Lecture 8: Knowledge 2
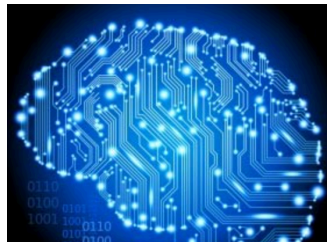
http://cs.nju.edu.cn/yuy/course_ai16.ashx

Previously...

```
function HYBRID-WUMPUS-AGENT(percept) returns an action
    inputs: percept, a list, [stench,breeze,glitter,bump,scream]
    persistent: KB, a knowledge base, initially the atemporal "wumpus physics"
                t, a counter, initially 0, indicating time
                plan, an action sequence, initially empty

    TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
    TELL the KB the temporal "physics" sentences for time t
```
$safe \leftarrow \{[x, y] : \text{ASK}(KB, OK^t_{x,y}) = true\}$
if $\text{ASK}(KB, Glitter^t) = true$ **then**
   $plan \leftarrow [Grab] + \text{PLAN-ROUTE}(current, \{[1,1]\}, safe) + [Climb]$
if $plan$ is empty **then**
   $unvisited \leftarrow \{[x, y] : \text{ASK}(KB, L^{t'}_{x,y}) = false \text{ for all } t' \leq t\}$
   $plan \leftarrow \text{PLAN-ROUTE}(current, unvisited \cap safe, safe)$
if $plan$ is empty and $\text{ASK}(KB, HaveArrow^t) = true$ **then**
   $possible\_wumpus \leftarrow \{[x, y] : \text{ASK}(KB, \neg W_{x,y}) = false\}$
   $plan \leftarrow \text{PLAN-SHOT}(current, possible\_wumpus, safe)$
if $plan$ is empty **then** // no choice but to take a risk
   $not\_unsafe \leftarrow \{[x, y] : \text{ASK}(KB, \neg OK^t_{x,y}) = false\}$
   $plan \leftarrow \text{PLAN-ROUTE}(current, unvisited \cap not\_unsafe, safe)$
if $plan$ is empty **then**
   $plan \leftarrow \text{PLAN-ROUTE}(current, \{[1, 1]\}, safe) + [Climb]$
$action \leftarrow \text{POP}(plan)$
$\text{TELL}(KB, \text{MAKE-ACTION-SENTENCE}(action, t))$
$t \leftarrow t + 1$
return $action$

---

function PLAN-ROUTE(current,goals,allowed) **returns** an action sequence
  inputs: $current$, the agent's current position
        $goals$, a set of squares; try to plan a route to one of them
        $allowed$, a set of squares that can form part of the route

$problem \leftarrow \text{ROUTE-PROBLEM}(current, goals, allowed)$
return A*-GRAPH-SEARCH($problem$)

Pros and cons of propositional logic

😄 Propositional logic is **declarative**: pieces of syntax correspond to facts

😄 Propositional logic allows partial/disjunctive/negated information
(unlike most data structures and databases)

😄 Propositional logic is **compositional**:
meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$

😄 Meaning in propositional logic is **context-independent**
(unlike natural language, where meaning depends on context)

☹ Propositional logic has very limited expressive power
(unlike natural language)
E.g., cannot say "pits cause breezes in adjacent squares"
    except by writing one sentence for each square

First-order logic

Whereas propositional logic assumes world contains **facts**,
first-order logic (like natural language) assumes the world contains

- Objects: people, houses, numbers, theories, Ronald McDonald, colors, baseball games, wars, centuries . . .

- Relations: red, round, bogus, prime, multistoried . . .,
brother of, bigger than, inside, part of, has color, occurred after, owns, comes between, . . .

- Functions: father of, best friend, third inning of, one more than, end of . . .

Logics in general

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief
Fuzzy logic	facts + degree of truth	known interval value

Syntax of FOL: Basic elements

Constants	$KingJohn, \ 2, \ UCB, \ldots$
Predicates	$Brother, \ >, \ldots$
Functions	$Sqrt, \ LeftLegOf, \ldots$
Variables	$x, \ y, \ a, \ b, \ldots$
Connectives	$\land \ \lor \ \lnot \ \Rightarrow \ \Leftrightarrow$
Equality	$=$
Quantifiers	$\forall \ \exists$

Atomic sentences

$$\text{Atomic sentence} \;=\; predicate(term_1, \ldots, term_n)$$
$$\text{or } term_1 = term_2$$

$$\text{Term} \;=\; function(term_1, \ldots, term_n)$$
$$\text{or } constant \text{ or } variable$$

E.g., $Brother(KingJohn, RichardTheLionheart)$
$> (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))$

Complex sentences

Complex sentences are made from atomic sentences using connectives

$$\neg S, \quad S_1 \wedge S_2, \quad S_1 \vee S_2, \quad S_1 \Rightarrow S_2, \quad S_1 \Leftrightarrow S_2$$

E.g. $Sibling(KingJohn, Richard) \Rightarrow Sibling(Richard, KingJohn)$
$>(1,2) \vee \leq(1,2)$
$>(1,2) \wedge \neg >(1,2)$

Truth in first-order logic

Sentences are true with respect to a model and an interpretation

Model contains ≥ 1 objects (domain elements) and relations among them
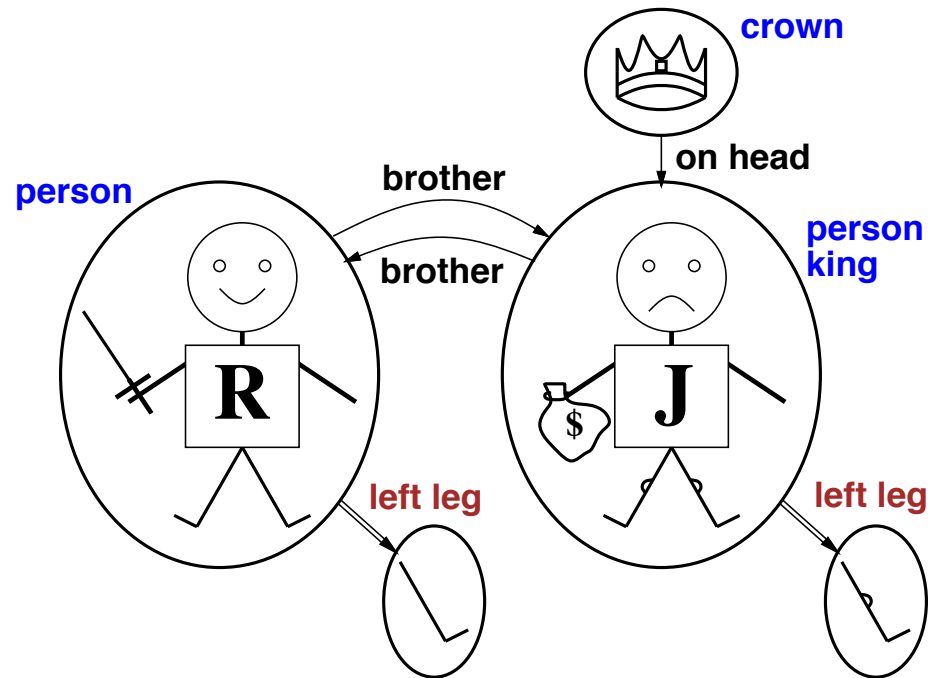
Interpretation specifies referents for
    constant symbols $\rightarrow$ objects
    predicate symbols $\rightarrow$ relations
    function symbols $\rightarrow$ functional relations

An atomic sentence $predicate(term_1, \ldots, term_n)$ is true
iff the objects referred to by $term_1, \ldots, term_n$
are in the relation referred to by $predicate$

Models for FOL: Example

Consider the interpretation in which
$Richard \rightarrow$ Richard the Lionheart
$John \rightarrow$ the evil King John
$Brother \rightarrow$ the brotherhood relation

Under this interpretation, $Brother(Richard, John)$ is true
just in case Richard the Lionheart and the evil King John
are in the brotherhood relation in the model

Models for FOL: Lots!

Entailment in propositional logic can be computed by enumerating models

We **can** enumerate the FOL models for a given KB vocabulary:

For each number of domain elements n from 1 to ∞
    For each k-ary predicate P_k in the vocabulary
        For each possible k-ary relation on n objects
            For each constant symbol C in the vocabulary
                For each choice of referent for C from n objects . . .

Computing entailment by enumerating FOL models is not easy!

Universal quantification

$\forall \langle variables \rangle \ \langle sentence \rangle$

Everyone at Berkeley is smart:
$\forall x \ \ At(x, Berkeley) \ \Rightarrow \ Smart(x)$

$\forall x \ \ P$ is true in a model m iff P is true with x being
each possible object in the model

Roughly speaking, equivalent to the conjunction of instantiations of P

$$(At(KingJohn, Berkeley) \ \Rightarrow \ Smart(KingJohn))$$
$$\wedge \ (At(Richard, Berkeley) \ \Rightarrow \ Smart(Richard))$$
$$\wedge \ (At(Berkeley, Berkeley) \ \Rightarrow \ Smart(Berkeley))$$
$$\wedge \ \dots$$

A common mistake to avoid

Typically, $\Rightarrow$ is the main connective with $\forall$

Common mistake: using $\wedge$ as the main connective with $\forall$:

$$\forall x \quad At(x, Berkeley) \wedge Smart(x)$$

means "Everyone is at Berkeley and everyone is smart"

Existential quantification

$\exists \langle variables \rangle \ \langle sentence \rangle$

Someone at Stanford is smart:
$\exists x \ \ At(x, Stanford) \wedge Smart(x)$

$\exists x \ \ P$ is true in a model m iff P is true with x being **some** possible object in the model

Roughly speaking, equivalent to the disjunction of instantiations of P

$$(At(KingJohn, Stanford) \wedge Smart(KingJohn))$$
$$\vee \ (At(Richard, Stanford) \wedge Smart(Richard))$$
$$\vee \ (At(Stanford, Stanford) \wedge Smart(Stanford))$$
$$\vee \ \ldots$$

Another common mistake to avoid

Typically, $\wedge$ is the main connective with $\exists$

Common mistake: using $\Rightarrow$ as the main connective with $\exists$:

$$\exists x \ At(x, Stanford) \Rightarrow Smart(x)$$

is true if there is anyone who is not at Stanford!

Properties of quantifiers

$\forall x \ \forall y$ is the same as $\forall y \ \forall x$ (<u>why??</u>)

$\exists x \ \exists y$ is the same as $\exists y \ \exists x$ (<u>why??</u>)

$\exists x \ \forall y$ is **not** the same as $\forall y \ \exists x$

$\exists x \ \forall y \ Loves(x, y)$
"There is a person who loves everyone in the world"

$\forall y \ \exists x \ Loves(x, y)$
"Everyone in the world is loved by at least one person"

Quantifier duality: each can be expressed using the other

$\forall x \ Likes(x, IceCream)$ $\neg \exists x \ \neg Likes(x, IceCream)$

$\exists x \ Likes(x, Broccoli)$ $\neg \forall x \ \neg Likes(x, Broccoli)$

Fun with sentences

Brothers are siblings

$$\forall x, y \ \ Brother(x, y) \Rightarrow Sibling(x, y).$$

"Sibling" is symmetric

$$\forall x, y \ \ Sibling(x, y) \Leftrightarrow Sibling(y, x).$$

One's mother is one's female parent

$$\forall x, y \ \ Mother(x, y) \Leftrightarrow (Female(x) \land Parent(x, y)).$$

A first cousin is a child of a parent's sibling

$$\forall x, y \ \ FirstCousin(x, y) \Leftrightarrow \exists p, ps \ \ Parent(p, x) \land Sibling(ps, p) \land Parent(ps, y)$$

Equality

$term_1 = term_2$ is true under a given interpretation
if and only if $term_1$ and $term_2$ refer to the same object

E.g., $1 = 2$ and $\forall x \ \times(Sqrt(x), Sqrt(x)) = x$ are satisfiable
$2 = 2$ is valid

E.g., definition of (full) $Sibling$ in terms of $Parent$:
$$\forall x, y \ Sibling(x, y) \ \Leftrightarrow \ [\neg(x = y) \wedge \exists m, f \ \neg(m = f) \wedge$$
$$Parent(m, x) \wedge Parent(f, x) \wedge Parent(m, y) \wedge Parent(f, y)]$$

Interacting with FOL KBs

Suppose a wumpus-world agent is using an FOL KB
and perceives a smell and a breeze (but no glitter) at $t = 5$:

$Tell(KB, Percept([Smell, Breeze, None], 5))$
$Ask(KB, \exists a \ Action(a, 5))$

I.e., does KB entail any particular actions at $t = 5$?

Answer: $Yes, \ \{a/Shoot\}$ $\leftarrow$ substitution (binding list)

Given a sentence S and a substitution σ,
$S\sigma$ denotes the result of plugging σ into S; e.g.,
$S = Smarter(x, y)$
$\sigma = \{x/Hillary, y/Bill\}$
$S\sigma = Smarter(Hillary, Bill)$

$Ask(KB, S)$ returns some/all σ such that $KB \models S\sigma$

Knowledge base for the wumpus world

"Perception"

$$\forall\, b, g, t \;\; Percept([Smell, b, g], t) \;\Rightarrow\; Smelt(t)$$
$$\forall\, s, b, t \;\; Percept([s, b, Glitter], t) \;\Rightarrow\; AtGold(t)$$

Reflex: $\forall\, t \;\; AtGold(t) \;\Rightarrow\; Action(Grab, t)$

Reflex with internal state: do we have the gold already?

$$\forall\, t \;\; AtGold(t) \wedge \neg Holding(Gold, t) \;\Rightarrow\; Action(Grab, t)$$

$Holding(Gold, t)$ cannot be observed
$\Rightarrow$ keeping track of change is essential

Deducing hidden properties

Properties of locations:
$$\forall\, x, t \ \ At(Agent, x, t) \wedge Smelt(t) \ \Rightarrow \ Smelly(x)$$
$$\forall\, x, t \ \ At(Agent, x, t) \wedge Breeze(t) \ \Rightarrow \ Breezy(x)$$

Squares are breezy near a pit:

Diagnostic rule—infer cause from effect
$$\forall\, y \ \ Breezy(y) \ \Rightarrow \ \exists\, x \ \ Pit(x) \wedge Adjacent(x, y)$$

Causal rule—infer effect from cause
$$\forall\, x, y \ \ Pit(x) \wedge Adjacent(x, y) \ \Rightarrow \ Breezy(y)$$

Neither of these is complete—e.g., the causal rule doesn't say whether squares far away from pits can be breezy

Definition for the $Breezy$ predicate:
$$\forall\, y \ \ Breezy(y) \ \Leftrightarrow \ [\exists\, x \ \ Pit(x) \wedge Adjacent(x, y)]$$

Keeping track of change

Facts hold in situations, rather than eternally
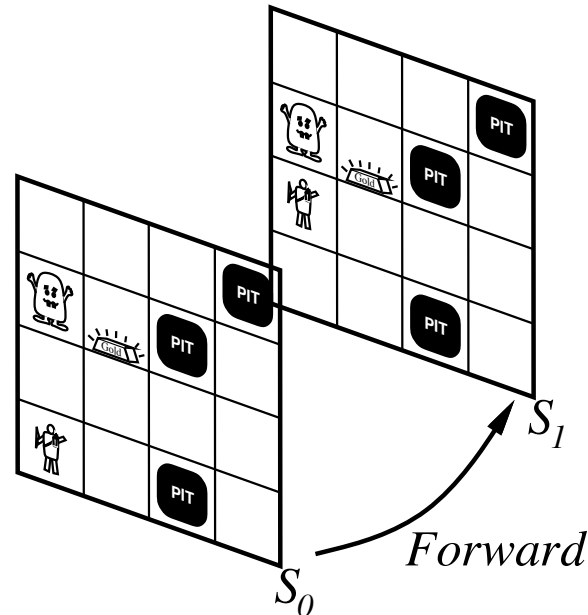E.g., $Holding(Gold, Now)$ rather than just $Holding(Gold)$

Situation calculus is one way to represent change in FOL:
      Adds a situation argument to each non-eternal predicate
      E.g., Now in $Holding(Gold, Now)$ denotes a situation

Situations are connected by the $Result$ function
$Result(a, s)$ is the situation that results from doing a in s

Describing actions I

"Effect" axiom—describe changes due to action

$$\forall\, s \;\; AtGold(s) \;\Rightarrow\; Holding(Gold, Result(Grab, s))$$

"Frame" axiom—describe **non-changes** due to action

$$\forall\, s \;\; HaveArrow(s) \;\Rightarrow\; HaveArrow(Result(Grab, s))$$

Frame problem: find an elegant way to handle non-change
      (a) representation—avoid frame axioms
      (b) inference—avoid repeated "copy-overs" to keep track of state

Qualification problem: true descriptions of real actions require endless caveats—
what if gold is slippery or nailed down or . . .

Ramification problem: real actions have many secondary consequences—
what about the dust on the gold, wear and tear on gloves, . . .

Describing actions II

Successor-state axioms solve the representational frame problem

Each axiom is "about" a **predicate** (not an action per se):

P true afterwards $\Leftrightarrow$ [an action made P true
$\lor$ P true already and no action made P false]

For holding the gold:
$$\forall a, s \ Holding(Gold, Result(a, s)) \Leftrightarrow$$
$$[(a = Grab \land AtGold(s))$$
$$\lor (Holding(Gold, s) \land a \neq Release)]$$

Making plans

Initial condition in KB:

$$At(Agent, [1, 1], S_0)$$
$$At(Gold, [1, 2], S_0)$$

Query: $Ask(KB, \exists s \; Holding(Gold, s))$
      i.e., in what situation will I be holding the gold?

Answer: $\{s/Result(Grab, Result(Forward, S_0))\}$
      i.e., go forward and then grab the gold

This assumes that the agent is interested in plans starting at S_0 and that S_0 is the only situation described in the KB

Making plans: A better way

Represent plans as action sequences $[a_1, a_2, \ldots, a_n]$

$PlanResult(p, s)$ is the result of executing p in s

Then the query $Ask(KB, \exists p \ Holding(Gold, PlanResult(p, S_0)))$
has the solution $\{p/[Forward, Grab]\}$

Definition of $PlanResult$ in terms of $Result$:
$$\forall s \ PlanResult([], s) = s$$
$$\forall a, p, s \ PlanResult([a|p], s) = PlanResult(p, Result(a, s))$$

Planning systems are special-purpose reasoners designed to do this type of inference more efficiently than a general-purpose reasoner

Summary

First-order logic:
  – objects and relations are semantic primitives
  – syntax: constants, functions, predicates, equality, quantifiers

Increased expressive power: sufficient to define wumpus world

Situation calculus:
  – conventions for describing actions and change in FOL
  – can formulate planning as inference on a situation calculus KB

A brief history of reasoning

450B.C.	Stoics	propositional logic, inference (maybe)
322B.C.	Aristotle	"syllogisms" (inference rules), quantifiers
1565	Cardano	probability theory (propositional logic + uncertainty)
1847	Boole	propositional logic (again)
1879	Frege	first-order logic
1922	Wittgenstein	proof by truth tables
1930	Gödel	$\exists$ complete algorithm for FOL
1930	Herbrand	complete algorithm for FOL (reduce to propositional)
1931	Gödel	$\neg\exists$ complete algorithm for arithmetic
1960	Davis/Putnam	"practical" algorithm for propositional logic
1965	Robinson	"practical" algorithm for FOL—resolution

Universal instantiation (UI)

Every instantiation of a universally quantified sentence is entailed by it:

$$\frac{\forall v \;\; \alpha}{\text{SUBST}(\{v/g\}, \alpha)}$$

for any variable v and ground term g

E.g., $\forall x \;\; King(x) \wedge Greedy(x) \Rightarrow Evil(x)$ yields

$$King(John) \wedge Greedy(John) \Rightarrow Evil(John)$$
$$King(Richard) \wedge Greedy(Richard) \Rightarrow Evil(Richard)$$
$$King(Father(John)) \wedge Greedy(Father(John)) \Rightarrow Evil(Father(John))$$

Existential instantiation (EI)

For any sentence α, variable v, and constant symbol k
that does not appear elsewhere in the knowledge base:

$$\frac{\exists v \; \alpha}{\text{SUBST}(\{v/k\}, \alpha)}$$

E.g., $\exists x \; Crown(x) \wedge OnHead(x, John)$ yields

$$Crown(C_1) \wedge OnHead(C_1, John)$$

provided C_1 is a new constant symbol, called a Skolem constant

Another example: from $\exists x \; d(x^y)/dy = x^y$ we obtain

$$d(e^y)/dy = e^y$$

provided e is a new constant symbol

Instantiation

UI can be applied several times to **add** new sentences;
the new KB is logically equivalent to the old

EI can be applied once to **replace** the existential sentence;
the new KB is **not** equivalent to the old,
but is satisfiable iff the old KB was satisfiable

Reduction to propositional inference

Suppose the KB contains just the following:

$$\forall\, x \;\; King(x) \wedge Greedy(x) \;\Rightarrow\; Evil(x)$$
$$King(John)$$
$$Greedy(John)$$
$$Brother(Richard, John)$$

Instantiating the universal sentence in **all possible** ways, we have

$$King(John) \wedge Greedy(John) \;\Rightarrow\; Evil(John)$$
$$King(Richard) \wedge Greedy(Richard) \;\Rightarrow\; Evil(Richard)$$
$$King(John)$$
$$Greedy(John)$$
$$Brother(Richard, John)$$

The new KB is propositionalized: proposition symbols are

$$King(John),\;\; Greedy(John),\;\; Evil(John), King(Richard)\;\text{etc.}$$

Reduction to propositional inference

Claim: a ground sentence* is entailed by new KB iff entailed by original KB

Claim: every FOL KB can be propositionalized so as to preserve entailment

Idea: propositionalize KB and query, apply resolution, return result

Problem: with function symbols, there are infinitely many ground terms,
e.g., $Father(Father(Father(John)))$

Theorem: Herbrand (1930). If a sentence α is entailed by an FOL KB,
it is entailed by a **finite** subset of the propositional KB

Idea: For $n = 0$ to ∞ do
create a propositional KB by instantiating with depth-n terms
see if α is entailed by this KB

Problem: works if α is entailed, loops if α is not entailed

Theorem: Turing (1936), Church (1936), entailment in FOL is semidecidable

Problems with propositionalization

Propositionalization seems to generate lots of irrelevant sentences.
E.g., from

$$\forall\, x \;\; King(x) \wedge Greedy(x) \;\Rightarrow\; Evil(x)$$
$$King(John)$$
$$\forall\, y \;\; Greedy(y)$$
$$Brother(Richard, John)$$

it seems obvious that $Evil(John)$, but propositionalization produces lots of facts such as $Greedy(Richard)$ that are irrelevant

With p k-ary predicates and n constants, there are $p \cdot n^k$ instantiations

With function symbols, it gets nuch much worse!

Unification

We can get the inference immediately if we can find a substitution θ such that $King(x)$ and $Greedy(x)$ match $King(John)$ and $Greedy(y)$

$\theta = \{x/John, y/John\}$ works

$\text{UNIFY}(\alpha, \beta) = \theta$ if $\alpha\theta = \beta\theta$

p	q	θ
$Knows(John, x)$	$Knows(John, Jane)$	$\{x/Jane\}$
$Knows(John, x)$	$Knows(y, OJ)$	$\{x/OJ, y/John\}$
$Knows(John, x)$	$Knows(y, Mother(y))$	$\{y/John, x/Mother(John)\}$
$Knows(John, x)$	$Knows(x, OJ)$	$fail$

Standardizing apart eliminates overlap of variables, e.g., $Knows(z_{17}, OJ)$

Generalized Modus Ponens (GMP)
（前件推理）

$$\frac{p_1{}',\ \ p_2{}',\ \ldots,\ p_n{}',\ \ (p_1 \wedge p_2 \wedge \ldots \wedge p_n \Rightarrow q)}{q\theta}$$

where $p_i{}'\theta = p_i\theta$ for all i

$p_1{}'$ is $King(John)$ p_1 is $King(x)$
$p_2{}'$ is $Greedy(y)$ p_2 is $Greedy(x)$
θ is $\{x/John, y/John\}$ q is $Evil(x)$
$q\theta$ is $Evil(John)$

GMP used with KB of definite clauses (**exactly** one positive literal)
All variables assumed universally quantified

Soundness of GMP

Need to show that

$$p_1', \ldots, p_n', \ (p_1 \wedge \ldots \wedge p_n \Rightarrow q) \models q\theta$$

provided that $p_i'\theta = p_i\theta$ for all i

Lemma: For any definite clause p, we have $p \models p\theta$ by UI

1. $(p_1 \wedge \ldots \wedge p_n \Rightarrow q) \models (p_1 \wedge \ldots \wedge p_n \Rightarrow q)\theta = (p_1\theta \wedge \ldots \wedge p_n\theta \Rightarrow q\theta)$

2. $p_1', \ldots, p_n' \models p_1' \wedge \ldots \wedge p_n' \models p_1'\theta \wedge \ldots \wedge p_n'\theta$

3. From 1 and 2, $q\theta$ follows by ordinary Modus Ponens

Example knowledge base

The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

Prove that Col. West is a criminal

... it is a crime for an American to sell weapons to hostile nations:
$$American(x) \land Weapon(y) \land Sells(x, y, z) \land Hostile(z) \Rightarrow Criminal(x)$$
Nono ... has some missiles, i.e., $\exists x \; Owns(Nono, x) \land Missile(x)$:
$$Owns(Nono, M_1) \text{ and } Missile(M_1)$$
... all of its missiles were sold to it by Colonel West
$$\forall x \; Missile(x) \land Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$$
Missiles are weapons:
$$Missile(x) \Rightarrow Weapon(x)$$
An enemy of America counts as "hostile":
$$Enemy(x, America) \Rightarrow Hostile(x)$$
West, who is American ...
$$American(West)$$
The country Nono, an enemy of America ...
$$Enemy(Nono, America)$$

Forward chaining algorithm

function FOL-FC-ASK(KB, α) **returns** a substitution or *false*

    repeat until *new* is empty

        new $\leftarrow \{\,\}$

        for each sentence r **in** KB **do**

            $(p_1 \wedge \ldots \wedge p_n \Rightarrow q) \leftarrow$ STANDARDIZE-APART(r)

            for each θ such that $(p_1 \wedge \ldots \wedge p_n)\theta = (p'_1 \wedge \ldots \wedge p'_n)\theta$

                    for some $p'_1, \ldots, p'_n$ in KB

              $q' \leftarrow$ SUBST(θ, q)

              if q' is not a renaming of a sentence already in KB or *new* **then do**

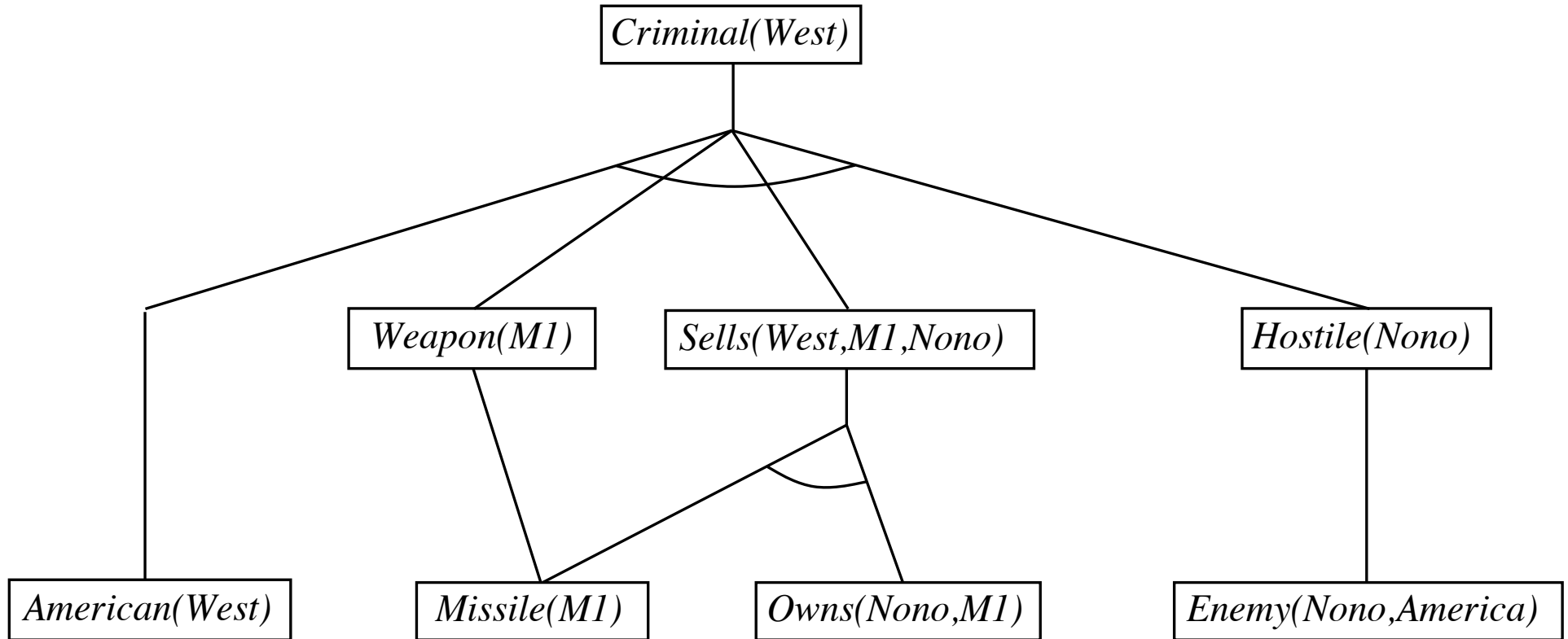   add q' to *new*

                    $\phi \leftarrow$ UNIFY(q', α)

                    if ϕ is not *fail* **then return** ϕ

        add *new* to KB

    return *false*

Forward chaining proof

Properties of forward chaining

Sound and complete for first-order definite clauses
(proof similar to propositional proof)

Datalog = first-order definite clauses + **no functions** (e.g., crime KB)
FC terminates for Datalog in poly iterations: at most $p \cdot n^k$ literals

May not terminate in general if α is not entailed

This is unavoidable: entailment with definite clauses is semidecidable

Efficiency of forward chaining

Simple observation: no need to match a rule on iteration k
if a premise wasn't added on iteration $k - 1$
$\Rightarrow$ match each rule whose premise contains a newly added literal
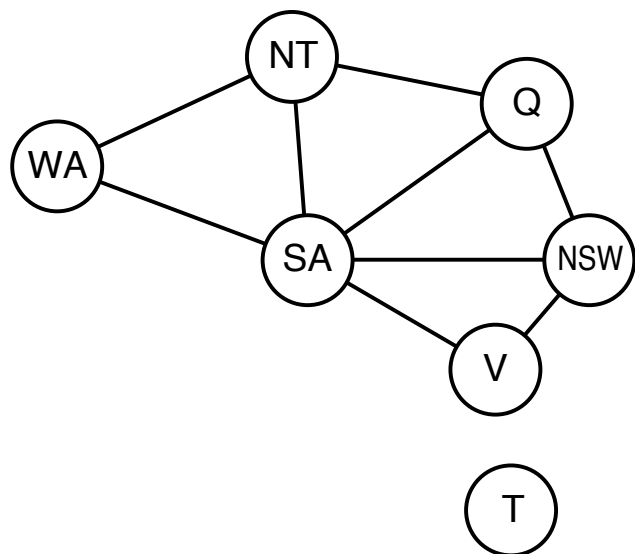
Matching itself can be expensive

Database indexing allows $O(1)$ retrieval of known facts
    e.g., query $Missile(x)$ retrieves $Missile(M_1)$

Matching conjunctive premises against known facts is NP-hard

Forward chaining is widely used in deductive databases

Hard matching example

$$Diff(wa, nt) \wedge Diff(wa, sa) \wedge$$
$$Diff(nt, q)\, Diff(nt, sa) \wedge$$
$$Diff(q, nsw) \wedge Diff(q, sa) \wedge$$
$$Diff(nsw, v) \wedge Diff(nsw, sa) \wedge$$
$$Diff(v, sa) \Rightarrow Colorable()$$

$$Diff(Red, Blue) \quad Diff(Red, Green)$$
$$Diff(Green, Red) \quad Diff(Green, Blue)$$
$$Diff(Blue, Red) \quad Diff(Blue, Green)$$

$Colorable()$ is inferred iff the CSP has a solution
CSPs include 3SAT as a special case, hence matching is NP-hard

Backward chaining algorithm

function FOL-BC-ASK($KB, goals, \theta$) **returns** a set of substitutions
   inputs: KB, a knowledge base
         $goals$, a list of conjuncts forming a query (θ already applied)
         θ, the current substitution, initially the empty substitution $\{\ \}$
   local variables: $answers$, a set of substitutions, initially empty

   if $goals$ is empty **then return** $\{\theta\}$
   $q' \leftarrow$ SUBST(θ, FIRST($goals$))
   for each sentence r **in** KB
        where STANDARDIZE-APART(r) = ($p_1\ \wedge\ \ldots\ \wedge\ p_n\ \Rightarrow\ q$)
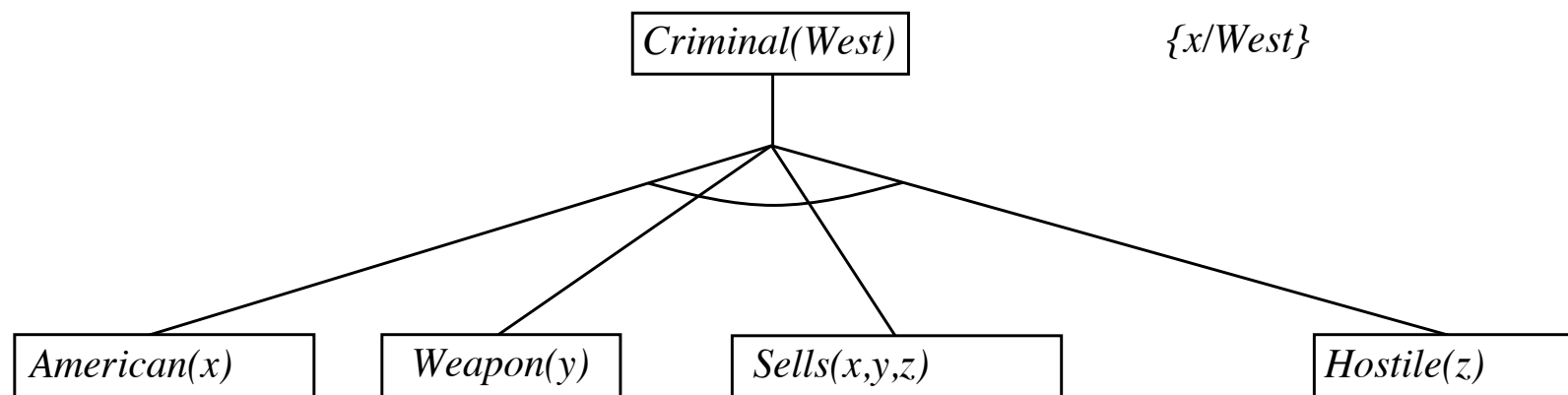   and $\theta' \leftarrow$ UNIFY(q, q') succeeds
     $new\_goals \leftarrow [\,p_1, \ldots, p_n | \text{REST}(goals)]$
     $answers \leftarrow$ FOL-BC-ASK(KB, new_goals, COMPOSE(θ', θ)) $\cup$ $answers$
   return $answers$

Backward chaining example
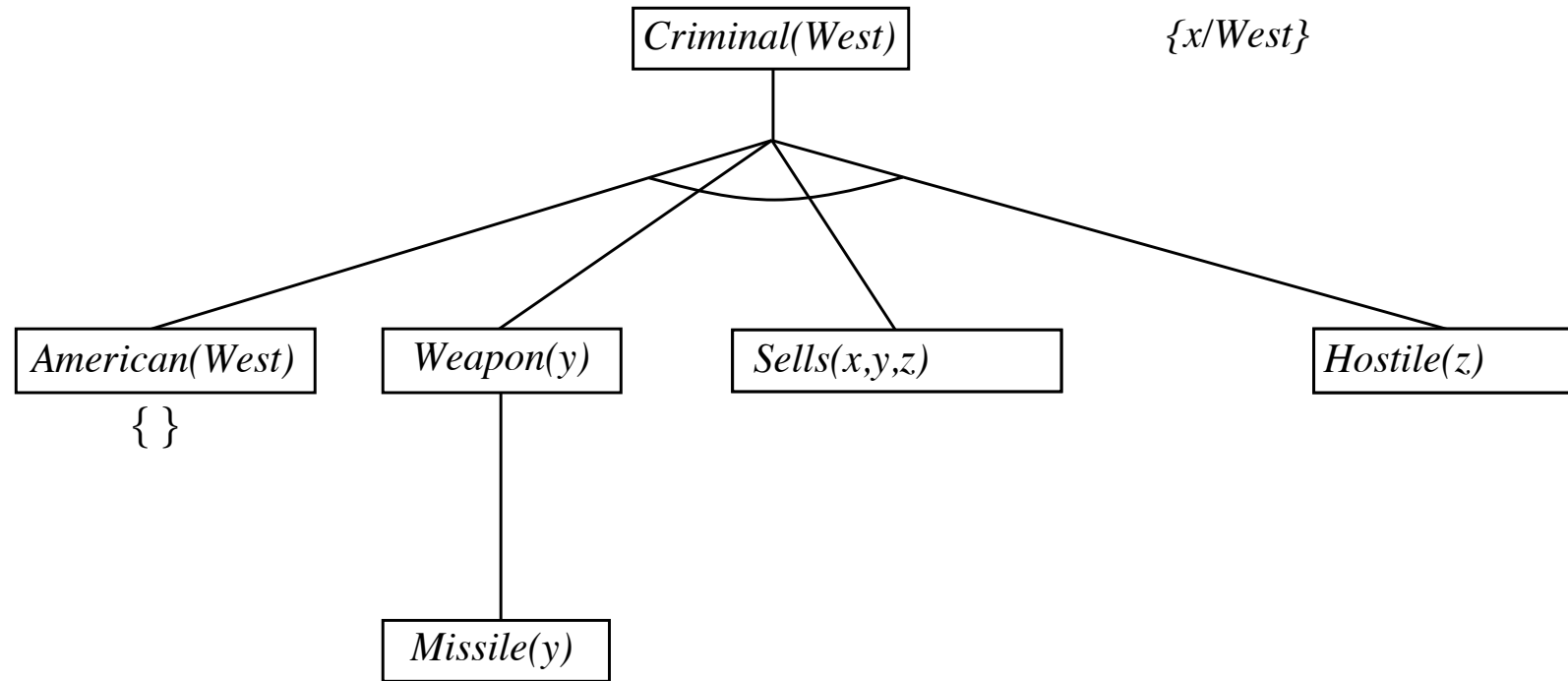
Criminal(West)   {x/West}

American(x)   Weapon(y)   Sells(x,y,z)   Hostile(z)

Backward chaining example

Criminal(West)   {x/West}

American(West)   Weapon(y)   Sells(x,y,z)   Hostile(z)
        { }

                  Missile(y)

Backward chaining example

Criminal(West)

{x/West, y/M1}

American(West)

{ }

Weapon(y)

Sells(x,y,z)

Hostile(z)

Missile(y)

{ y/M1}

Backward chaining example

$\{x/West, y/M1, z/Nono\}$

```
                        Criminal(West)
                              |
        ┌──────────┬──────────┼──────────────────────┐
  American(West)  Weapon(y)  Sells(West,M1,z)      Hostile(z)
       { }          |          { z/Nono }
                    |              |
                    |        ┌─────┴─────┐
                Missile(y)  Missile(M1)  Owns(Nono,M1)
                 { y/M1}
```

Backward chaining example

Criminal(West)

$\{x/West, y/M1, z/Nono\}$

American(West)
$\{\,\}$

Weapon(y)

Sells(West,M1,z)
$\{\,z/Nono\,\}$

Hostile(Nono)

Missile(y)
$\{\,y/M1\,\}$

Missile(M1)
$\{\,\}$

Owns(Nono,M1)
$\{\,\}$

Enemy(Nono,America)
$\{\,\}$
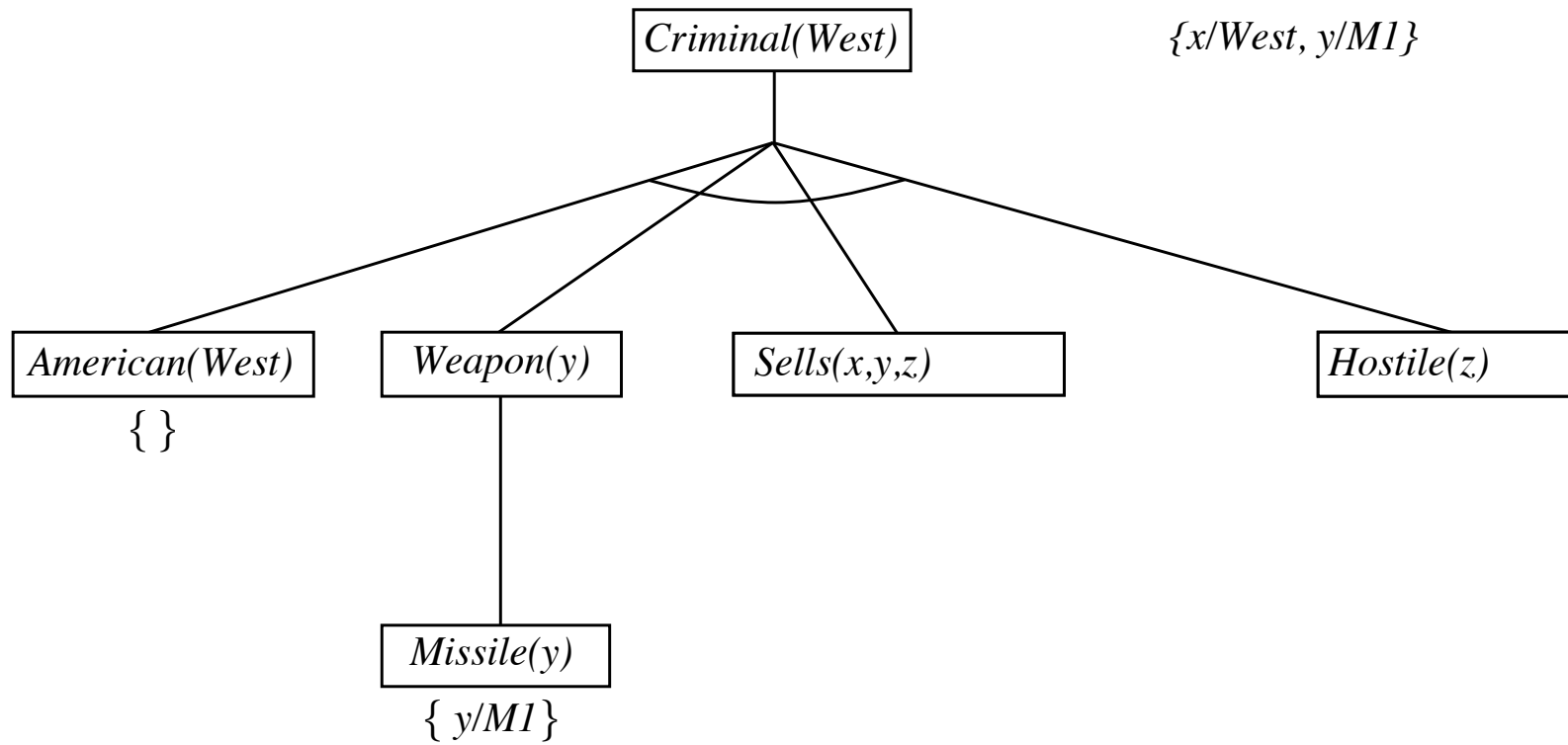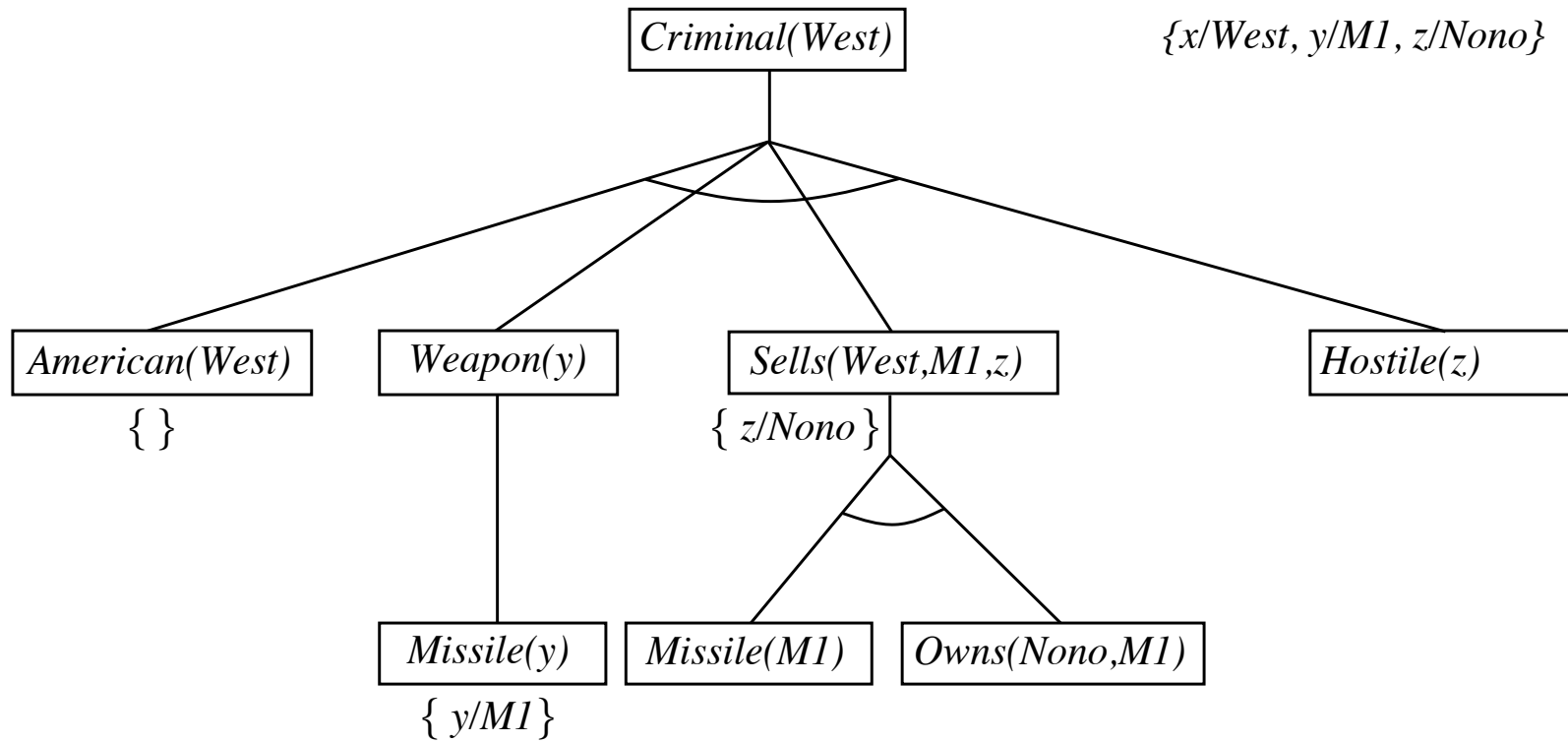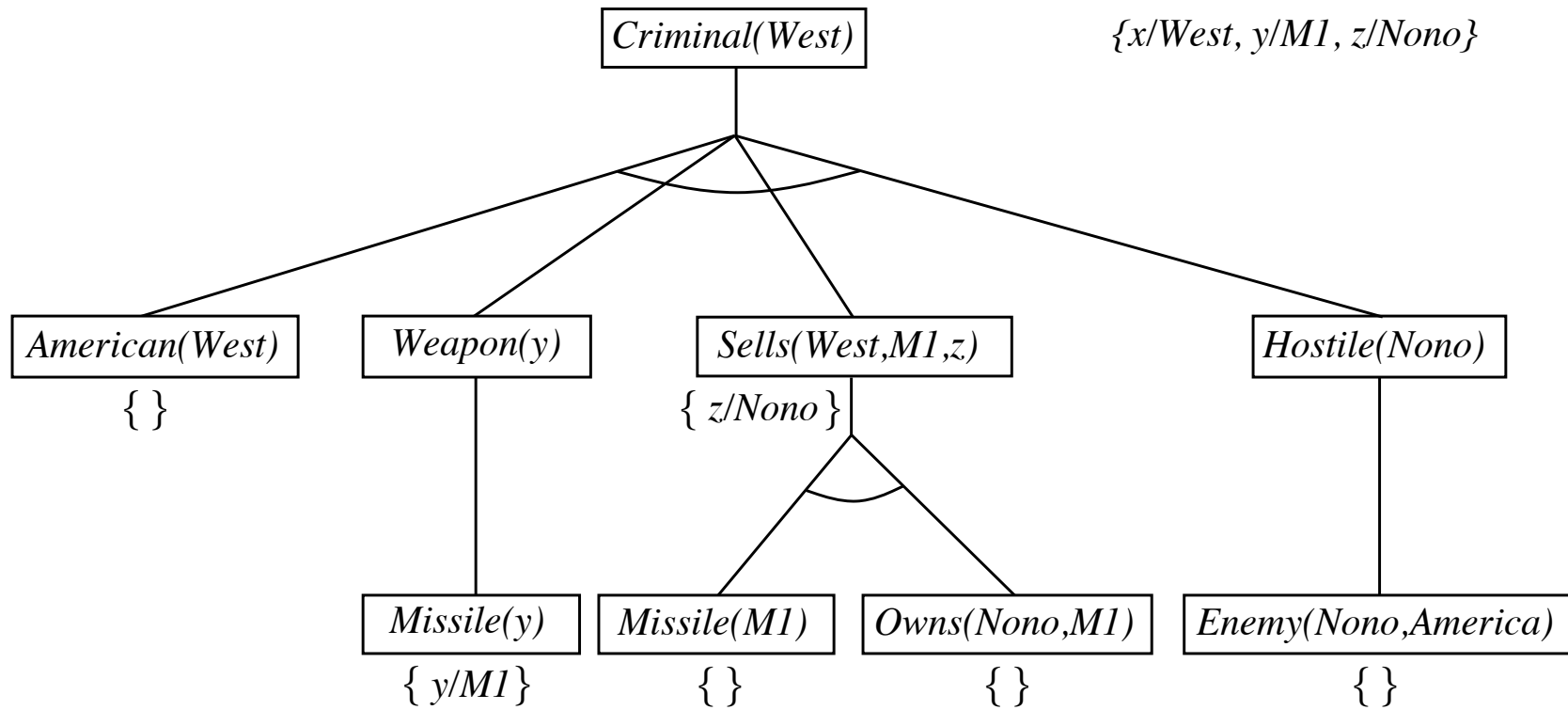
Properties of backward chaining

Depth-first recursive proof search: space is linear in size of proof

Incomplete due to infinite loops
$\Rightarrow$ fix by checking current goal against every goal on stack

Inefficient due to repeated subgoals (both success and failure)
$\Rightarrow$ fix using caching of previous results (extra space!)

Widely used (without improvements!) for logic programming

Logic programming

Sound bite: computation as inference on logical KBs

Logic programming	Ordinary programming
1. Identify problem	Identify problem
2. Assemble information	Assemble information
3. Tea break	Figure out solution
4. Encode information in KB	Program solution
5. Encode problem instance as facts	Encode problem instance as data
6. Ask queries	Apply program to data
7. Find false facts	Debug procedural errors

Should be easier to debug $Capital(NewYork, US)$ than $x := x + 2$!

Prolog systems

Basis: backward chaining with Horn clauses + bells & whistles
Widely used in Europe, Japan (basis of 5th Generation project)
Compilation techniques $\Rightarrow$ approaching a billion LIPS

Program = set of clauses = `head :- literal`$_1$`, ... literal`$_n$`.`

```
criminal(X) :- american(X), weapon(Y), sells(X,Y,Z), hostile(Z).
```

Efficient unification by open coding
Efficient retrieval of matching clauses by direct linking
Depth-first, left-to-right backward chaining
Built-in predicates for arithmetic etc., e.g., `X is Y*Z+3`
Closed-world assumption ("negation as failure")
    e.g., given `alive(X) :- not dead(X).`
    `alive(joe)` succeeds if `dead(joe)` fails

Prolog examples

Depth-first search from a start state X:

```prolog
dfs(X) :- goal(X).
dfs(X) :- successor(X,S),dfs(S).
```

No need to loop over S: successor succeeds for each

Appending two lists to produce a third:

```prolog
append([],Y,Y).
append([X|L],Y,[X|Z]) :- append(L,Y,Z).
```

```prolog
query:    append(A,B,[1,2]) ?
answers: A=[]     B=[1,2]
         A=[1]    B=[2]
         A=[1,2]  B=[]
```

Prolog example

Let's try

member(1,[1,2,3,4,5])

query: grandfather(X,yuqing)?

male(di).
male(jianbo).
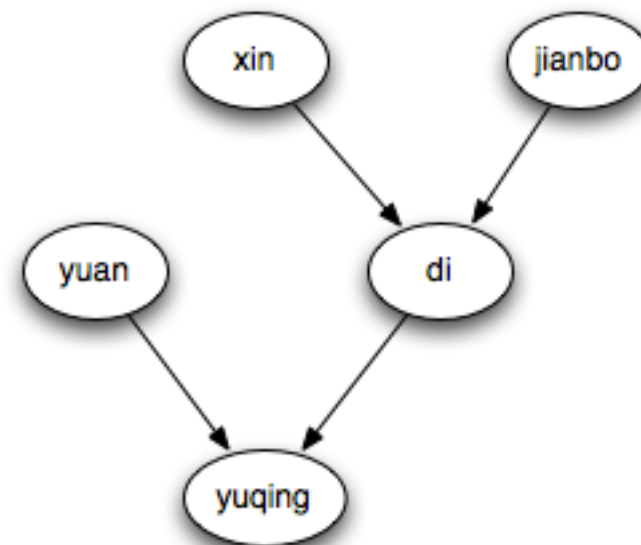female(xin).
female(yuan).
female(yuqing).
father(jianbo,di).
father(di,yuqing).
mother(xin,di).
mother(yuan,yuqing).
grandfather(X,Y):-father(X,Z),father(Z,Y).
grandmother(X,Y):-mother(X,Z),father(Z,Y).
daughter(X,Y):-father(X,Y),female(Y).

Resolution: brief summary

Full first-order version:

$$\frac{\ell_1 \vee \cdots \vee \ell_k, \qquad m_1 \vee \cdots \vee m_n}{(\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n)\theta}$$

where $\text{UNIFY}(\ell_i, \neg m_j) = \theta$.

For example,

$$\frac{\neg Rich(x) \vee Unhappy(x)}{Unhappy(Ken)}$$
$$Rich(Ken)$$

with $\theta = \{x/Ken\}$

Apply resolution steps to $CNF(KB \wedge \neg\alpha)$; complete for FOL

Conversion to CNF

Everyone who loves all animals is loved by someone:
$$\forall x \; [\forall y \; Animal(y) \Rightarrow Loves(x,y)] \Rightarrow [\exists y \; Loves(y,x)]$$

1. Eliminate biconditionals and implications

$$\forall x \; [\neg \forall y \; \neg Animal(y) \vee Loves(x,y)] \vee [\exists y \; Loves(y,x)]$$

2. Move $\neg$ inwards: $\neg \forall x, p \; \equiv \exists x \; \neg p, \quad \neg \exists x, p \; \equiv \forall x \; \neg p$:

$$\forall x \; [\exists y \; \neg(\neg Animal(y) \vee Loves(x,y))] \vee [\exists y \; Loves(y,x)]$$
$$\forall x \; [\exists y \; \neg\neg Animal(y) \wedge \neg Loves(x,y)] \vee [\exists y \; Loves(y,x)]$$
$$\forall x \; [\exists y \; Animal(y) \wedge \neg Loves(x,y)] \vee [\exists y \; Loves(y,x)]$$

Conversion to CNF

3. Standardize variables: each quantifier should use a different one

$$\forall x \; [\exists y \; Animal(y) \wedge \neg Loves(x, y)] \vee [\exists z \; Loves(z, x)]$$

4. Skolemize: a more general form of existential instantiation. Each existential variable is replaced by a Skolem function of the enclosing universally quantified variables:

$$\forall x \; [Animal(F(x)) \wedge \neg Loves(x, F(x))] \vee Loves(G(x), x)$$

5. Drop universal quantifiers:

$$[Animal(F(x)) \wedge \neg Loves(x, F(x))] \vee Loves(G(x), x)$$

6. Distribute $\wedge$ over $\vee$:

$$[Animal(F(x)) \vee Loves(G(x), x)] \wedge [\neg Loves(x, F(x)) \vee Loves(G(x), x)]$$

Resolution proof: definite clauses

¬ *American(x)* ∨ ¬ *Weapon(y)* ∨ ¬ *Sells(x,y,z)* ∨ ¬ *Hostile(z)* ∨ *Criminal(x)*

¬ *Criminal(West)*

American(West)

¬ *American(West)* ∨ ¬ *Weapon(y)* ∨ ¬ *Sells(West,y,z)* ∨ ¬ *Hostile(z)*

¬ *Missile(x)* ∨ *Weapon(x)*

¬ *Weapon(y)* ∨ ¬ *Sells(West,y,z)* ∨ ¬ *Hostile(z)*

Missile(M1)

¬ *Missile(y)* ∨ ¬ *Sells(West,y,z)* ∨ ¬ *Hostile(z)*

¬ *Missile(x)* ∨ ¬ *Owns(Nono,x)* ∨ *Sells(West,x,Nono)*

¬ *Sells(West,M1,z)* ∨ ¬ *Hostile(z)*

Missile(M1)

¬ *Missile(M1)* ∨ ¬ *Owns(Nono,M1)* ∨ ¬ *Hostile(Nono)*

Owns(Nono,M1)

¬ *Owns(Nono,M1)* ∨ ¬ *Hostile(Nono)*

¬ *Enemy(x,America)* ∨ *Hostile(x)*

¬ *Hostile(Nono)*

Enemy(Nono,America)

¬ *Enemy(Nono,America)*