PCIC 2025

南京大學 NANJING UNIVERSITY

LAMDA Learning And Mining from DatA

# 第七届泛太平洋因果推断大会

## The 7th Pacific Causal Inference Conference

# Estimating Causal Effects within Markov Equivalence Class in the Presence of Latent Confounders


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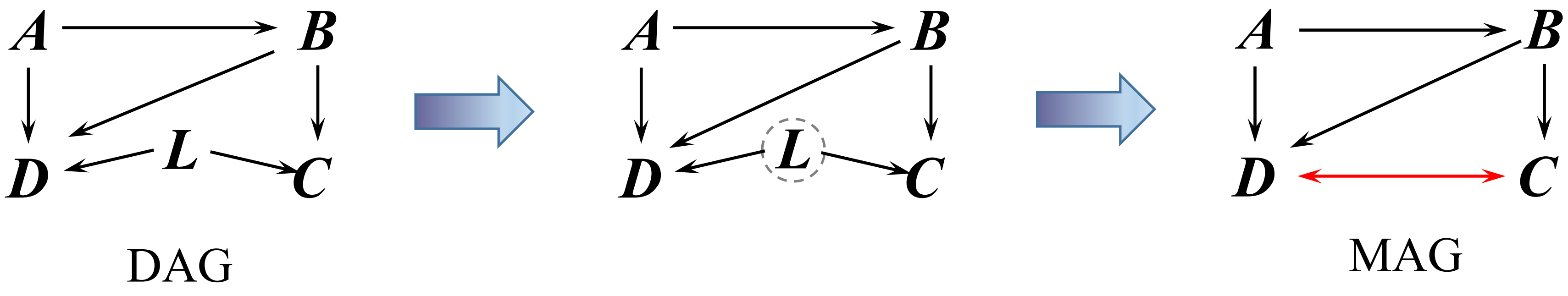
# Causal graph

- In Pearl's causality framework, we generally use a **causal graph** to characterize the causal relations among the variables

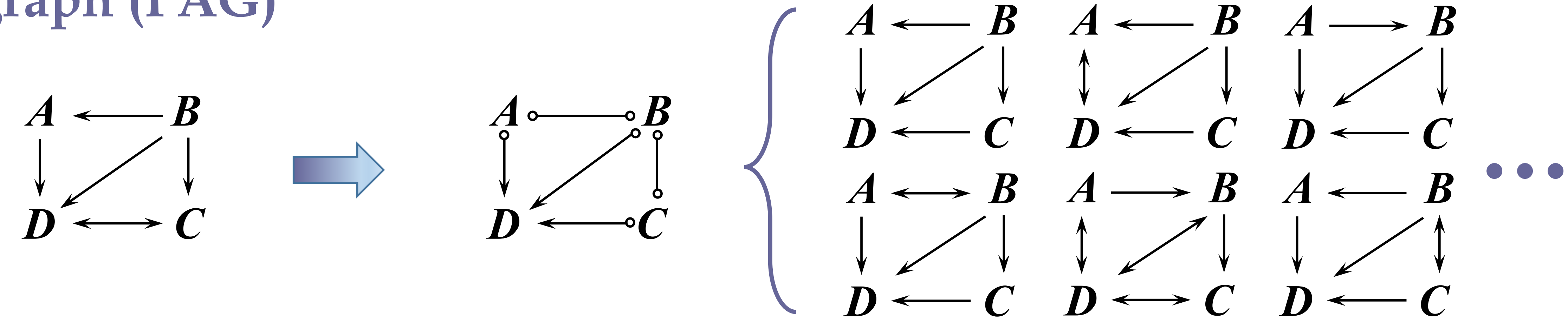



- **Maximal ancestral graph (MAGs)** is a graphical model to characterize causal relations among observable variables in the presence of latent variables

(AOS'02) Thomas Richardson, Peter Spirtes. Ancestral Graph Markov models

# Partial ancestral graph

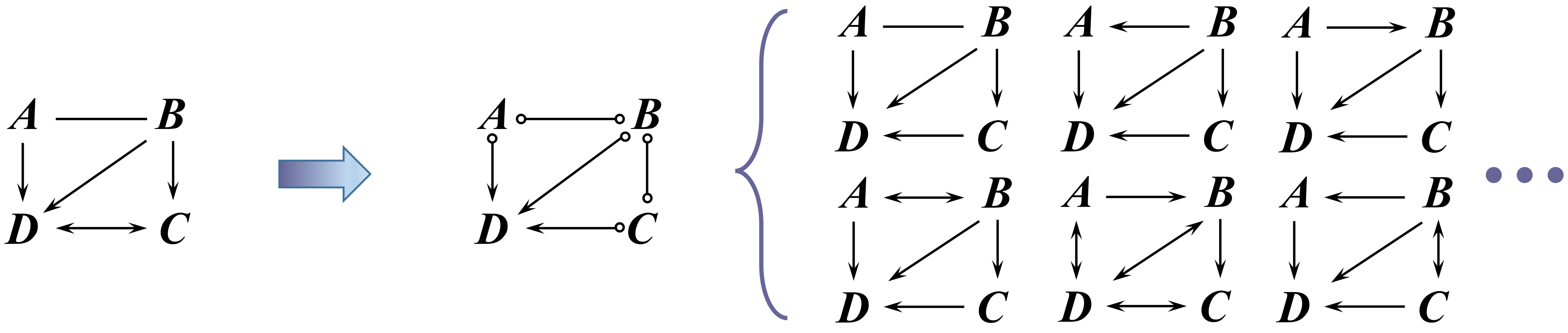
- With observational data and under mild assumptions, we generally cannot identify a MAG, instead, only identify a **partial ancestral graph (PAG)**



MAG PAG An MEC of MAGs

- A PAG represents a **Markov equivalence class (MEC)** of MAGs

# Partial ancestral graph




MAG PAG An MEC of MAGs

**Facts**

1. There are **a large number** of MAGs represented by the PAG.
2. The causal effect in a PAG is possibly **unidentifiable**, since different MAGs are possibly associated with different causal effects.

# Partial ancestral graph

## Facts

1. There are **a large number** of MAGs represented by the PAG.
2. The causal effect in a PAG is possibly **unidentifiable**, since different MAGs are possibly associated with different causal effects.

## Task

Given a PAG, how to determine **the set of causal effects** in all the causal graphs within the Markov equivalence class? – **Set determination**

# IDA: addressing similar task for DAGs

- Input: A **CPDAG** learned from obser. data via constraint-based method
- Target: output set of causal effects $P(Y|do(X))$ in all the DAGs within a MEC represented by a CPDAG without latent confounding
- Challenge: the number of DAGs in a MEC is $O(2^{d^2})$. Determining the set by enumerating all the DAGs is inefficient

ESTIMATING HIGH-DIMENSIONAL INTERVENTION EFFECTS FROM OBSERVATIONAL DATA

BY MARLOES H. MAATHUIS, MARKUS KALISCH AND PETER BÜHLMANN

*ETH Zürich*

We assume that we have observational data generated from an unknown underlying directed acyclic graph (DAG) model. A DAG is typically not identifiable from observational data, but it is possible to consistently estimate the equivalence class of a DAG. Moreover, for any given DAG, causal effects can be estimated using intervention calculus. In this paper, we combine these two

→ Perkovic et al. 2017, Fang & He 2020, Witte et al. 2020, Liu et al. 2020, Guo & Perkovic 2021…

variables, these CPDAGs may no longer be interpreted causally. Relaxing the assumption of unmeasured confounders is possible by extending our methodology to ancestral graphs (see [26, 27] and [34]) which allow for hidden variables. However, deriving bounds for causal effects when the underlying ancestral graph is unknown is an open issue.

→ We relax the causal sufficiency assumption

# IDA under latent confounders

- Input: A **PAG** learned from obser. data via constraint-based method
- Target: output the set of causal effects $P(Y|do(X))$ in all the MAGs represented by a PAG
- Challenge: the number of MAGs in a MEC is $O(3^{d^2})$

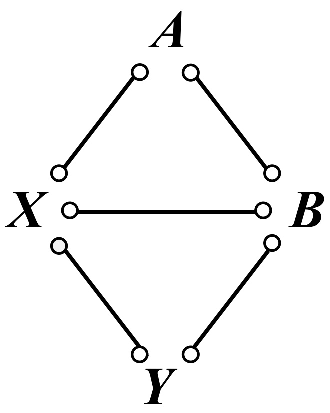
**A direct method**

1. Enumerate all the MAGs
2. In each MAG, find **adjustment set $\boldsymbol{Z}$**, through which there is $P(Y|do(X)) = \int P(\boldsymbol{Z})P(Y|X,\boldsymbol{Z})d\boldsymbol{Z}$

**Main Idea**

Could we enumerate each **variable set** instead of **MAGs**, and **determine whether each set could be an adjustment set** in some MAGs in the MEC?

# An example




PAG

Target: output the set of causal effects of $X$ on $Y$

Enumerate: $\emptyset, \{A\}, \{B\}, \{A, B\} \subseteq 2^{\{A,B\}}$

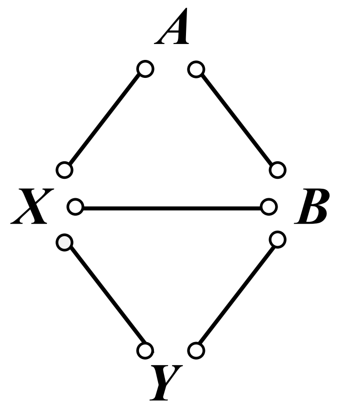
For $\boldsymbol{W} = \emptyset, \{A\}, \{B\}, \{A, B\}$, determine the existence of MAGs represented by the PAG with $\boldsymbol{W}$ as an adjustment set for $(X, Y)$

**The main challenge**

Given any a set $\boldsymbol{W} \subseteq \{\boldsymbol{A}, \boldsymbol{B}\}$, how to determine **the existence of** MAGs represented by the PAG with adjustment set for $(X, Y)$ being $\boldsymbol{W}$

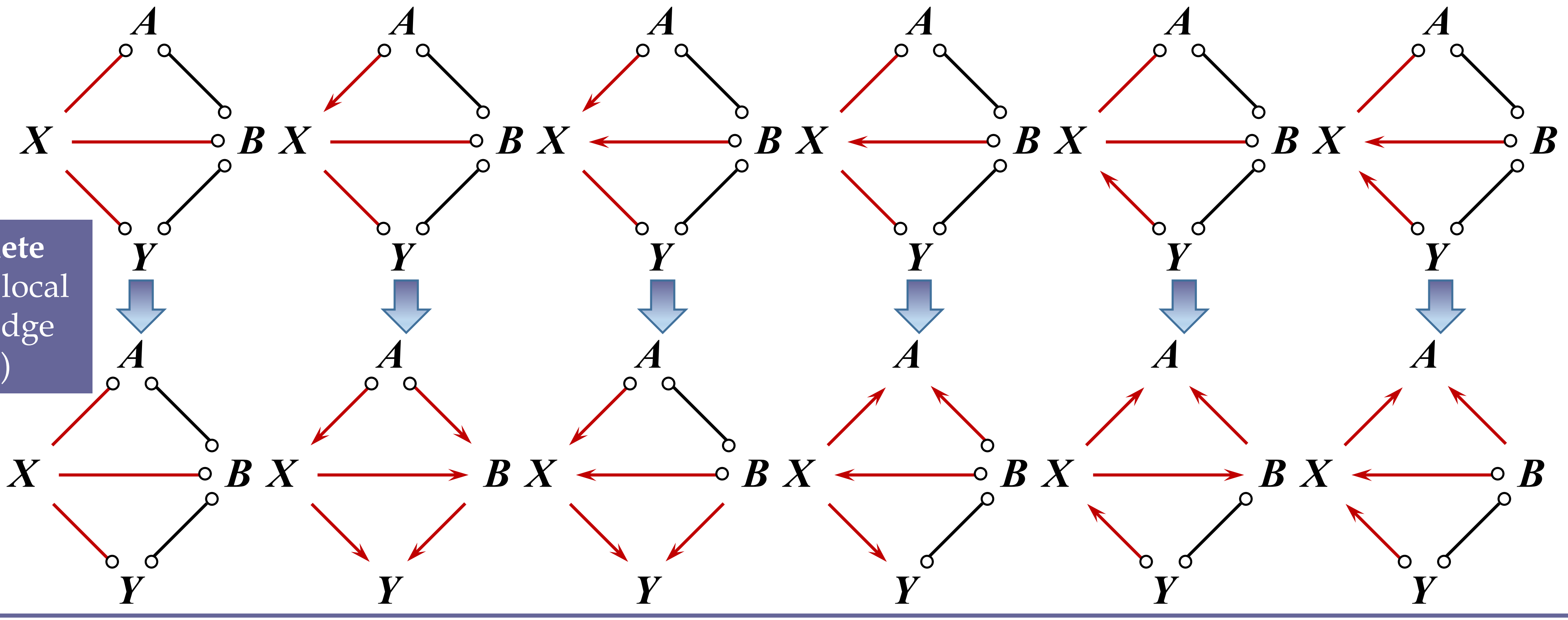
**It cannot be achieved based on only a PAG!**

# Two step procedure



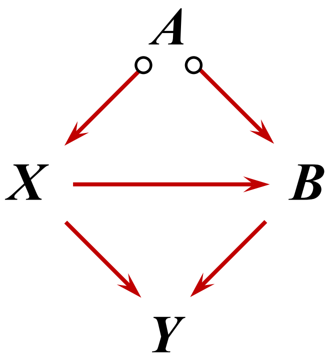

PAG

- Step 1: enumerate the local structures of $X$
  - We propose the **sufficient** and **necessary** condition for determining the **validity of each local structure**
  - For example, 3 circles -> enumerate $2^3$ local structures




Utilizing the **complete orientation rules** for local background knowledge (Wang et al. 2023)

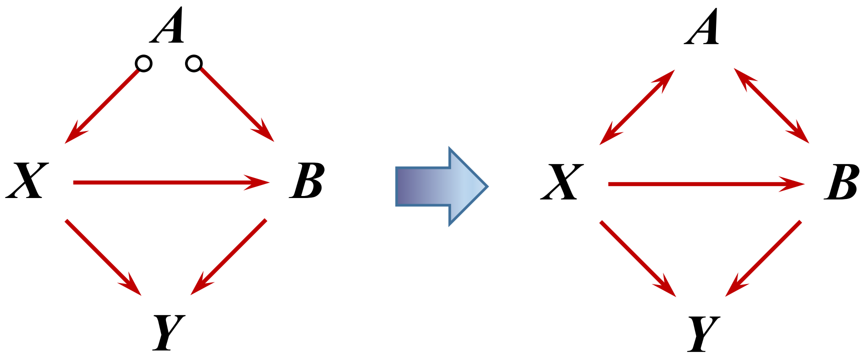
# Two step procedure




$\mathbb{M}$

- Step 2: for any a set $\boldsymbol{W} \subseteq \{A, B\}$, determining whether $\boldsymbol{W}$ could be an adjustment set in MAG consistent with $\mathbb{M}$
- Idea: Determine whether a MAG can be **constructed** based on $\mathbb{M}$ with adjustment set $\boldsymbol{W}$ for $(X, Y)$

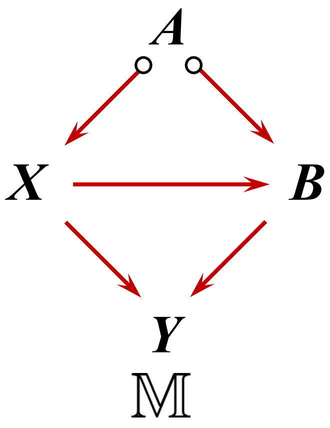
**An example**




**To construct a MAG with adjustment set $W = \emptyset$:**
We need **to prevent some specific variables belonging to $\mathrm{Anc}(Y)$** in the constructed MAG, for otherwise $A$ belongs to the adjustment set for $(X, Y)$ such that $\boldsymbol{W} \neq \emptyset$

**If we construct such a MAG, some variables are restricted to be non-ancestors of $Y$**

# Two step procedure




- Step 2: for any a set $\boldsymbol{W} \subseteq \{A, B\}$, determining whether $\boldsymbol{W}$ could be an adjustment set in MAG consistent with $\mathbb{M}$
- Idea: Determine whether a MAG can be **constructed** based on $\mathbb{M}$ with adjustment set $\boldsymbol{W}$ for $(X, Y)$

**If we construct such a MAG, some variables are restricted to be non-ancestors of $Y$**

In a partial graph, if we want to transform some circles to ensure that some specific vertices are not ancestors of $Y$, there are **exponential number of manners** to achieve in general

We use a variable set $\boldsymbol{S}$ (called **block set**) to characterize each manner

**One main theorem**

**Theorem 2.** Given a maximal local MAG $\mathbb{M}$, for any potential adjustment set $\mathbf{W}$, there exists a MAG $\mathcal{M}$ valid to $\mathbb{M}$ such that $\mathbf{W}$ is an adjustment set in $\mathcal{M}$ if there exists a block set $\mathbf{S}$ such that

(1) $\mathrm{PossDe}(\bar{\mathbf{W}}, \mathbb{M}[-\mathbf{S}]) \cap \mathrm{Pa}(\mathbf{S}, \mathbb{M}) = \emptyset$;

(2) $\mathbb{M}[\mathbf{S}_V]$ is a complete graph for any $V \in \bar{\mathbf{W}}$, where $\mathbf{S}_V = \{V' \in \mathbf{S} | V \circ\!\!\to\!\!* V' \text{ in } \mathbb{M}\}$;

(3) $\mathbb{M}[\mathrm{PossDe}(\bar{\mathbf{W}}, \mathbb{M}[-\mathbf{S}])]$ is bridged relative to $\mathbf{S}$ in $\mathbb{M}$.

For each $S$, it can be verified in **polynomial** time

# Algorithm

**Algorithm 1:** PAGcauses.

**Input:** PAG $\mathcal{P}$, $X$, $Y$

$\widehat{\mathrm{AS}}(\mathcal{P}) = \emptyset$ // Record all the valid adjustment sets;
**if** $X \notin \mathrm{PossAn}(Y, \mathcal{P})$ **then return** No causal effects;
**if** *the conditions in Proposition 1 are satisfied for $\mathcal{P}$* **then**
  **return** $\widehat{\mathrm{AS}}(\mathcal{P}) \leftarrow \{\text{D-SEP}(X, Y, \mathcal{P}_{\underline{X}})\}$
**for** *each set $\mathbf{C} \subseteq \{V \mid V \ast\!\!-\!\!\circ\, X \text{ in } \mathcal{P}\}$* **do**
  **if** *the three conditions in Proposition 2 are satisfied* **then**
    Obtain a maximal local MAG $\mathbb{M}$ based on $\mathcal{P}$ and $\mathbf{C}$;
    Find all potential adjustment sets $\mathbf{W}_1, \mathbf{W}_2, \cdots$ given $\mathbb{M}$ according to Definition 6;
    **for** *each potential adjustment set $\mathbf{W}_i$* **do**
      **for** *each block set* $\mathbf{S}$ **do**
        **if** *the three conditions in Theorem 2 are satisfied given* $\mathbf{S}$ **then**
          $\widehat{\mathrm{AS}}(\mathcal{P}) \leftarrow \widehat{\mathrm{AS}}(\mathcal{P}) \cup \{\mathbf{W}_i\}$;
          **break** // Break the loop of $\mathbf{S}$;

**Output:** Set of causal effects via adjustment in the given PAG $\mathcal{P}$ identified with $\widehat{\mathrm{AS}}(\mathcal{P})$ by (1)

If the causal effect is identifiable given a PAG, stop

Enumerate each valid local structure

Enumerate each subset $\boldsymbol{W}$ possibly being adjustment set

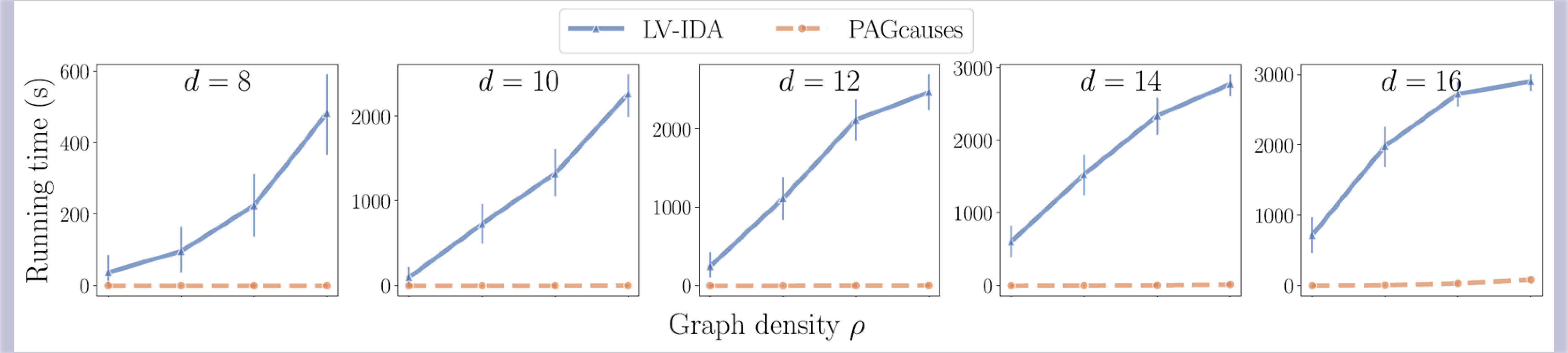
Enumerate each **block set $\boldsymbol{S}$**

Determine the existence of MAGs with adjustment set $\boldsymbol{W}$ given circles transformation dictated by $\boldsymbol{S}$

## Theoretical results

1. Our method can return the **identical** set as the SOTA MAG enumeration method;
2. Our method takes **super-exponentially** less time than SOTA MAG enumeration method

# Simulations



# Limitation

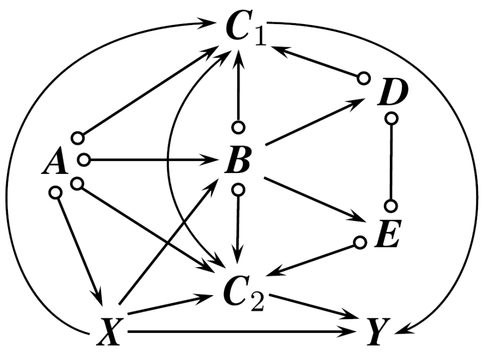
- In the presence of latent confounders, it is possible that the true causal effect cannot be obtained from observational data, even if for a given MAG


- The determined set in our method only includes the causal effect value that is obtainable from observational data

# Improved version

- Note in Step 2, we determine whether we can construct a MAG with adjustment set $\boldsymbol{W}$ based on $\mathbb{M}$

- In this process, we need to ensure that some specific vertices are not ancestors of $Y$, where there are exponential number of block set **S** to potentially achieve it. Could we find proper **S** directly if possible?



$\mathbb{M}$

If $W = \emptyset$ is an adjustment set in some MAG consistent with $\mathbb{M}$, then $A$ cannot be an ancestor of $Y$, to achieve this, we

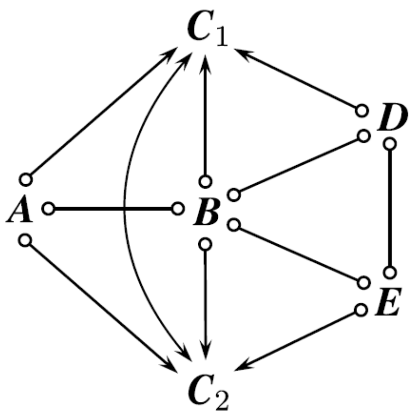
1. Transforming $A \circ\!\!\rightarrow C_1/C_2$ to $A \Leftrightarrow C_1/C_2$
2. Some circles can be further oriented given the BK

Background knowledge

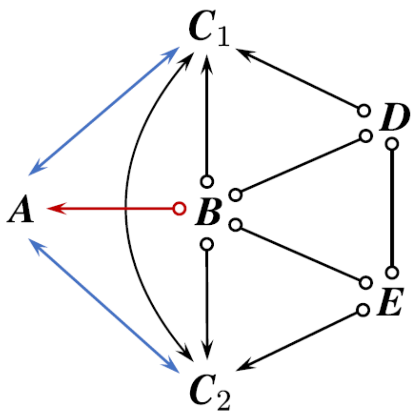
# Orienting graphs with BK

- What causal relations are identifiable in a PAG when incorporating additional background knowledge (BK)
  - ☐ Ten rules in FCI algorithm can be utilized, but are they sound and complete to incorporate BK?
  - ☐ No.

Counterexample 1




PAG

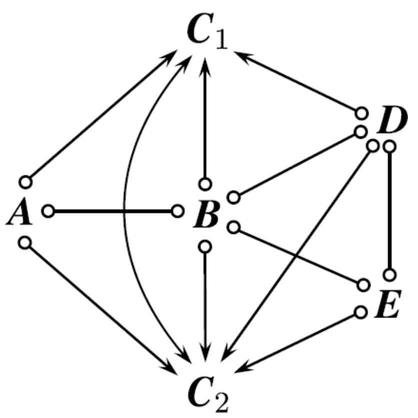



Incor. **BK**

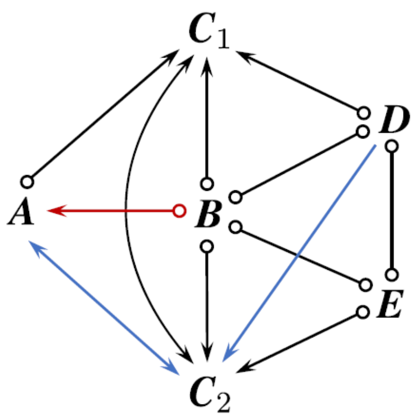
# Orienting graphs with BK

- What causal relations are identifiable in a PAG when incorporating additional background knowledge (BK)
  - Ten rules in FCI algorithm can be utilized, but are they sound and complete to incorporate BK?
  - No.

Counterexample 2



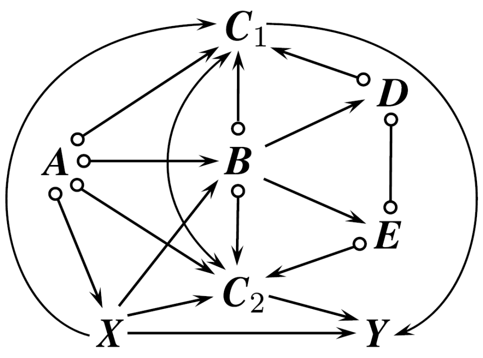

PAG




Incor. **BK**

# Orienting graphs with BK

- What causal relations are identifiable in a PAG when incorporating additional background knowledge (BK)
  - Ten rules in FCI algorithm can be utilized, but are they sound and complete to incorporate BK?
  - No.
- We propose two novel orientation rules to incorporate BK into a PAG

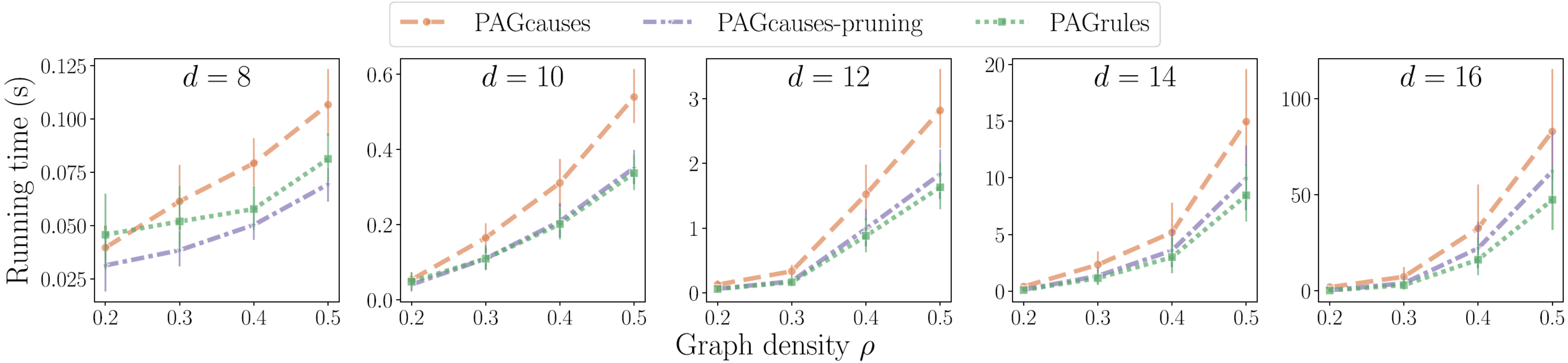



$\mathbb{M}$

Whether $W = \emptyset$ is an adjustment set in some MAG consistent with $\mathbb{M}$

1. Transforming $A \circ\!\!\rightarrow C_1/C_2$ to $A \Leftrightarrow C_1/C_2$
2. Some circles can be further oriented given the novel orientation rules: transforming $A \circ\!\!\rightarrow B$ to $A \Leftrightarrow B$
3. Detecting whether the obtained blocking set $S$ satisfy the conditions in Thm. 2 (in polynomial time)

# Simulations



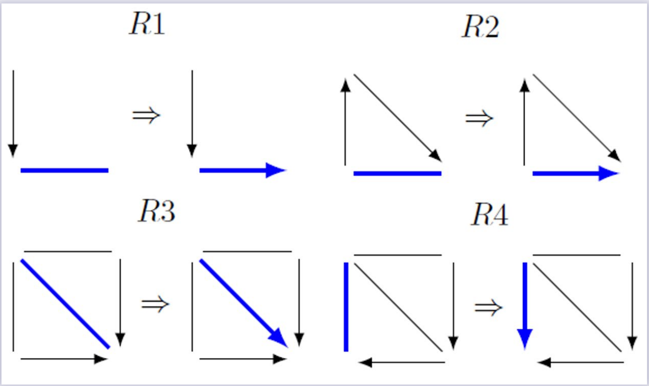

- All the methods can find the same set of causal effects
- The application of the novel rules can help reduce the complexity

# Incorporating BK

- It has been an open problem for a long time what causal relations are identifiable given observational data and BK in the presence of latent variables

Causal sufficiency

Meek rules




(From Perkovic et al. 2017)

- The current rules are not complete for incorporating BK
- We propose seven new rules [**<u>Wang</u>**-Du-Zhou, ICML 2025]

# References

## Set determination:


- ✓ Tian-Zuo Wang, Lue Tao, Tian Qin, Zhi-Hua Zhou. Estimating possible causal effects with latent variables via adjustment and novel rule orientation. **Artificial Intelligence**, 2025.
- ✓ Tian-Zuo Wang, Tian Qin, Zhi-Hua Zhou. Estimating possible causal effects with latent variables via adjustment. **ICML 2023**.


## Orientation rules:


- ✓ Tian-Zuo Wang, Wen-Bo Du, Zhi-Hua Zhou. Polynomial-delay MAG listing with novel locally complete orientation rules. **ICML 2025**. Oral (0.99%).

# Joint works with

Lue Tao
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Tian Qin
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Zhi-Hua Zhou
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***Thanks !***

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*Thanks!*