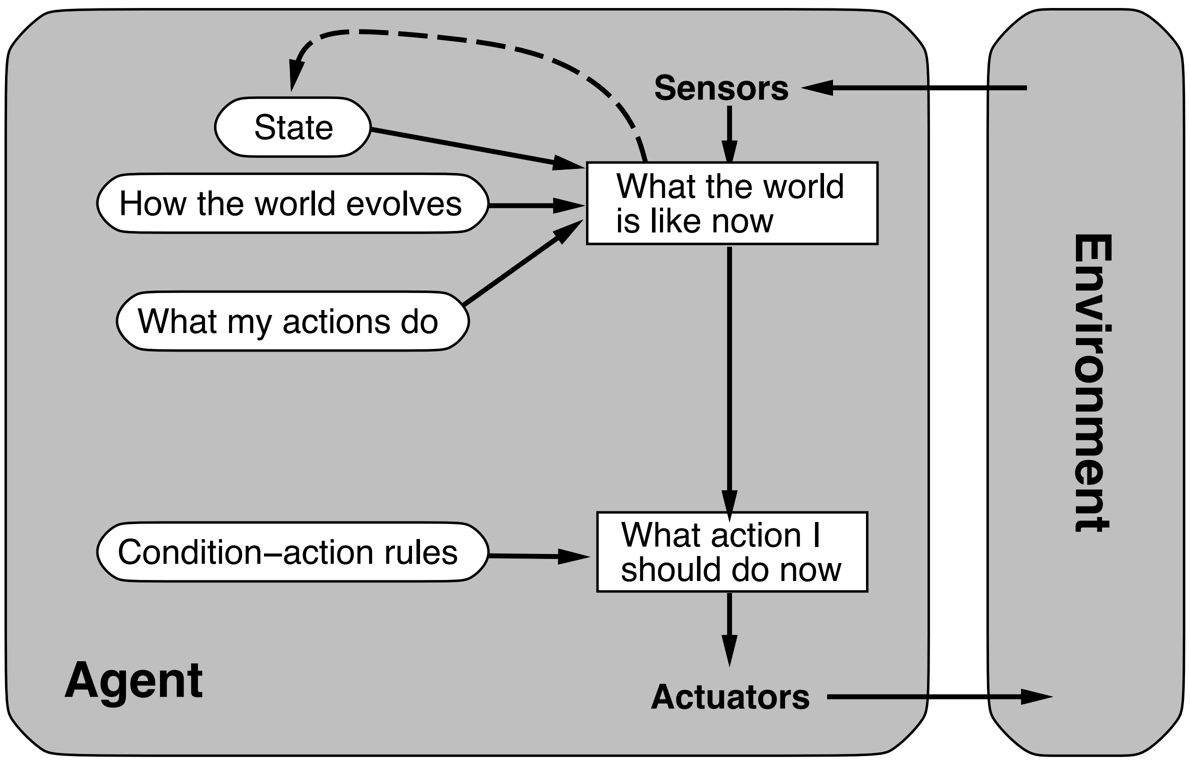
can simple reflex agents do the search?

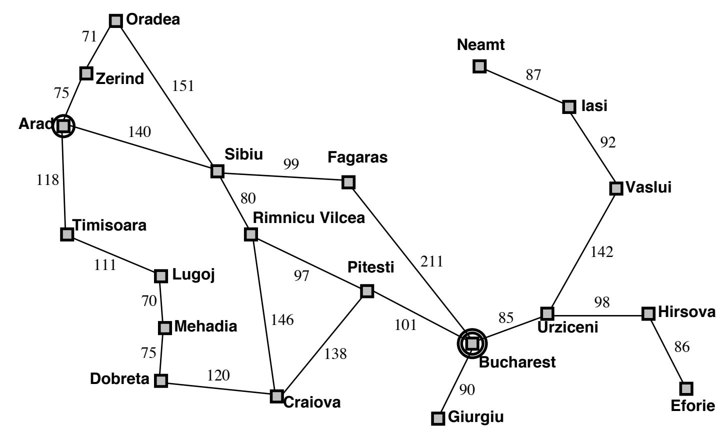




Agent that searches

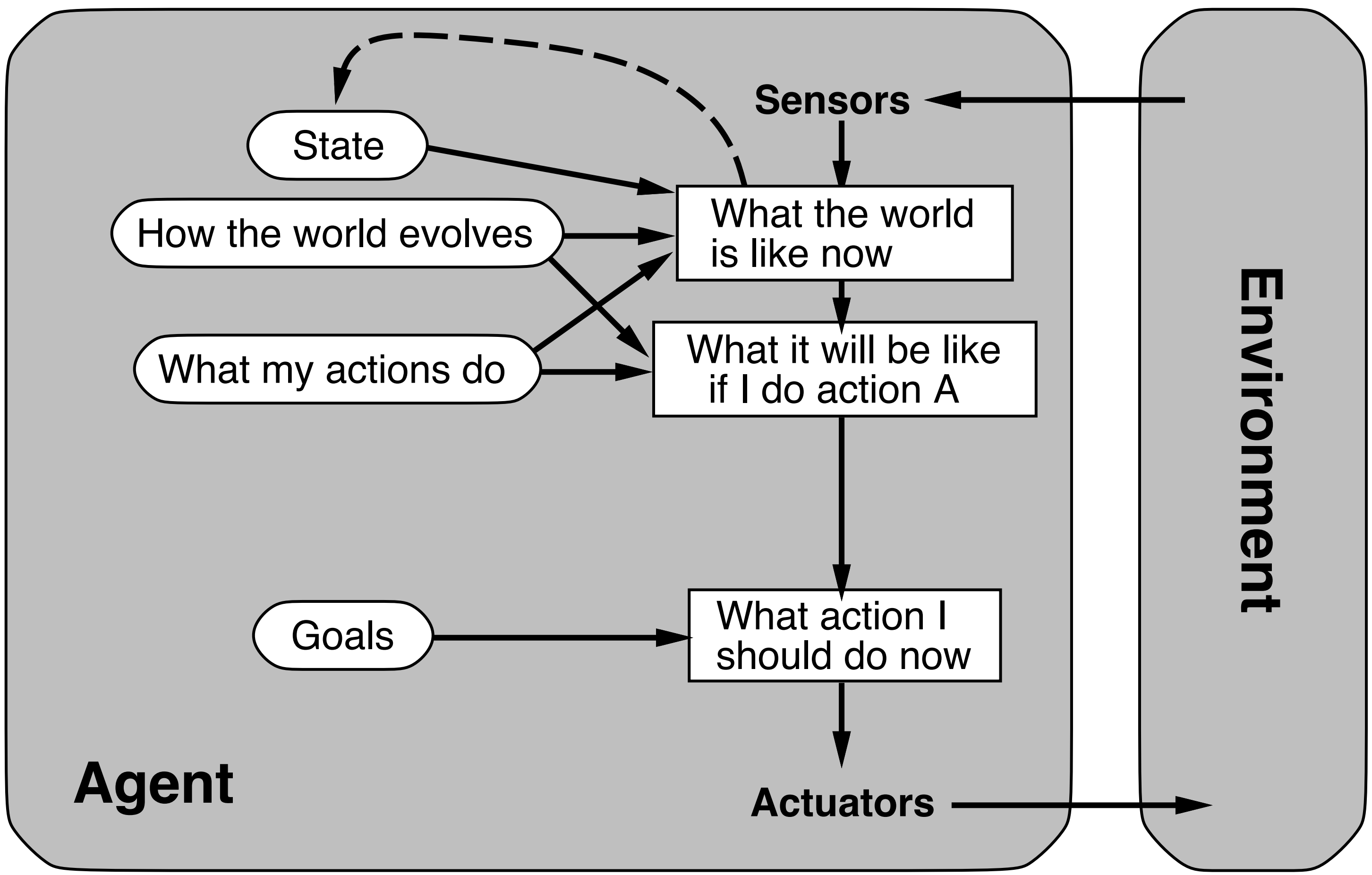
can reflex agents with state do the search?





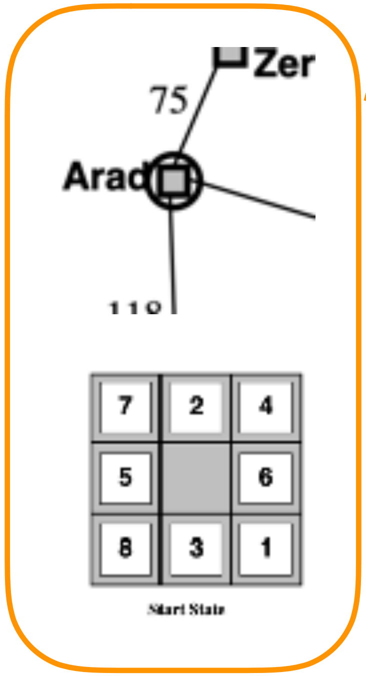
Agent that searches

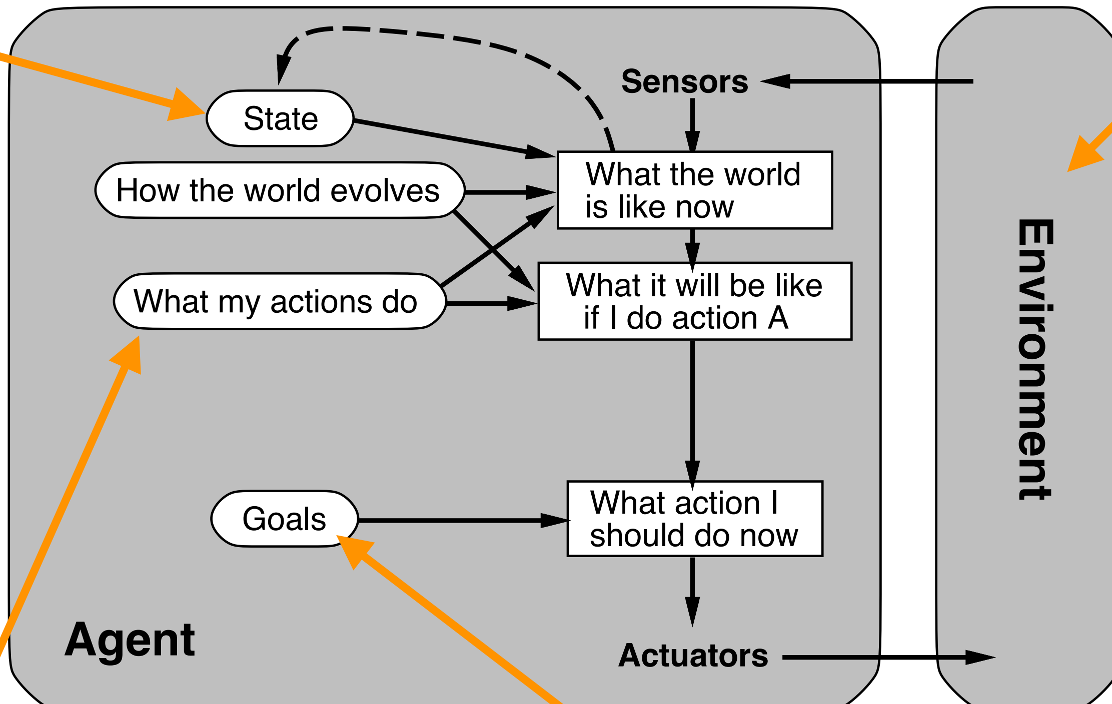
consider goal-based agents

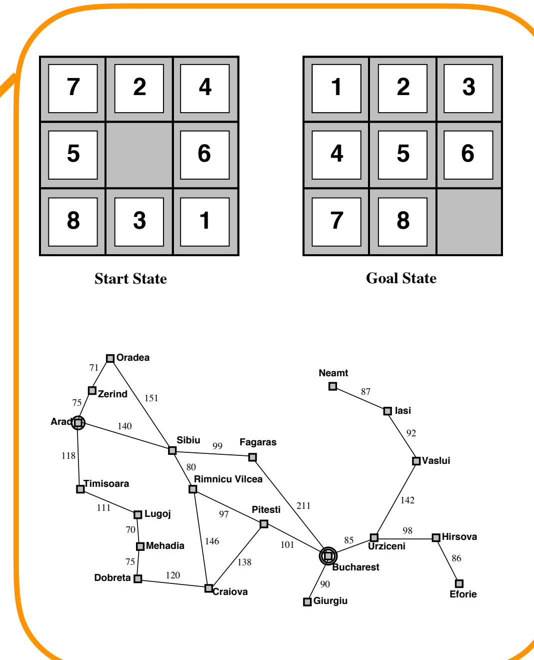


Agent that searches

consider goal-based agents



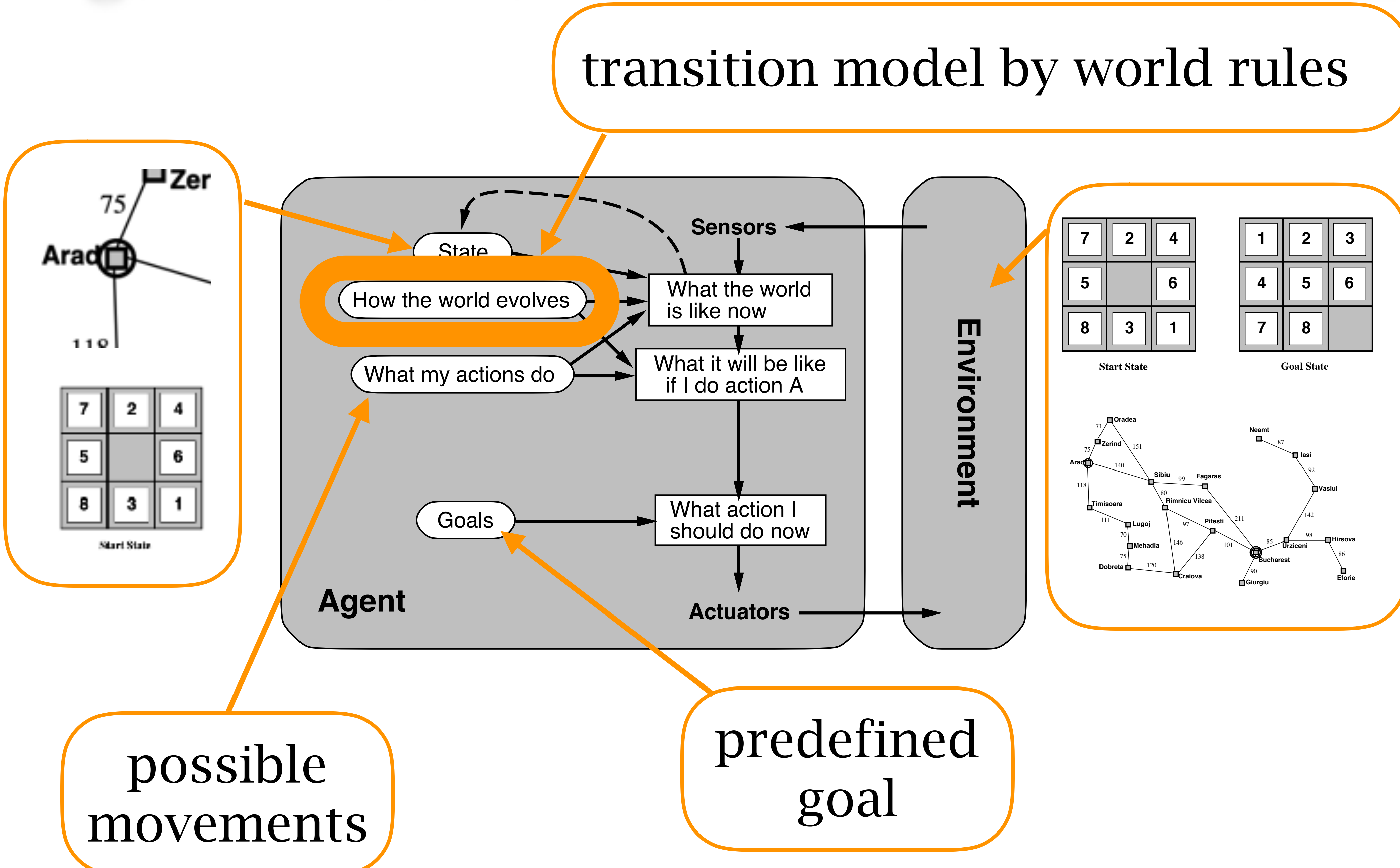




possible movements

predefined goal

Agent that searches



Extra knowledge

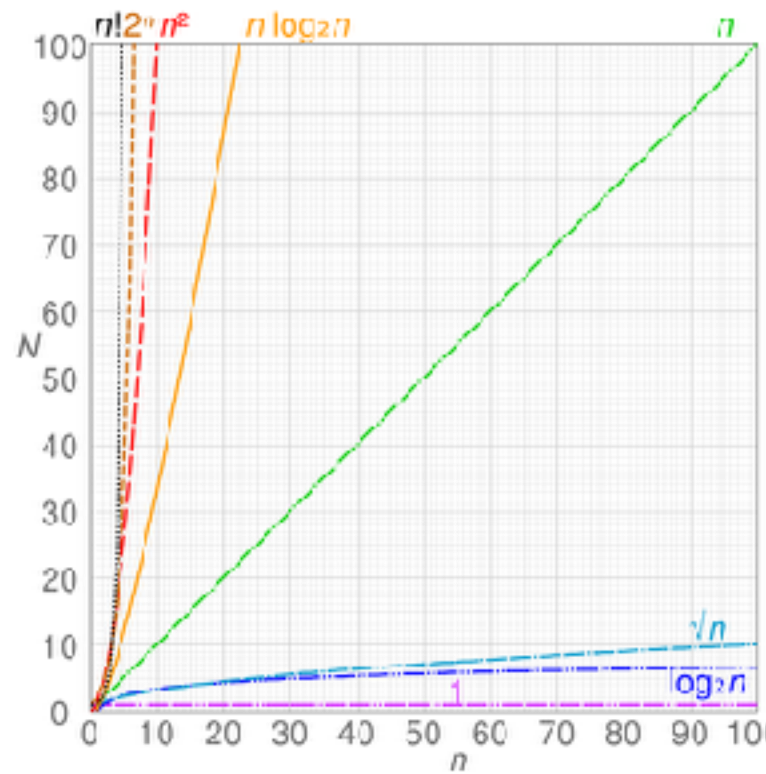
time complexity: number of key operations

space complexity: number of key bits stored

the big O representation:

$O(1)$ $O(\ln n)$ $O(n)$ $O(n^2)$

$O(2^n)$ $O(n^n)$

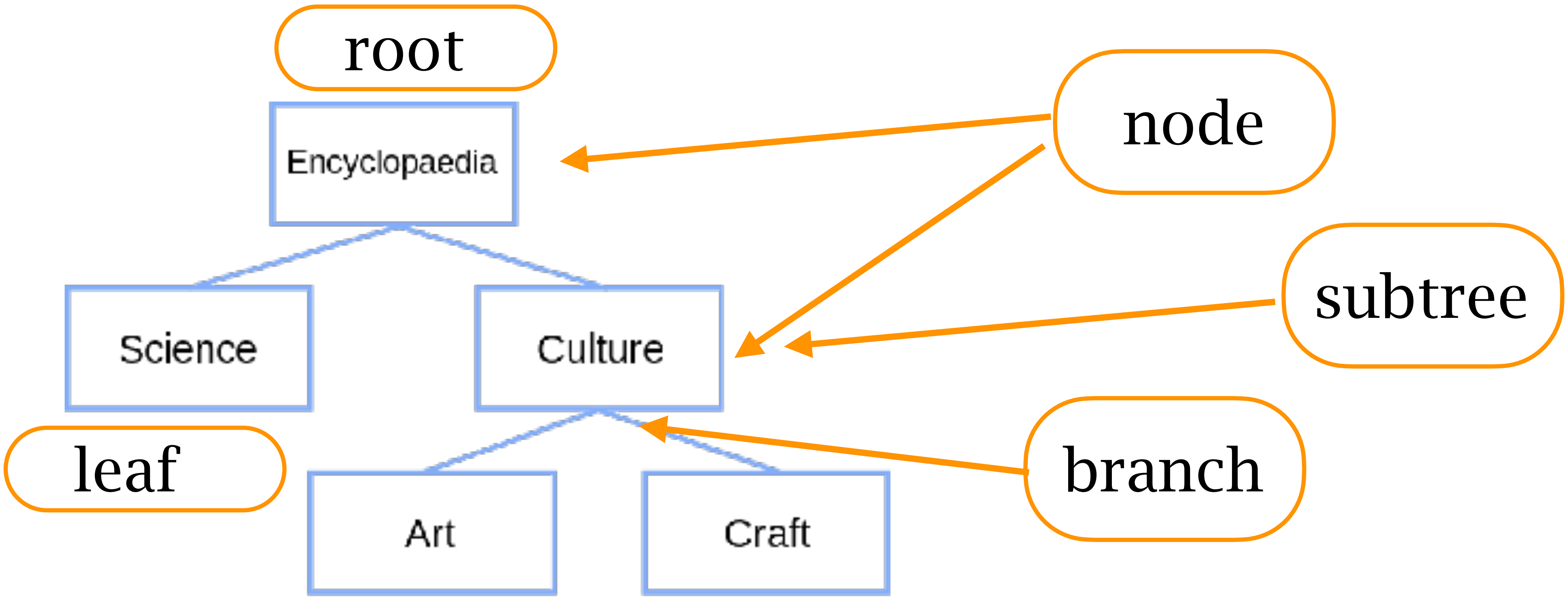


[from wikipedia: "Big O"]

NP-hardness and NP-completness

Search Algorithms on Graphs

Tree structure



[from wikipedia: “Tree structure”]

binary tree: each node has at most two branches

search tree: a tree data structure for search

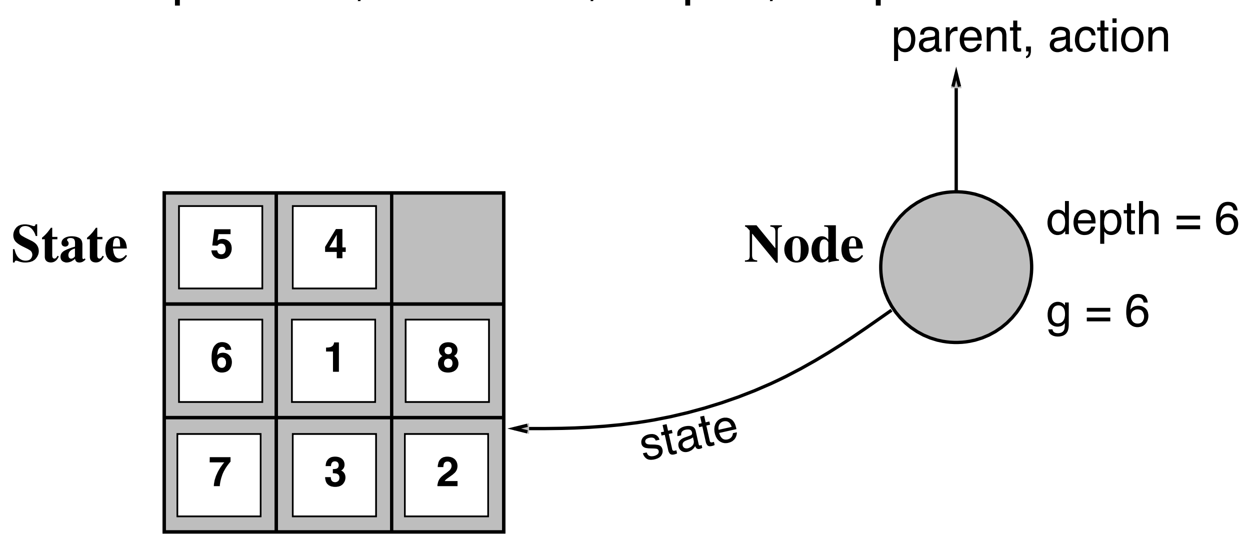
State v.s. node

A state is a (representation of) a physical configuration

A node is a data structure constituting part of a search tree

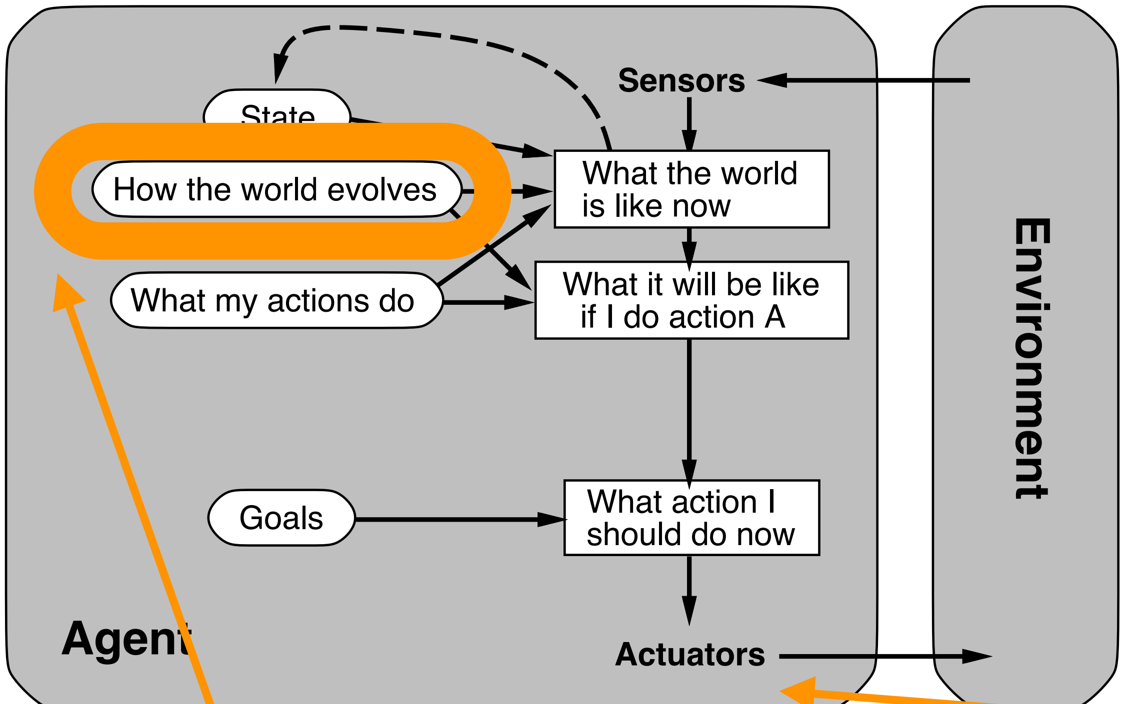
includes parent, children, depth, path cost $g(x)$

States do not have parents, children, depth, or path cost!



The EXPAND function creates new nodes, filling in the various fields and using the SUCCESSORFN of the problem to create the corresponding states.

Agent that searches



the search does NOT change the world!

only actions change the world

evolves in a tree structure:
use tree search to find the goal

Tree search

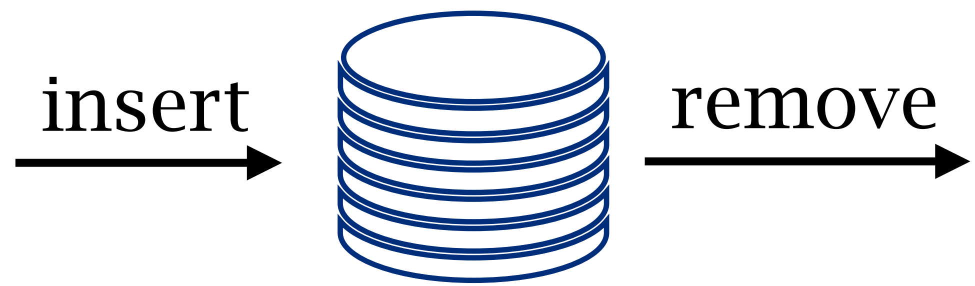
1. start from the initial state (root)
2. expand the current state

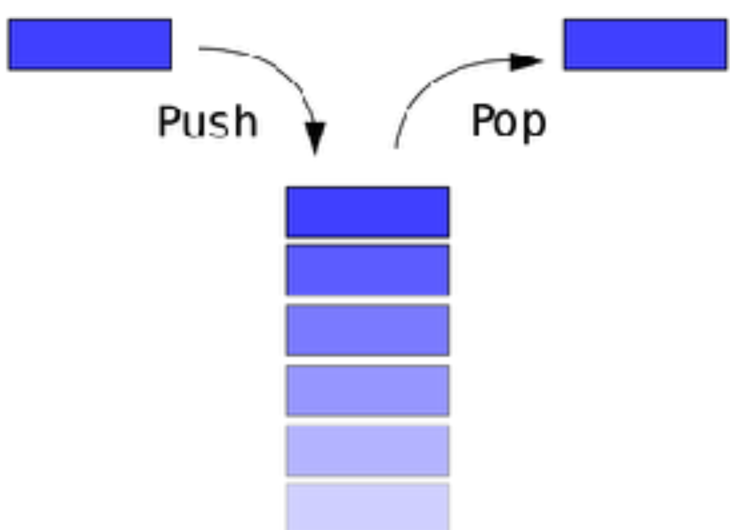
essence of search: following up one option now and putting the others aside

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

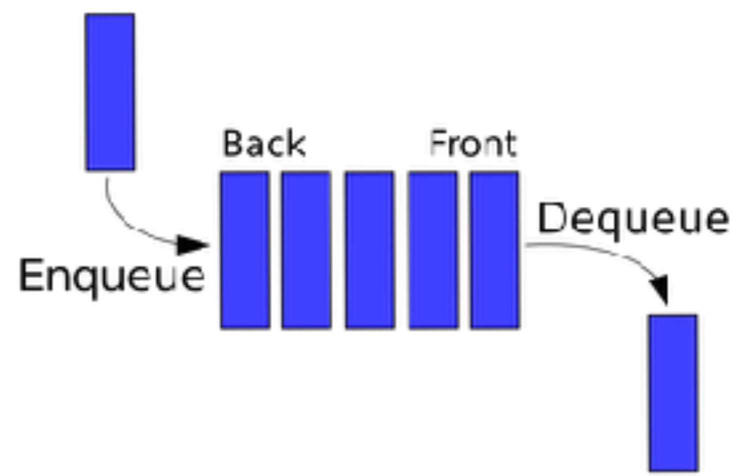
all search algorithms share this tree search structure
they vary primarily according to how they choose which state to expand --- the so-called search strategy

Storage data structure





stack



queue

[images from https://stackoverflow.com/questions/10974922/what-is-the-basic-difference-between-stack-and-queue]

General tree search

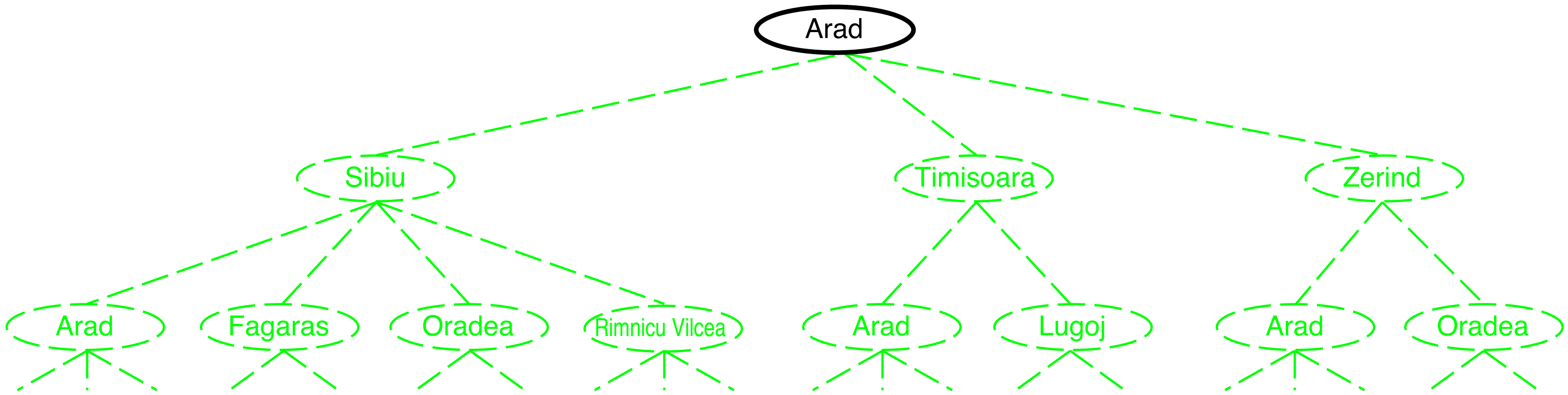
function TREE-SEARCH(*problem, fringe*) **returns** a solution, or failure
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)
  loop do
    if *fringe* is empty **then return** failure
    node ← REMOVE-FRONT(*fringe*)
    if GOAL-TEST(*problem*, STATE(*node*)) **then return** *node*
    fringe ← INSERTALL(EXPAND(*node*, *problem*), *fringe*)

note the time of goal-test: expanding time not generating time

---

function EXPAND(*node, problem*) **returns** a set of nodes
  successors ← the empty set
  for each *action, result* **in** SUCCESSOR-FN(*problem*, STATE[*node*]) **do**
    s ← a new NODE
    PARENT-NODE[*s*] ← *node*; ACTION[*s*] ← *action*; STATE[*s*] ← *result*
    PATH-COST[*s*] ← PATH-COST[*node*] + STEP-COST(*node*, *action*, *s*)
    DEPTH[*s*] ← DEPTH[*node*] + 1
    add *s* to *successors*
  return *successors*

Example



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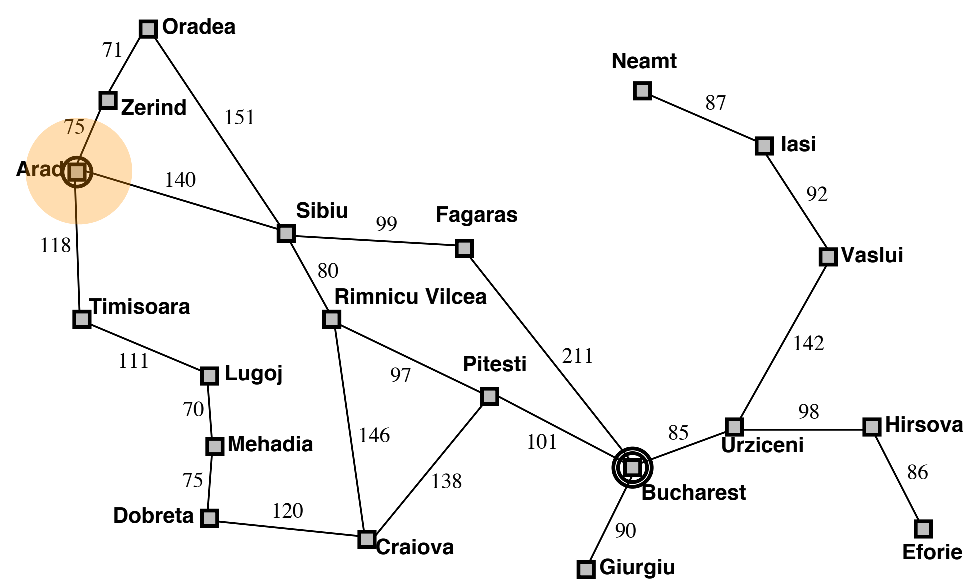

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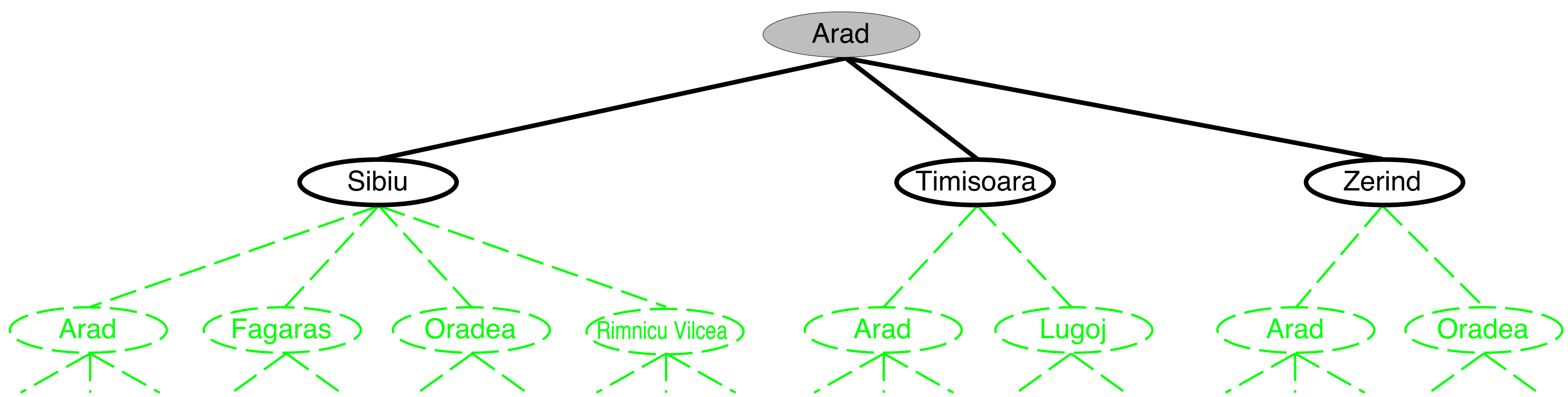
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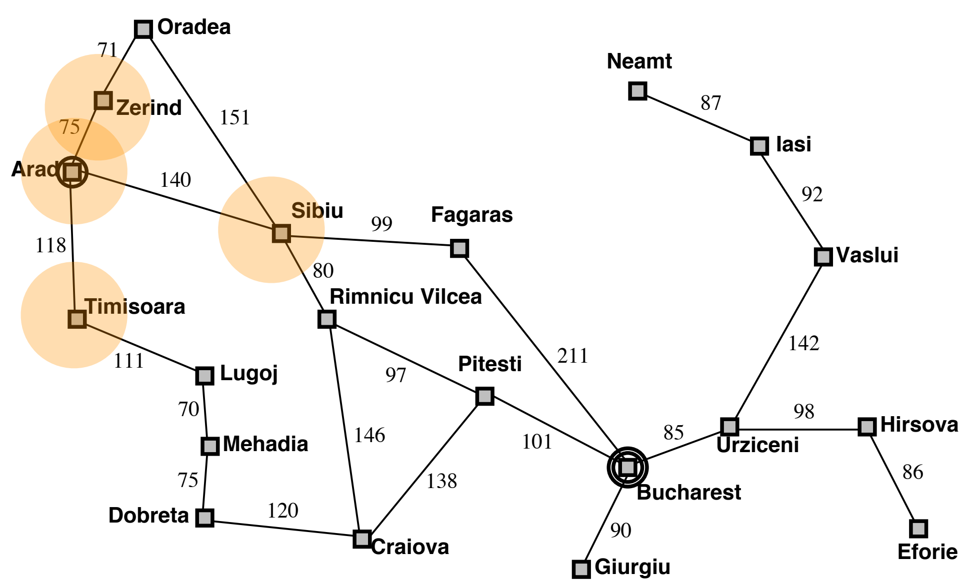
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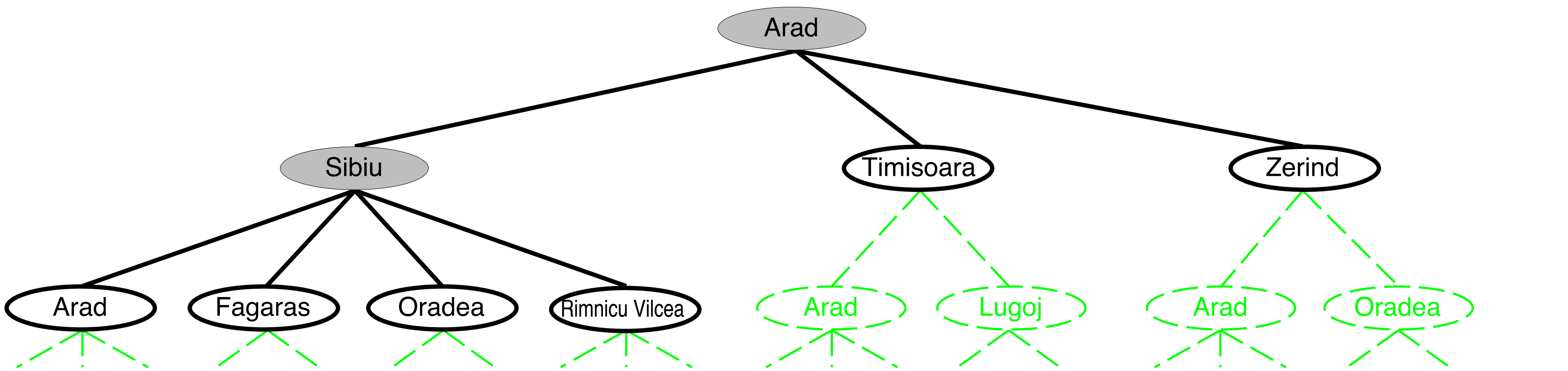
Example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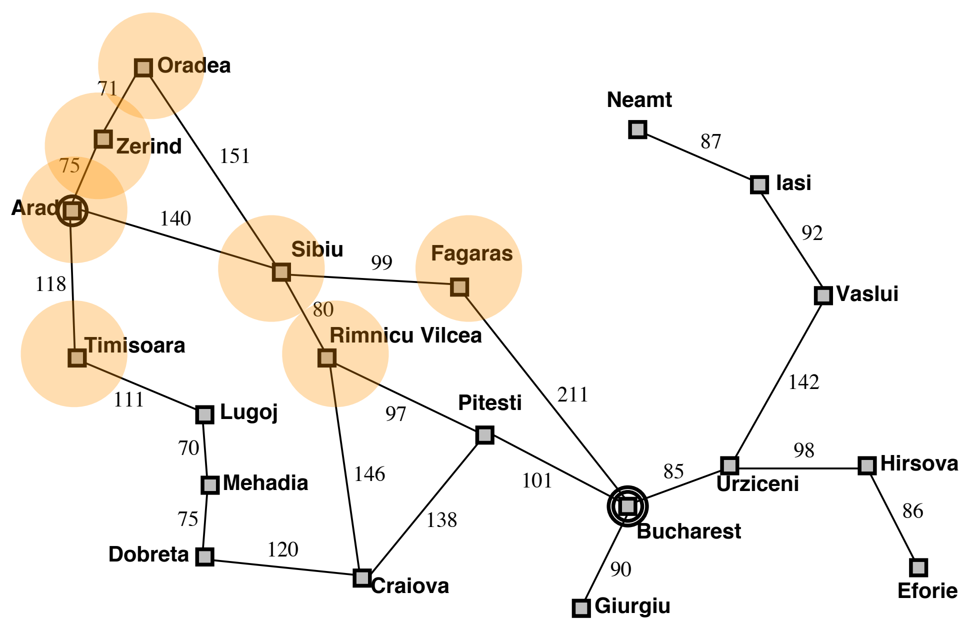
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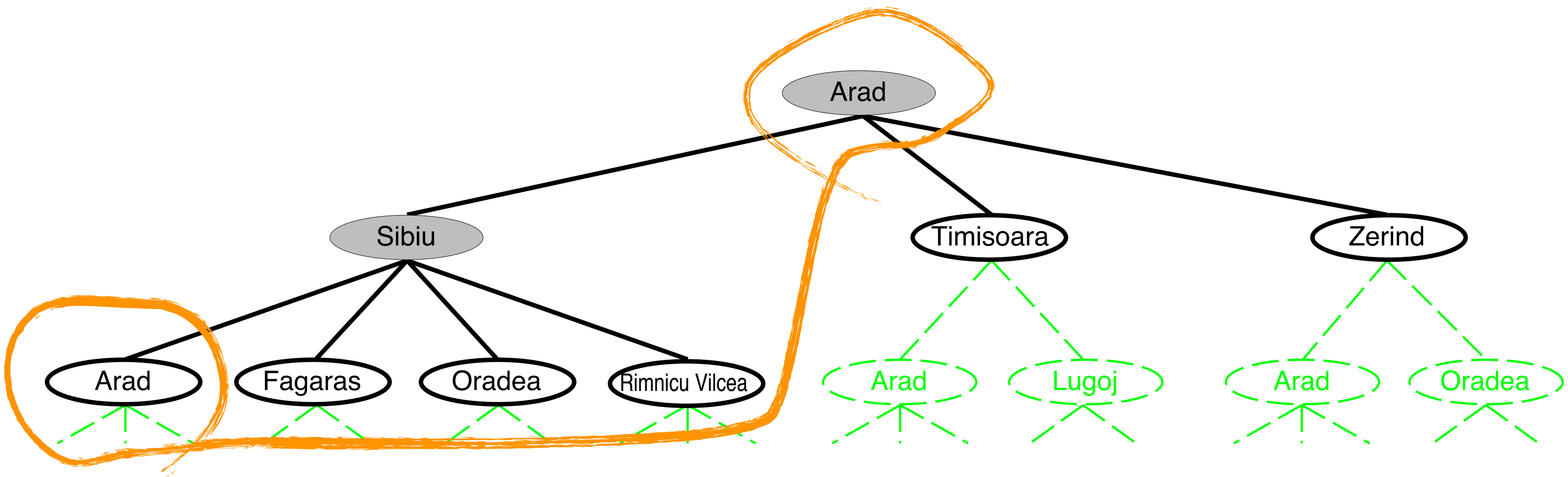
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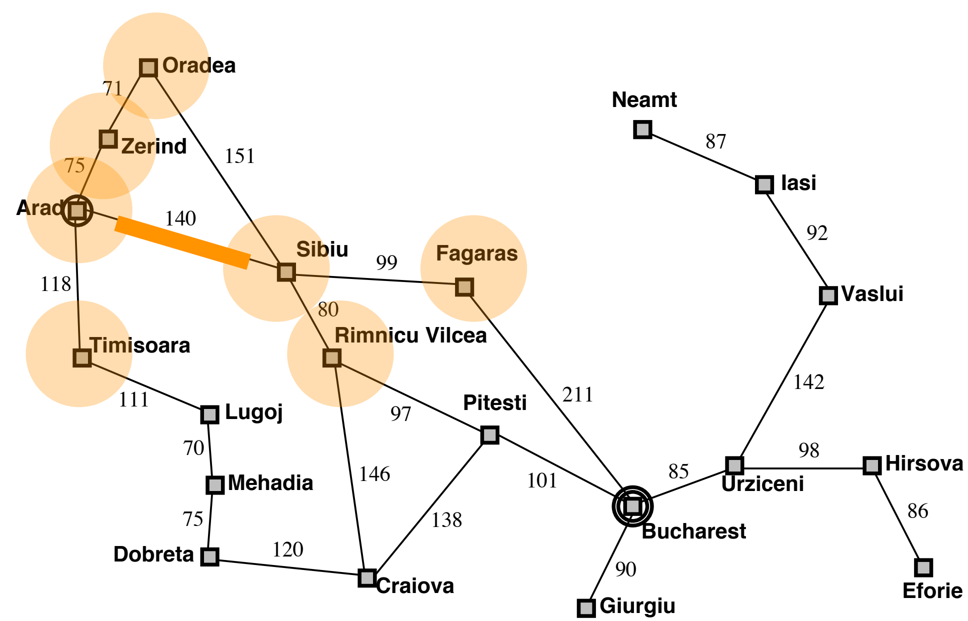
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Graph search

```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE(node)) then return node
        fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        if STATE[node] is not in closed then
            add STATE[node] to closed
            fringe ← INSERTALL(EXPAND(node, problem), fringe)
    end
```

Graph separation property

the frontier (expandable leaf nodes) separates the visited and the unexplored nodes

Search strategies

A strategy is defined by picking the **order of node expansion**

Strategies are evaluated along the following dimensions:

completeness—does it always find a solution if one exists?
time complexity—number of nodes generated/expanded
space complexity—maximum number of nodes in memory
optimality—does it always find a least-cost solution?

Time and space complexity are measured in terms of

b—maximum branching factor of the search tree
d—depth of the least-cost solution
m—maximum depth of the state space (may be ∞)

Uninformed Search Strategies

Uninformed strategies use only the information available in the problem definition

Breadth-first search

Uniform-cost search

Depth-first search

Depth-limited search

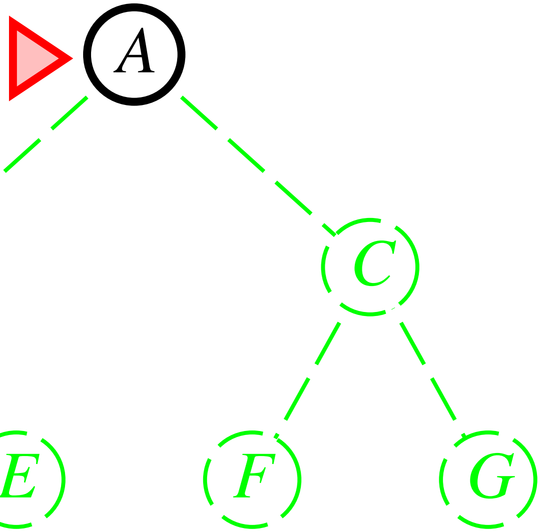
Iterative deepening search

Breadth-first search

Expand shallowest unexpanded node

Implementation:

fringe is a FIFO queue, i.e., new successors go at end

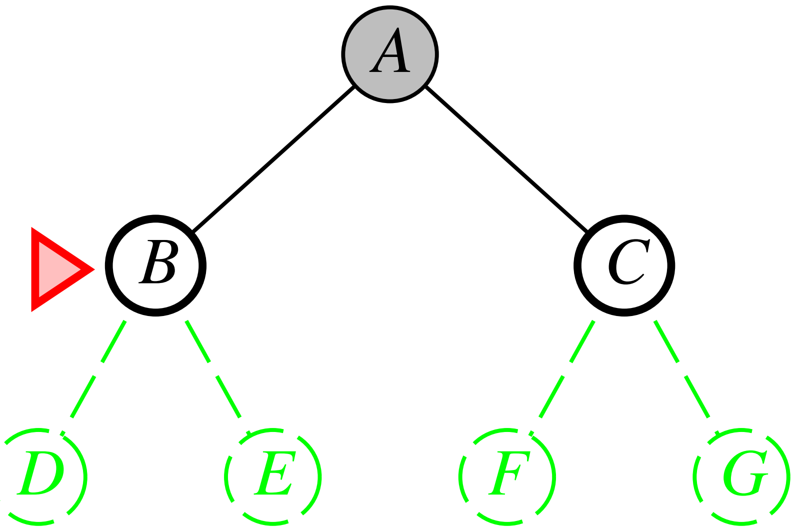


Breadth-first search

Expand shallowest unexpanded node

Implementation:

fringe is a FIFO queue, i.e., new successors go at end



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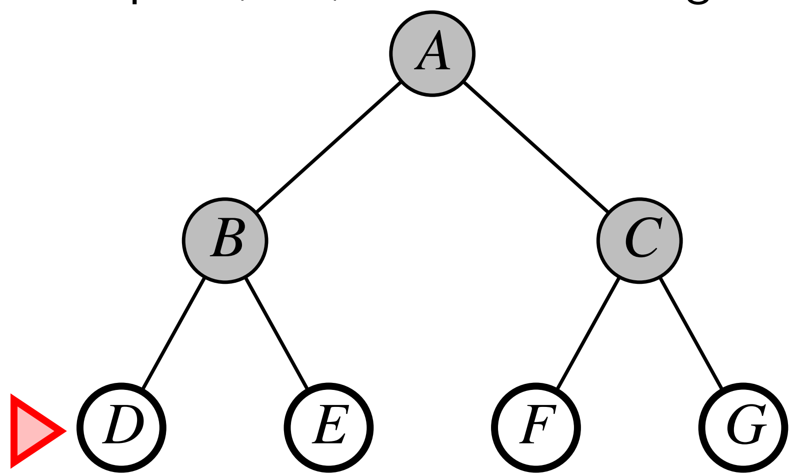


Properties

Complete?? Yes (if b is finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space?? $O(b^{d+1})$ (keeps every node in memory)

Optimal?? Yes (if cost $= 1$ per step); not optimal in general

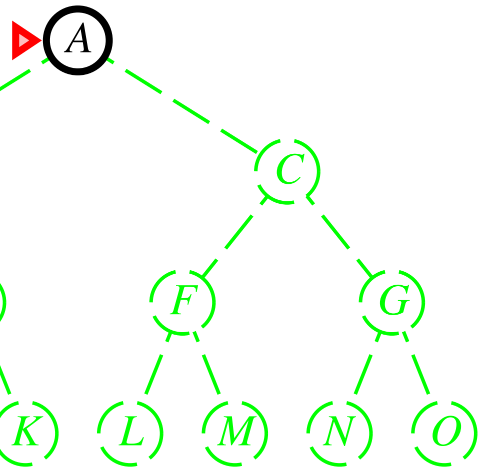
Space is the big problem; can easily generate nodes at 100MB/sec
so 24hrs $=$ 8640GB.

Depth-first search

Expand deepest unexpanded node

Implementation:

$fringe$ = LIFO queue, i.e., put successors at front



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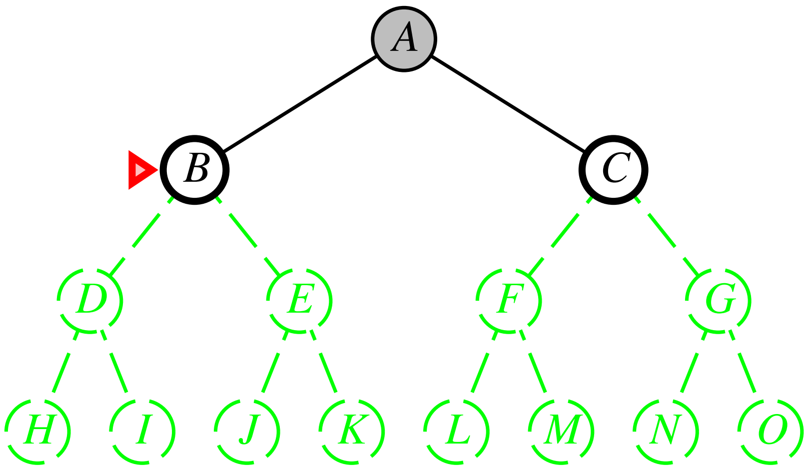

Depth-first search

Expand deepest unexpanded node

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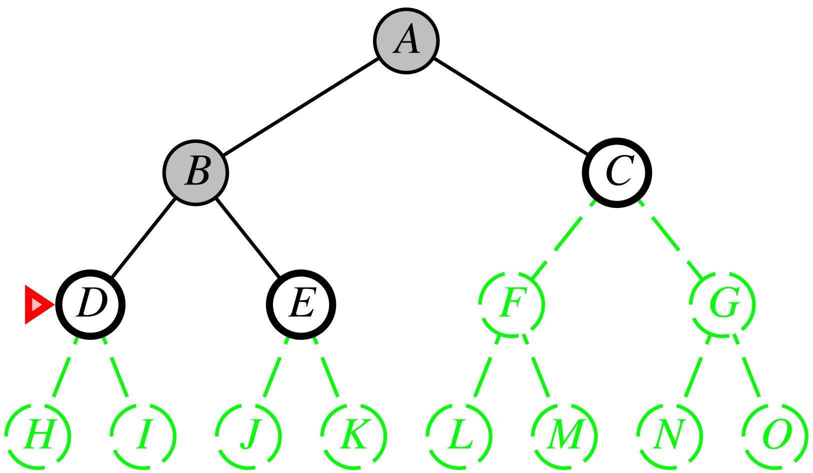

Depth-first search

Implementation:

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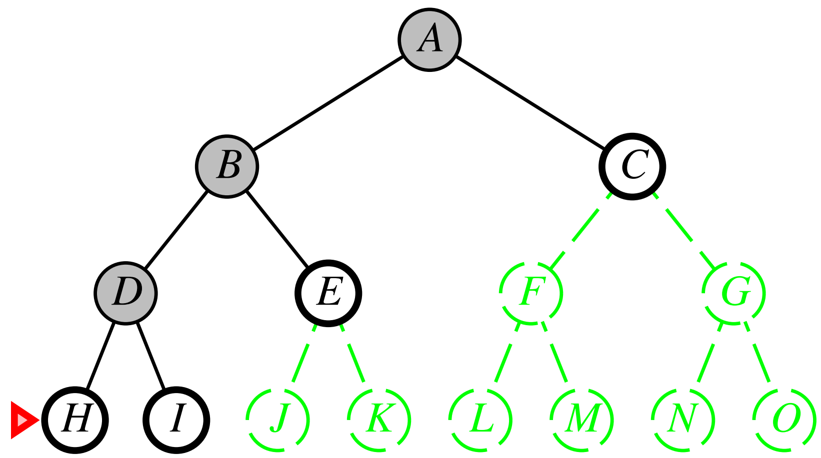

Depth-first search

Expand deepest unexpanded node

Implementation:

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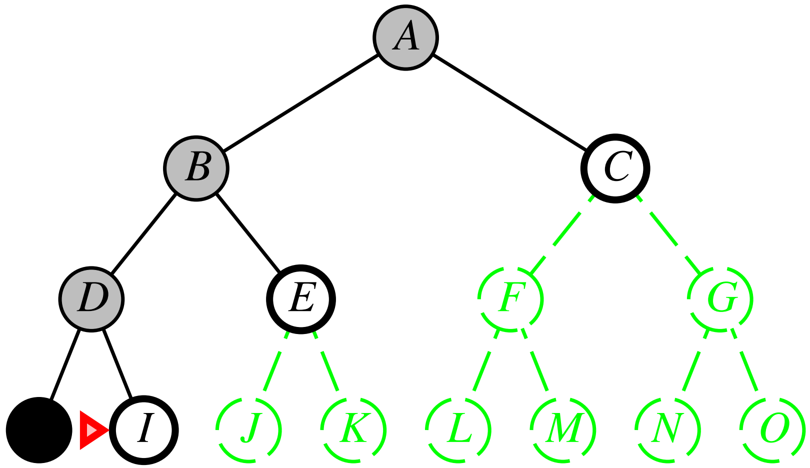

Depth-first search

Expand deepest unexpanded node

Implementation:

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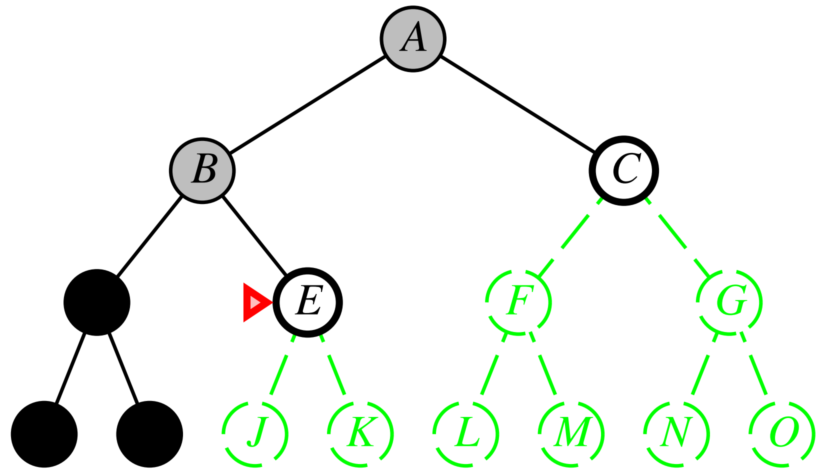

Depth-first search

Expand deepest unexpanded node

Implementation:

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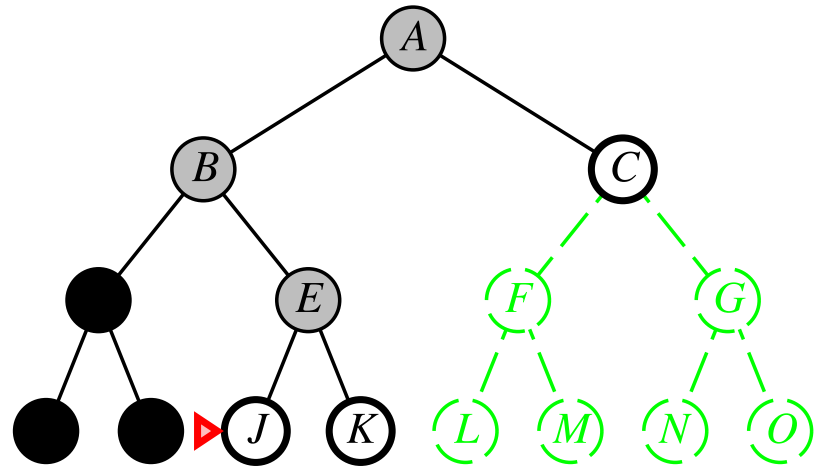

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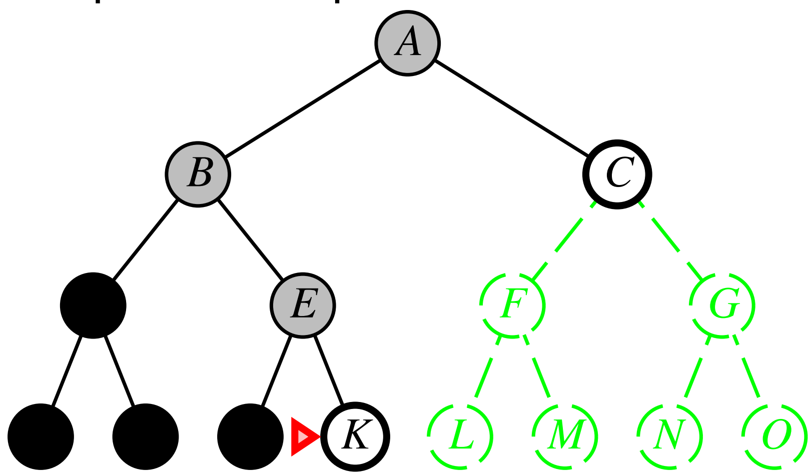

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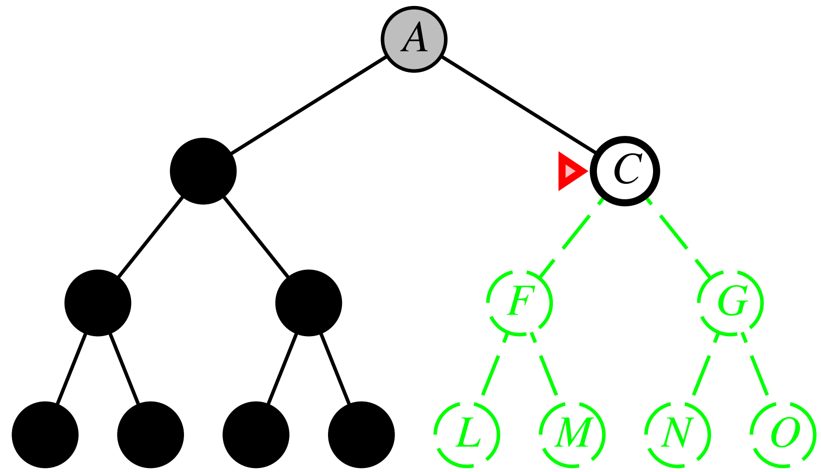


Properties

Complete?? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces with repeated states avoid

Time?? $O(b^m)$: terrible if m is much larger than d
but if solutions are dense, may be much faster than breadth-first

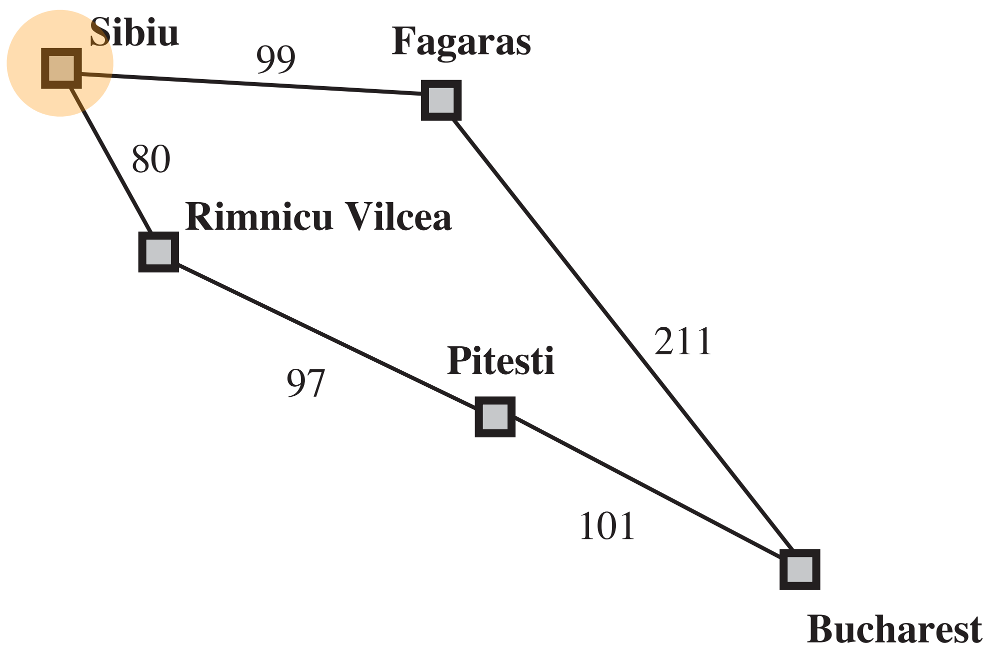
Space?? $O(bm)$, i.e., linear space!

Optimal?? No

Uniform-cost search

Breadth-first search: First In First Out queue
Depth-first search: Last In First Out queue (stack)
Uniform-cost search: Priority queue (least cost out)

part of the map



Uniform-cost search

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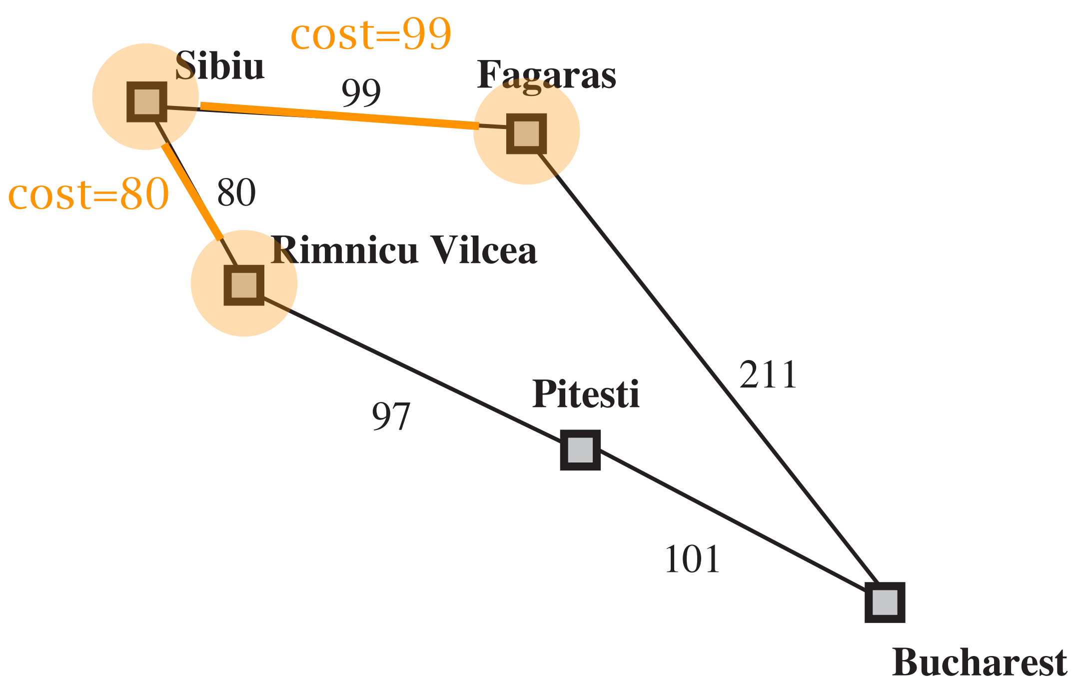
part of the map



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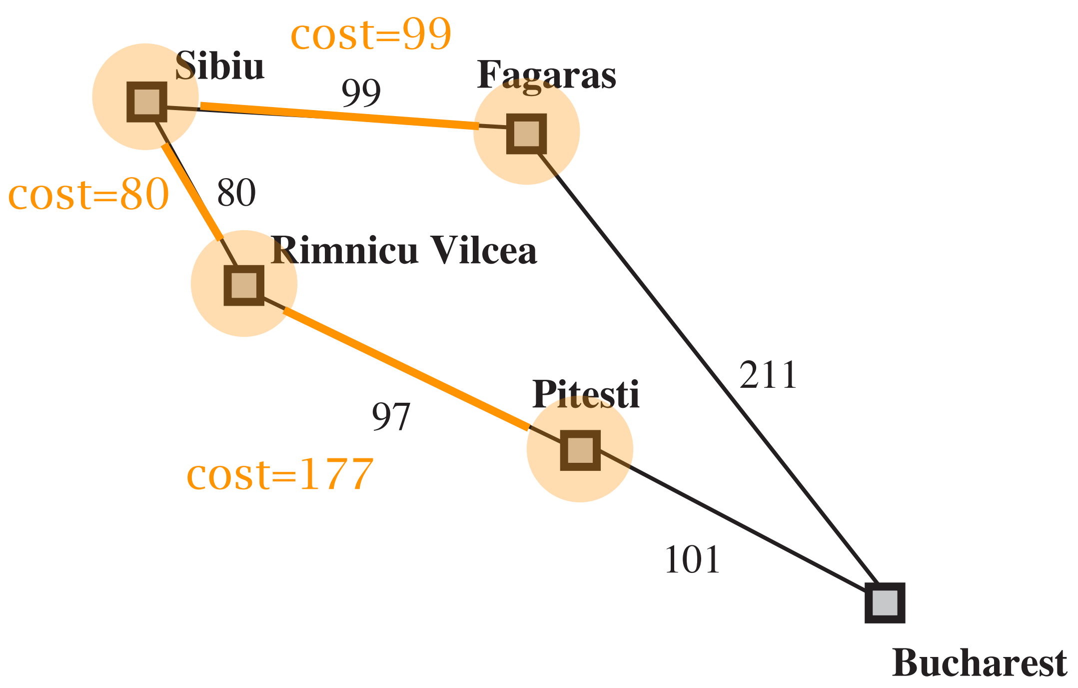

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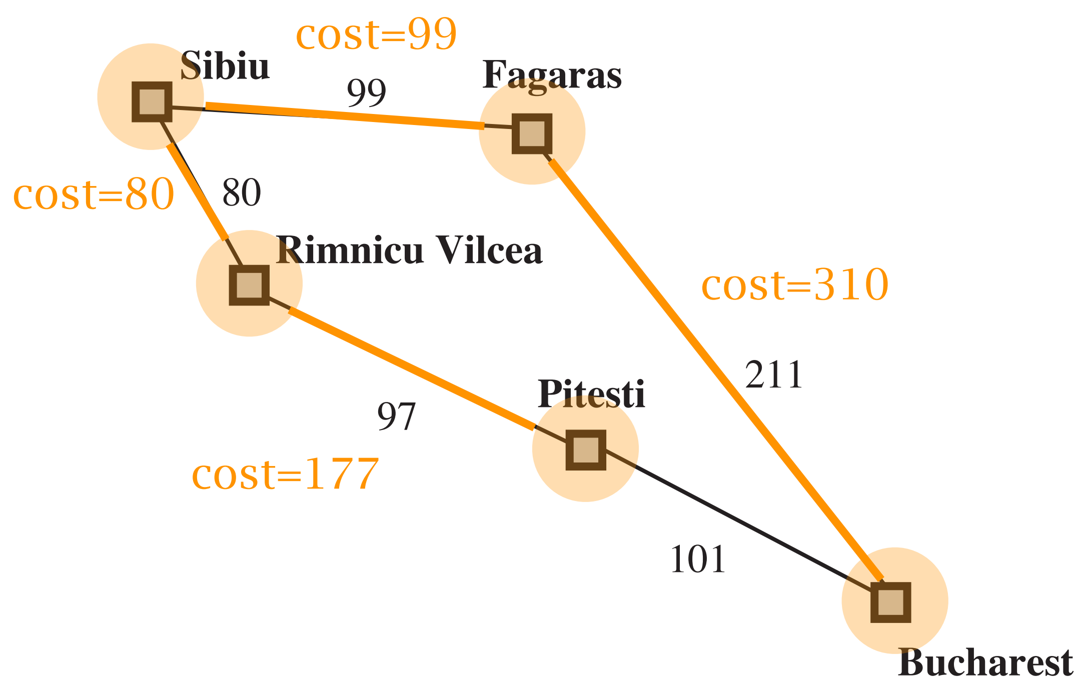

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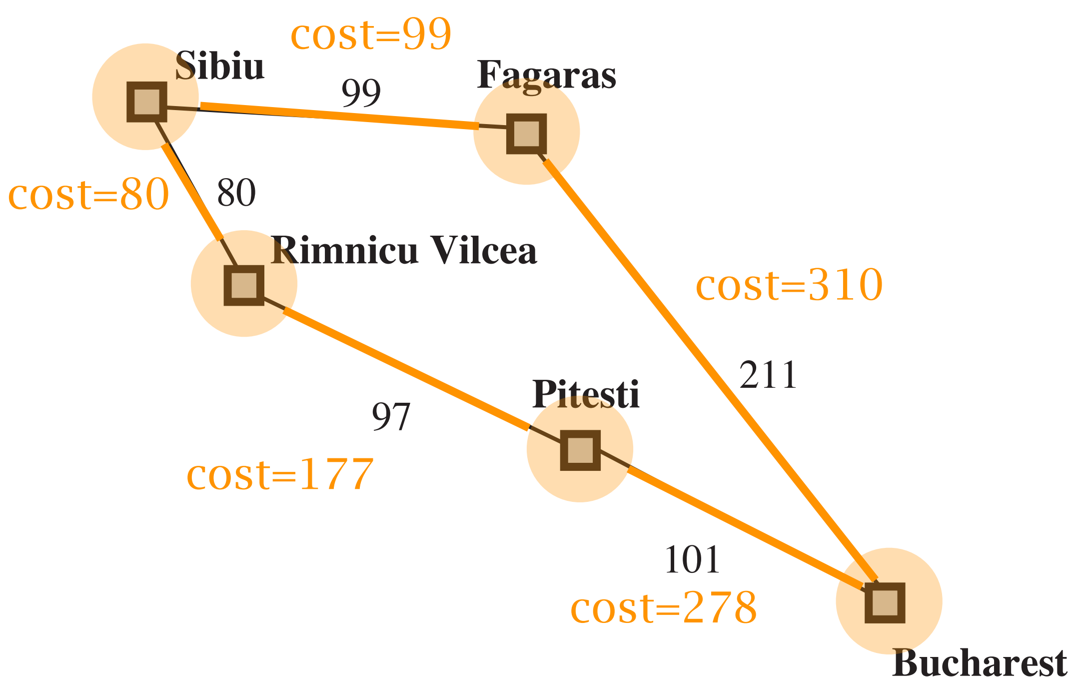
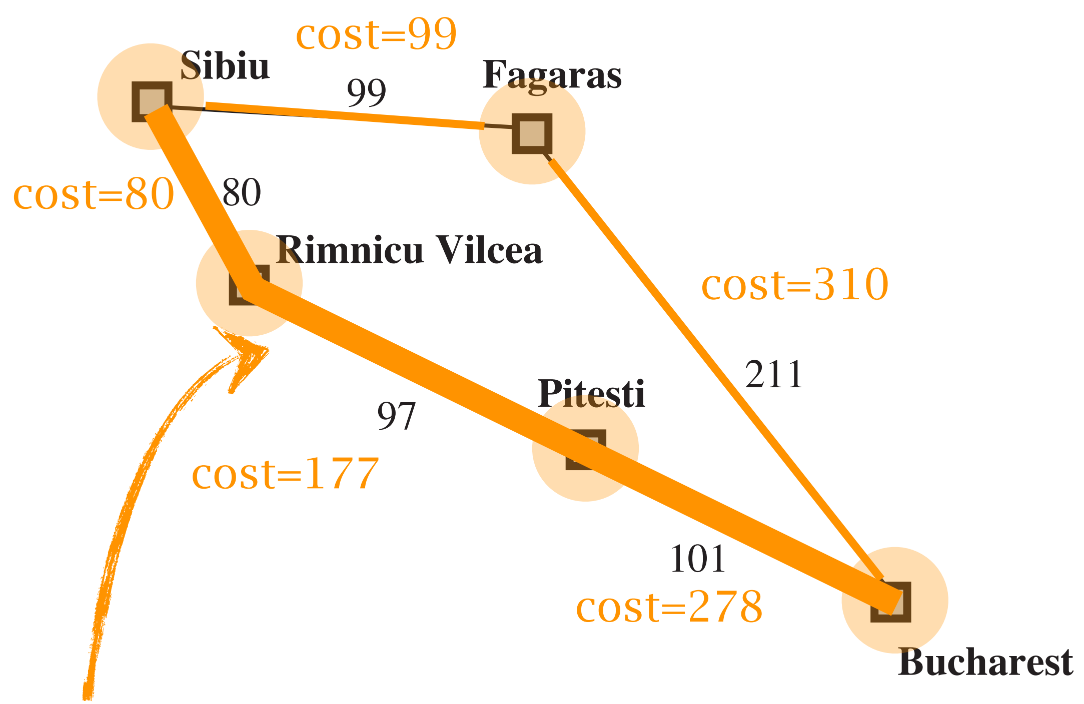


Uniform-cost search

Breadth-first search: First In First Out queue
Depth-first search: Last In First Out queue (stack)
Uniform-cost search: Priority queue (least cost out)

Equivalent to breadth-first if step costs all equal

part of the map



best path from Sibiu to Bucharest

Properties

Complete?? Yes, if step cost $\geq \epsilon$

Time?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$
where C^* is the cost of the optimal solution

Space?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$

Optimal?? Yes—

Question: why it is optimal?

Breadth-first v.s. depth-first

Breadth-first: faster, larger memory
Depth-first: less memory, slower

Question: how to better balance time and space?

Depth-limited search

limit the maximum depth of the depth-first search

i.e., nodes at depth l have no successors

```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
    RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
    cutoff-occurred? ← false
    if GOAL-TEST(problem, STATE[node]) then return node
    else if DEPTH[node] = limit then return cutoff
    else for each successor in EXPAND(node, problem) do
        result ← RECURSIVE-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
    if cutoff-occurred? then return cutoff else return failure
```

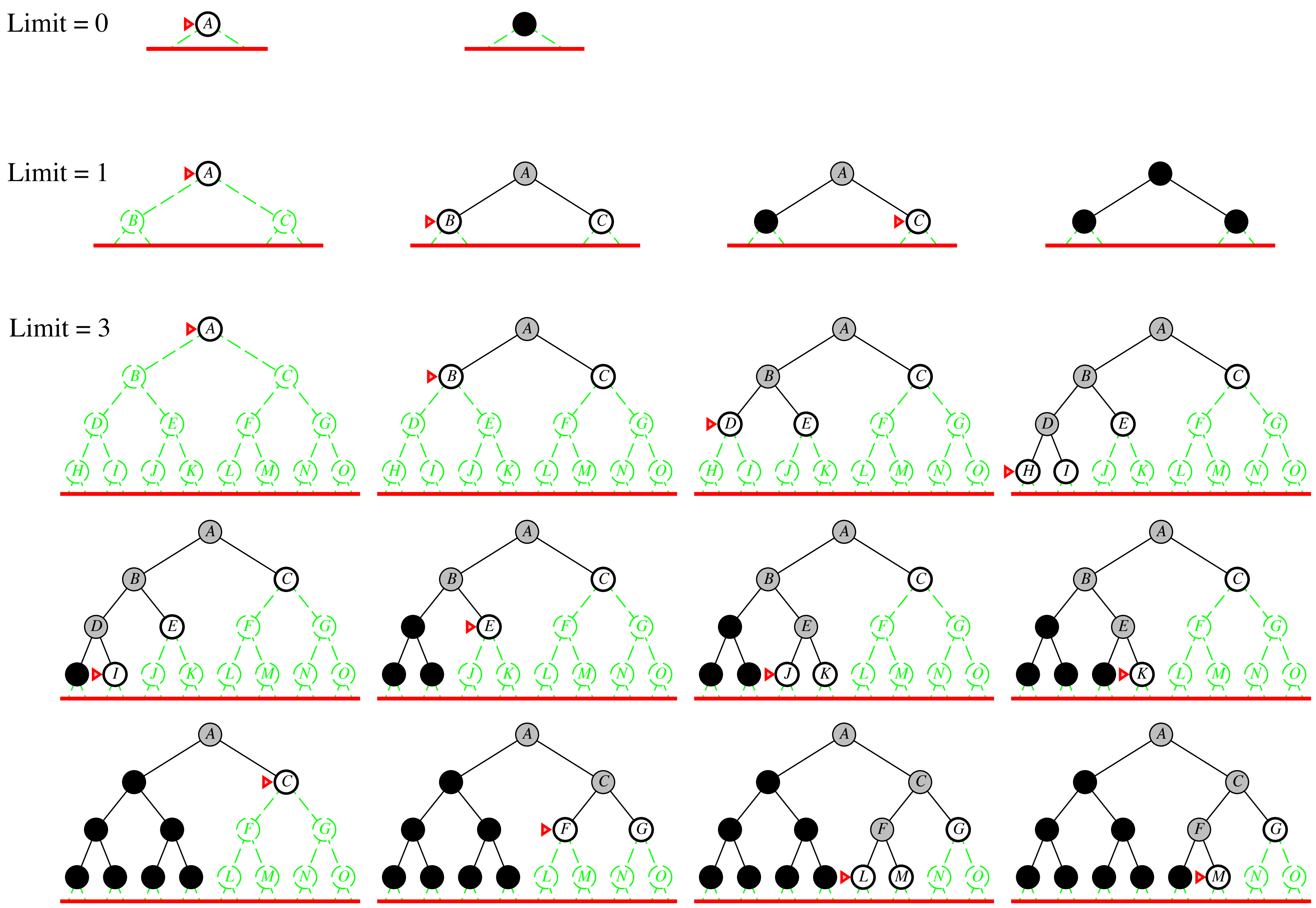
Iterative deepening search

try depth-limited search with increasing limit

restart the search at each time

```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution
    inputs: problem, a problem
    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH( problem, depth)
        if result ≠ cutoff then return result
    end
```

Example



wasteful searching the beginning nodes many times?

Properties

in the same order as the breadth-first search

Complete?? Yes

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

small space as depth-first search

Space?? $O(bd)$

Optimal?? Yes, if step cost $= 1$

Can be modified to explore uniform-cost tree

Numerical comparison for $b = 10$ and $d = 5$, solution at far right leaf:

$$N(\text{IDS}) = 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450$$
$$N(\text{BFS}) = 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 = 1,111,100$$

IDS does better because other nodes at depth d are not expanded

BFS can be modified to apply goal test when a node is **generated**

Summary

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes*	Yes*	No	Yes, if $l \geq d$	Yes
Time	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	b^m	b^l	b^d
Space	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	bm	bl	bd
Optimal?	Yes*	Yes	No	No	Yes*

HW1

作业截止日期：9月28日23:59