Lecture 3: Search 2

http://cs.nju.edu.cn/yuy/course_ai15.ashx

Previously...

function TREE-SEARCH(*problem, fringe*) **returns** a solution, or failure
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)
  loop do
    if *fringe* is empty **then return** failure
    node ← REMOVE-FRONT(*fringe*)
    if GOAL-TEST(*problem*, STATE(*node*)) **then return** *node*
    fringe ← INSERTALL(EXPAND(*node*, *problem*), *fringe*)

note the time of goal-test: expanding time not generating time

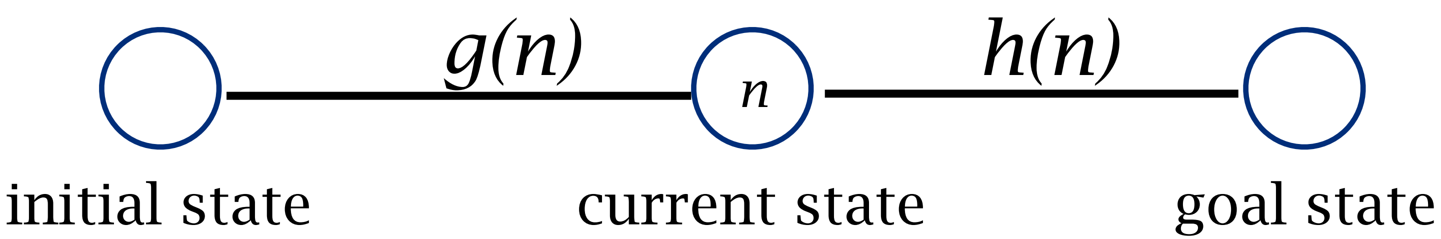
---

function EXPAND(*node, problem*) **returns** a set of nodes
  successors ← the empty set
  for each *action*, *result* **in** SUCCESSOR-FN(*problem*, STATE[*node*]) **do**
    s ← a new NODE
    PARENT-NODE[*s*] ← *node*; ACTION[*s*] ← *action*; STATE[*s*] ← *result*
    PATH-COST[*s*] ← PATH-COST[*node*] + STEP-COST(*node*, *action*, *s*)
    DEPTH[*s*] ← DEPTH[*node*] + 1
    add *s* to *successors*
  return *successors*

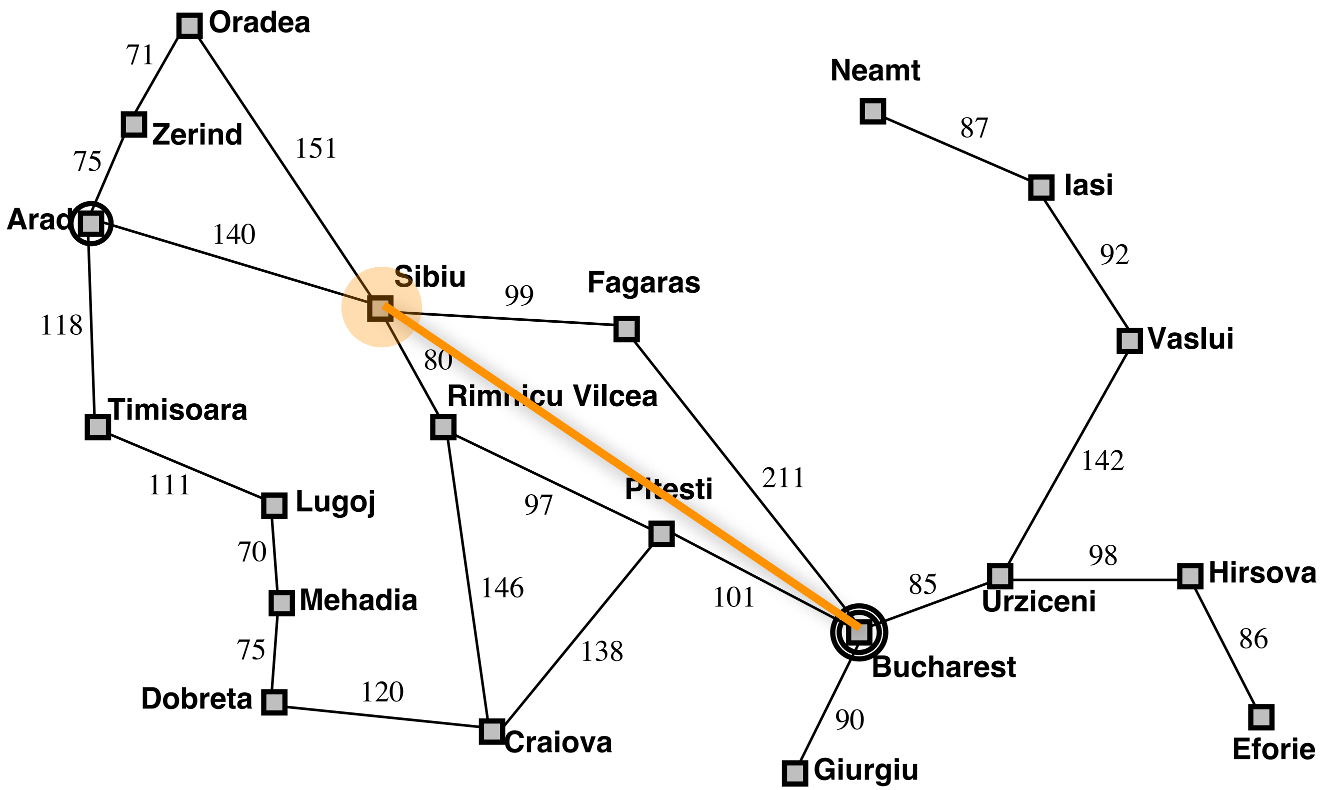
Informed Search Strategies

best-first search: f — but what is best?

uniform cost search: cost function g
heuristic function: *h*



Example: h_{SLD}



Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

Figure 3.22 Values of h_{SLD}—straight-line distances to Bucharest.

Greedy search

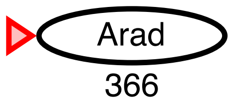
Evaluation function $h(n)$ (**h**euristic)

$= $ estimate of cost from n to the closest goal

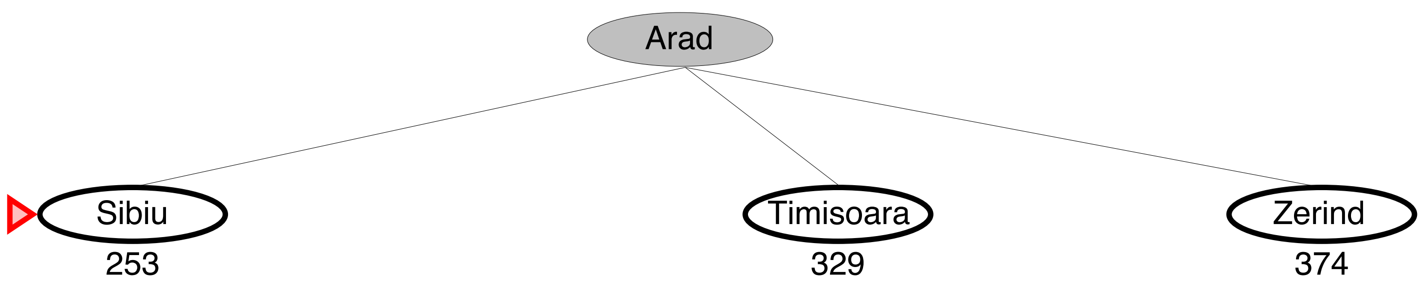
E.g., $h_{\rm SLD}(n) = $ straight-line distance from n to Bucharest

Greedy search expands the node that **appears** to be closest to goal

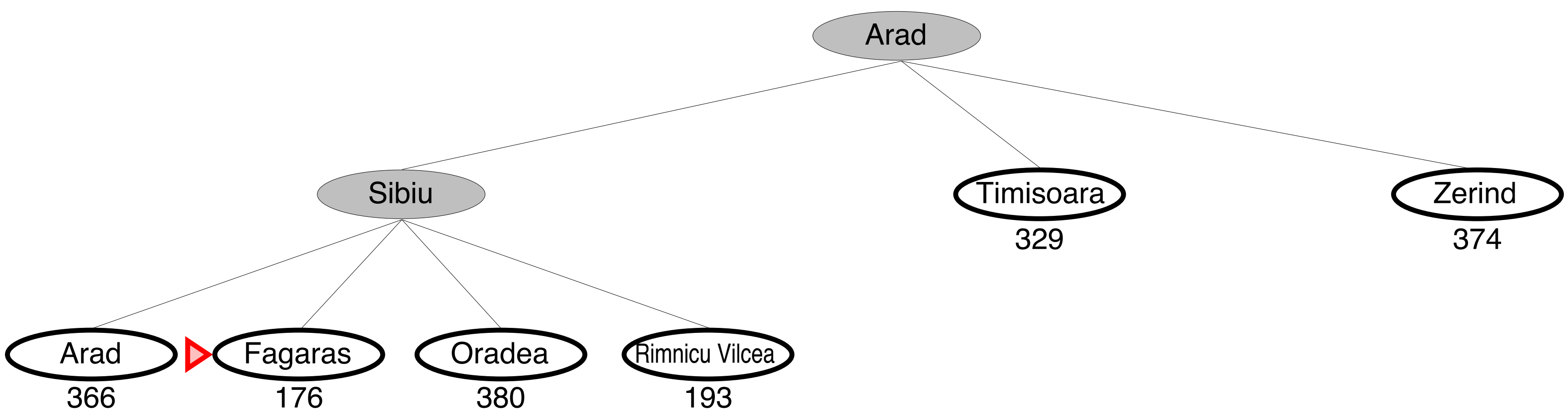
Example



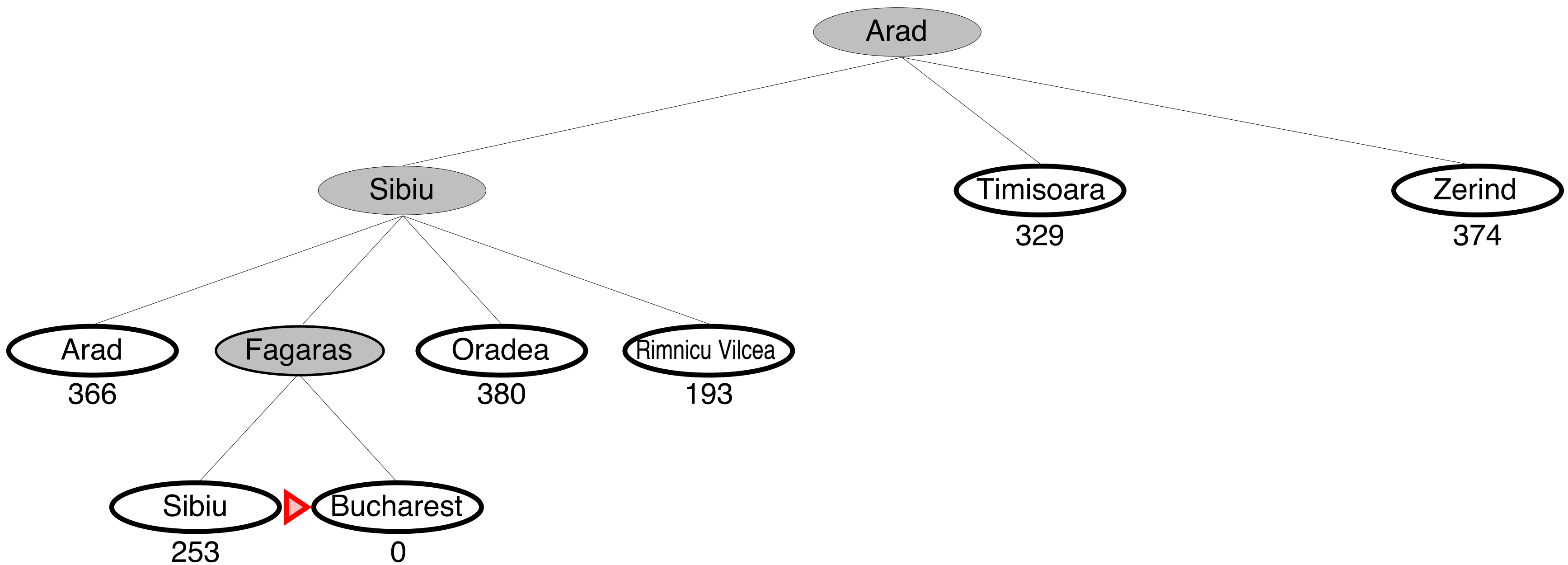
Example



Example


Arad
Sibiu
Timisoara
329
Zerind
374
Arad
366
Fagaras
176
Oradea
380
Rimnicu Vilcea
193

Example


Arad
Sibiu
Timisoara
329
Zerind
374
Arad
366
Fagaras
Oradea
380
Rimnicu Vilcea
193
Sibiu
253
Bucharest
0

Properties

Complete?? No–can get stuck in loops, e.g.,

Iasi $\to$ Neamt $\to$ Iasi $\to$ Neamt $\to$

Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space?? $O(b^m)$—keeps all nodes in memory

Optimal?? No

A* search

Idea: avoid expanding paths that are already expensive

Evaluation function $f(n) = g(n) + h(n)$

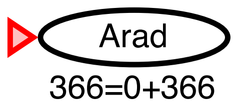
$g(n)$ = cost so far to reach n
$h(n)$ = estimated cost to goal from n
$f(n)$ = estimated total cost of path through n to goal

A* search uses an admissible heuristic
i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the **true** cost from n.
(Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal G.)

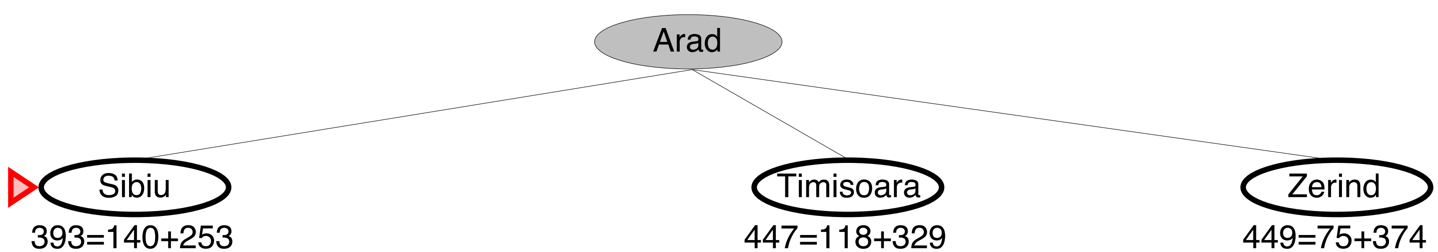
E.g., $h_{\mathrm{SLD}}(n)$ never overestimates the actual road distance

Theorem: A* search is optimal

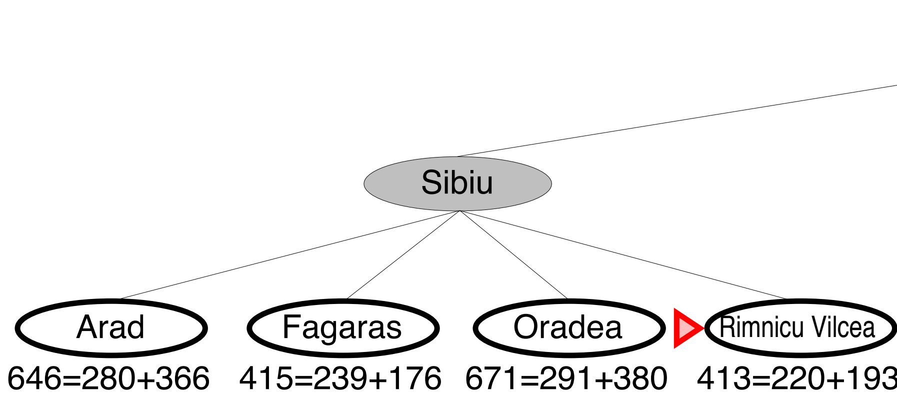
Example



Example

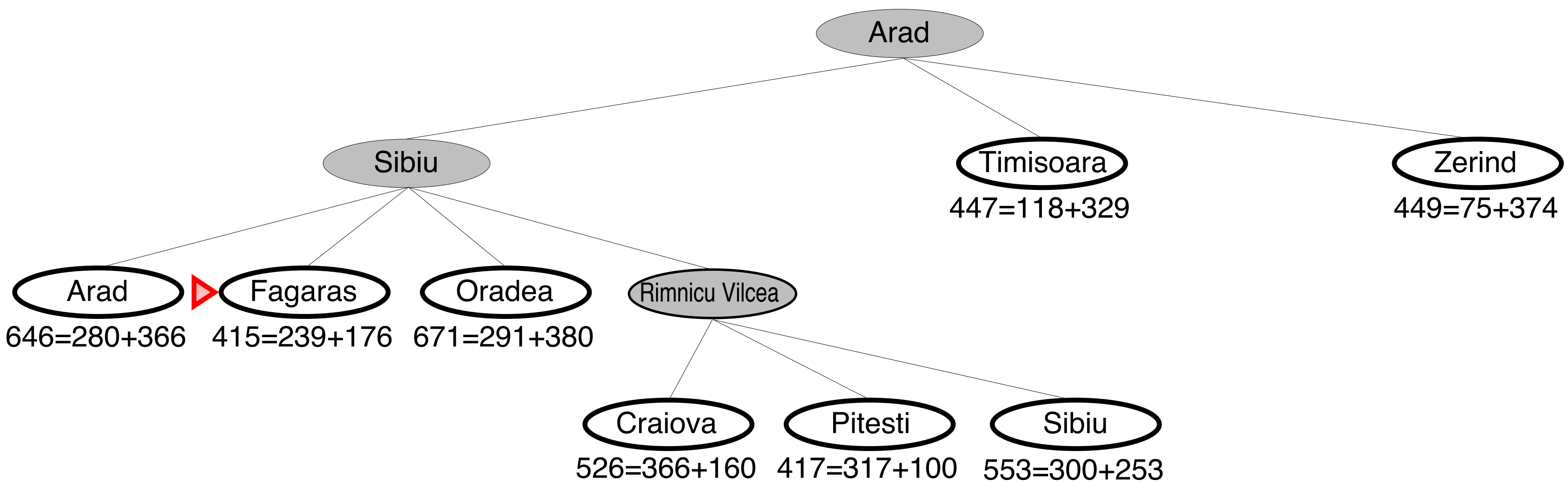

Arad
Sibiu
393=140+253
Timisoara
447=118+329
Zerind
449=75+374

Example

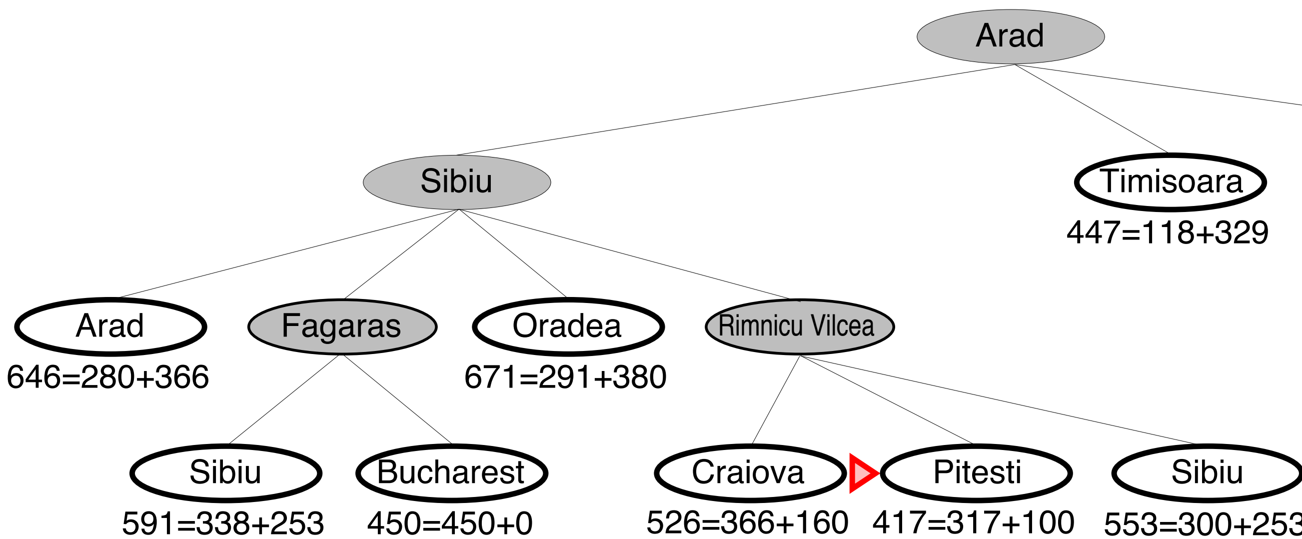


Arad
Sibiu
Timisoara
447=118+329
Zerind
449=75+374
Arad
646=280+366
Fagaras
415=239+176
Oradea
671=291+380
Rimnicu Vilcea
413=220+193

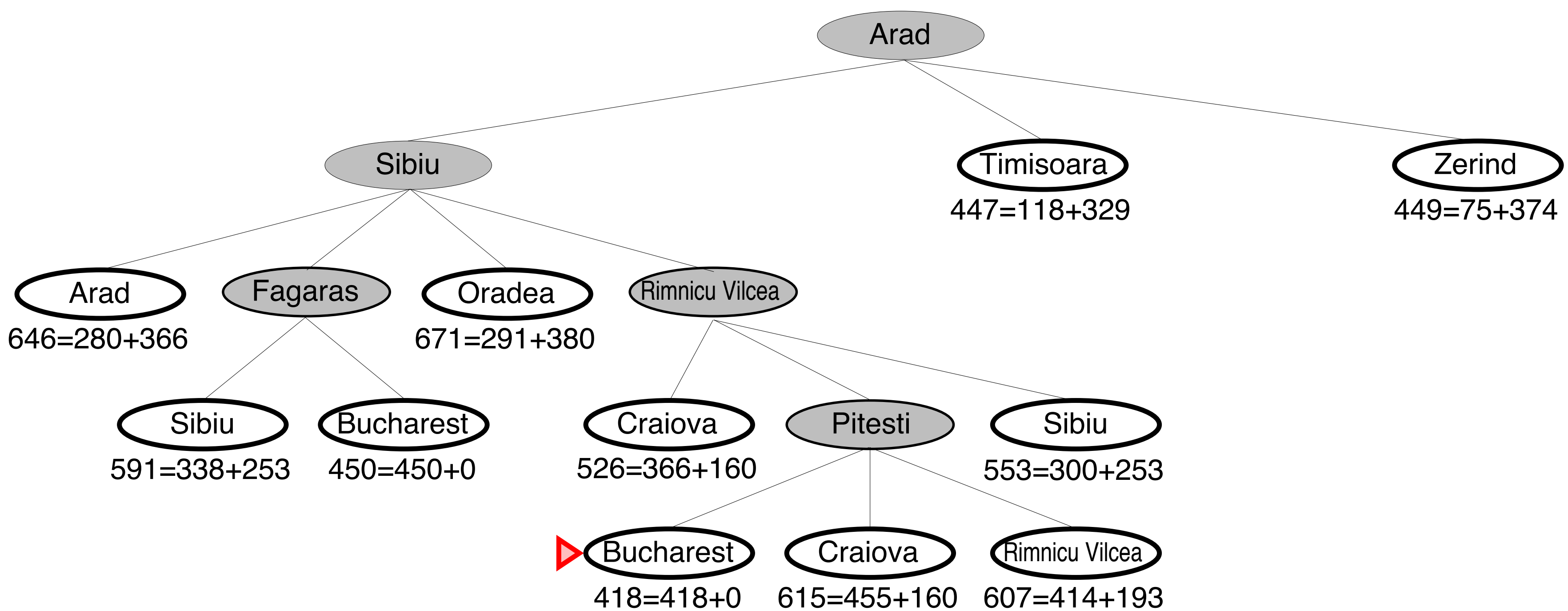
Example


Arad
Sibiu
Timisoara
447=118+329
Zerind
449=75+374
Arad
646=280+366
Fagaras
415=239+176
Oradea
671=291+380
Rimnicu Vilcea
Craiova
526=366+160
Pitesti
417=317+100
Sibiu
553=300+253

Example


Arad
Sibiu
Timisoara
447=118+329
Zerind
449=75+374
Arad
646=280+366
Fagaras
Oradea
671=291+380
Rimnicu Vilcea
Sibiu
591=338+253
Bucharest
450=450+0
Craiova
526=366+160
Pitesti
417=317+100
Sibiu
553=300+253

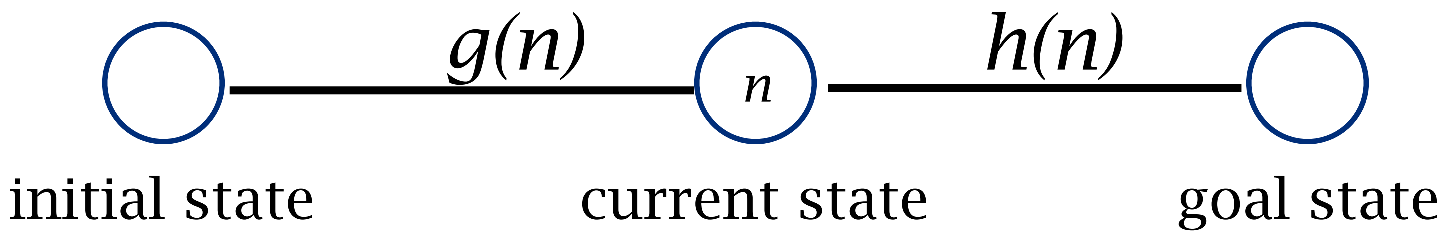
Example



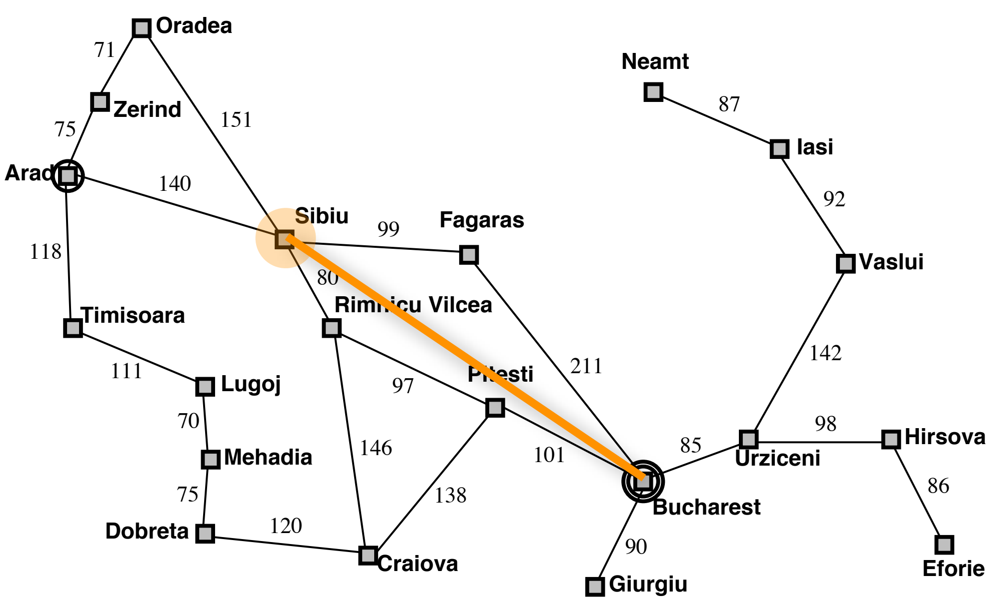
Arad
Sibiu
Timisoara
447=118+329
Zerind
449=75+374
Arad
646=280+366
Fagaras
Oradea
671=291+380
Rimnicu Vilcea
Sibiu
591=338+253
Bucharest
450=450+0
Craiova
526=366+160
Pitesti
Sibiu
553=300+253
Bucharest
418=418+0
Craiova
615=455+160
Rimnicu Vilcea
607=414+193

A* is optimal: Admissible and consistency

Admissible: never over estimate the cost



no larger than the cost
of the optimal path
from *n* to the goal



A* is optimal: Admissible and consistency

A* is optimal with admissible heuristic

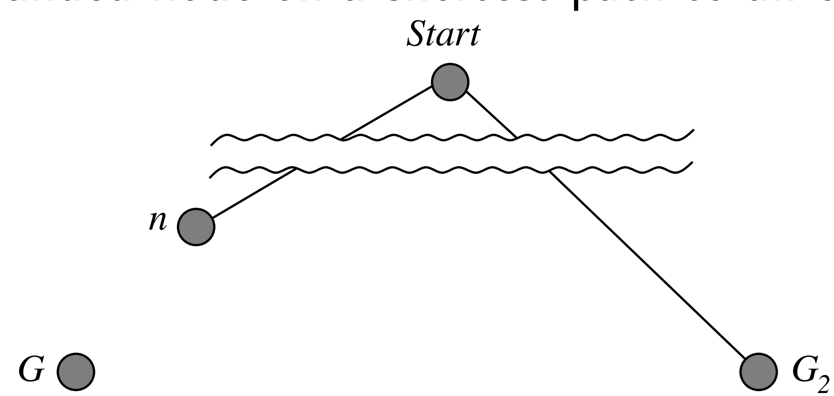
why?

A* is optimal: Admissible and consistency

A* is optimal with admissible heuristic

why?

Suppose some suboptimal ~~goal~~ solution G_2 has been generated and is in the queue. Let n be an unexpanded node on a shortest path to an optimal goal G_1.



$$\begin{aligned} f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\ &> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\ &\geq f(n) && \text{since } h \text{ is admissible} \end{aligned}$$

Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

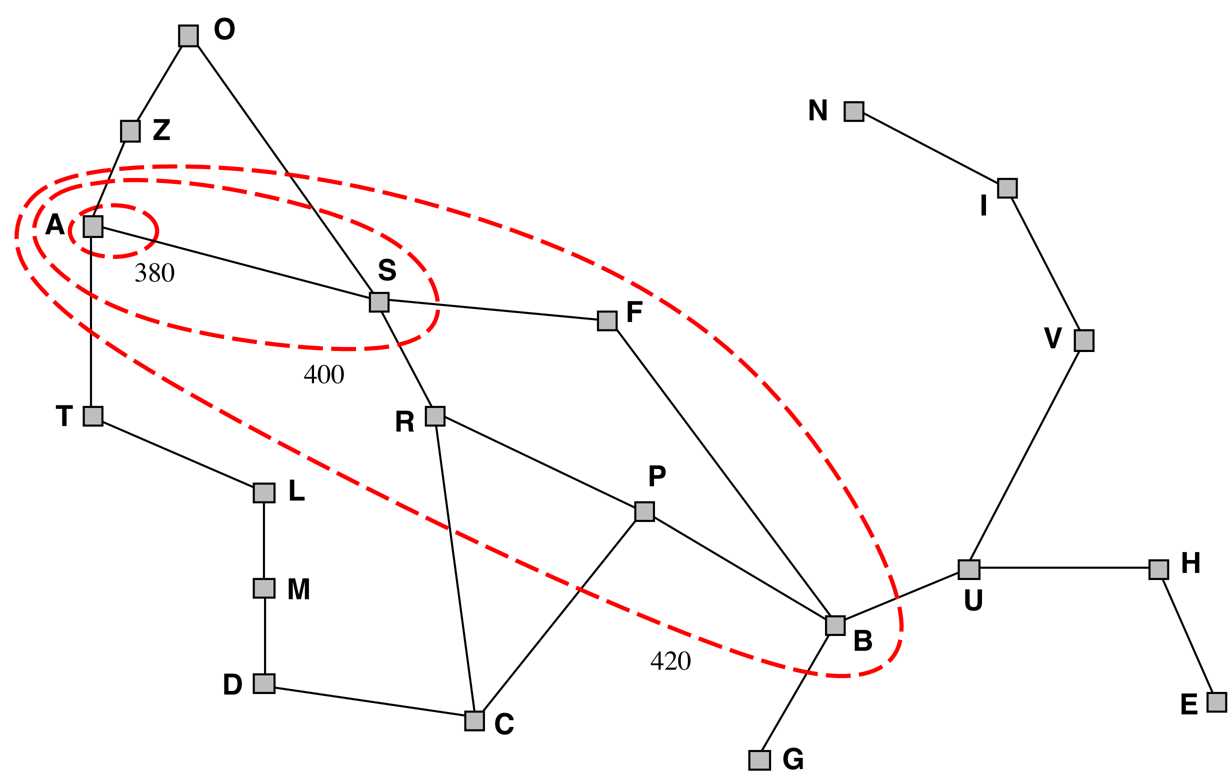
A* is optimal: Admissible and consistency

A* is optimal with admissible heuristic

why?

Lemma: A* expands nodes in order of increasing f value*

Gradually adds "f-contours" of nodes (cf. breadth-first adds layers)
Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



A* is optimal: Admissible and consistency

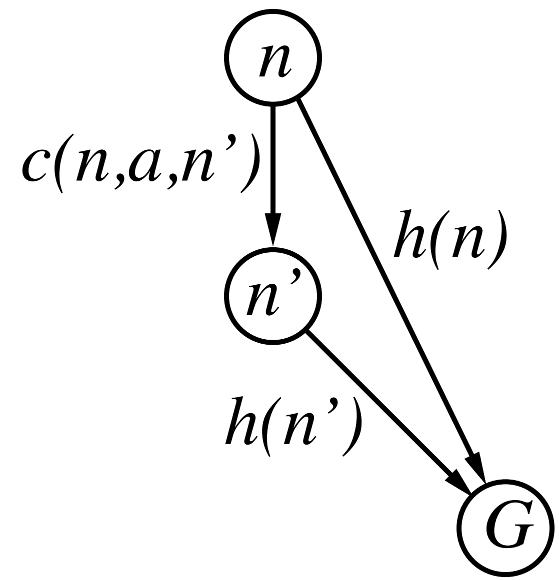
Admissible is not the best condition

A heuristic is consistent if

$$h(n) \leq c(n, a, n') + h(n')$$

If h is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n, a, n') + h(n') \\ &\geq g(n) + h(n) \\ &= f(n) \end{aligned}$$



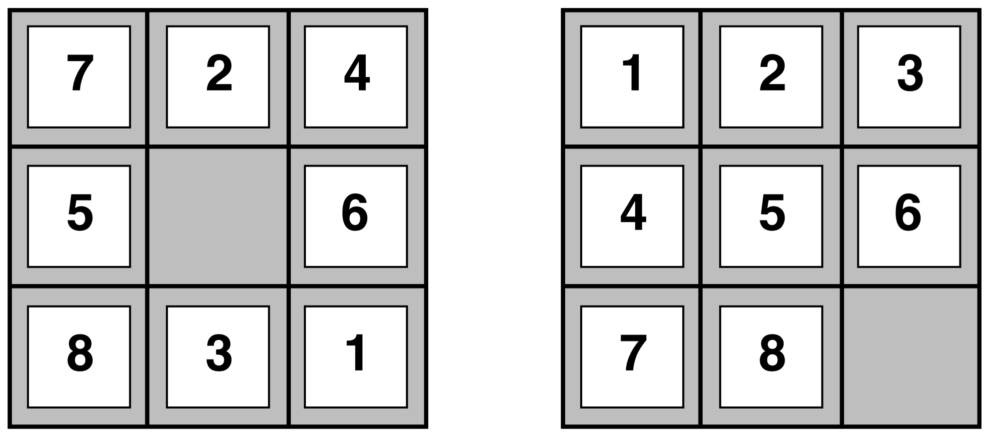
I.e., $f(n)$ is nondecreasing along any path.

Proof is similar with that of admissible

Example

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)



Start State Goal State

$h_1(S) =$?? 6
$h_2(S) =$?? $4+0+3+3+1+0+2+1 = 14$

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible)
then h_2 **dominates** h_1 and is better for search

why?

Typical search costs:

$d = 14$ IDS = 3,473,941 nodes
$A^*(h_1) = 539$ nodes
$A^*(h_2) = 113$ nodes
$d = 24$ IDS $\approx$ 54,000,000,000 nodes
$A^*(h_1) = 39{,}135$ nodes
$A^*(h_2) = 1{,}641$ nodes

Given any admissible heuristics h_a, h_b,

$$h(n) = \max(h_a(n), h_b(n))$$

is also admissible and dominates h_a, h_b

Admissible heuristics from relaxed problem

Admissible heuristics can be derived from the **exact** solution cost of a **relaxed** version of the problem

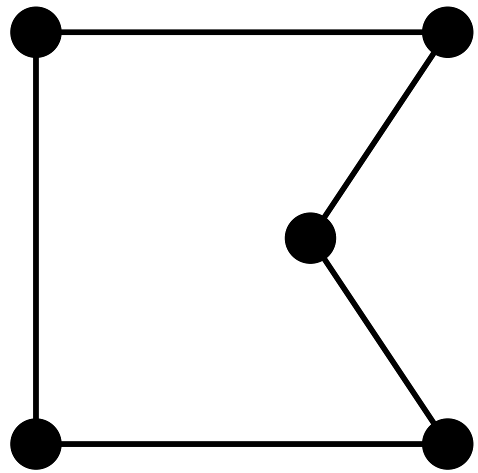
If the rules of the 8-puzzle are relaxed so that a tile can move **anywhere**, then $h_1(n)$ gives the shortest solution

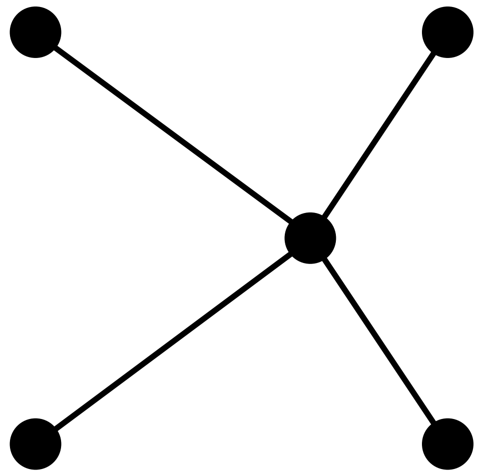
If the rules are relaxed so that a tile can move to **any adjacent square**, then $h_2(n)$ gives the shortest solution

Key point: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

Example

Well-known example: travelling salesperson problem (TSP)
Find the shortest tour visiting all cities exactly once





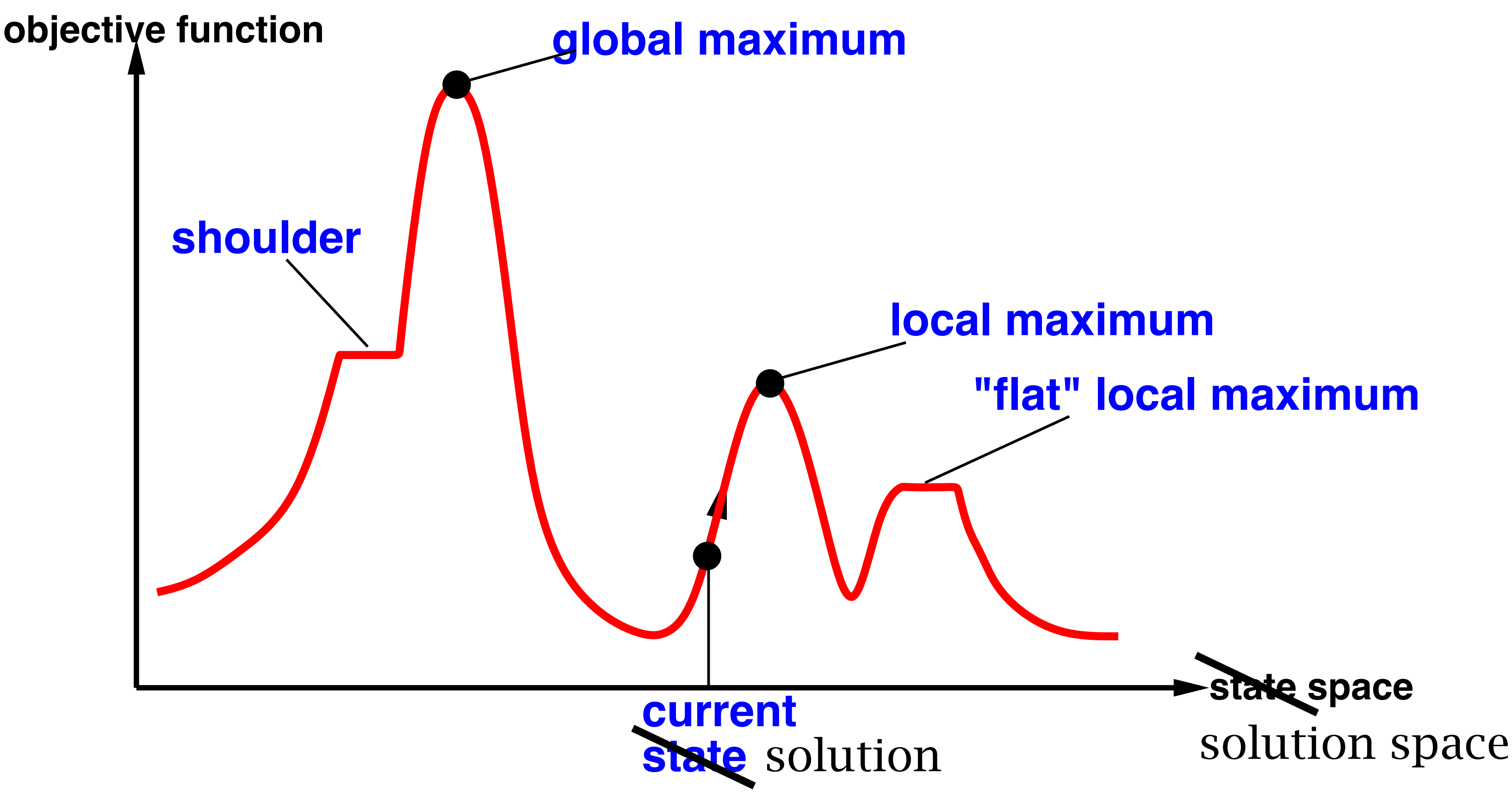
Minimum spanning tree can be computed in $O(n^2)$
and is a lower bound on the shortest (open) tour

Beyond Classical Search

Iterative-improvement search

a higher level perspective of optimization

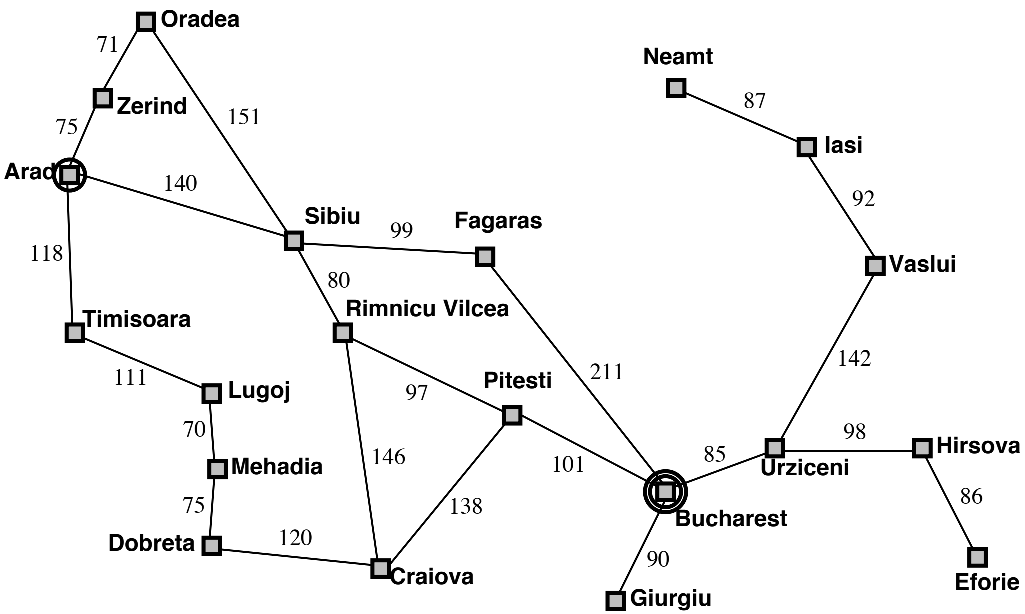
$$\max_{x \in X} \quad \text{objective-function}(x)$$



Different with path search

Uniform-cost, A* --> path search

path search v.s. iterative improvement search



by A*: search the path one-step by one-step

by iterative improvement: improve a path

Hill climbing

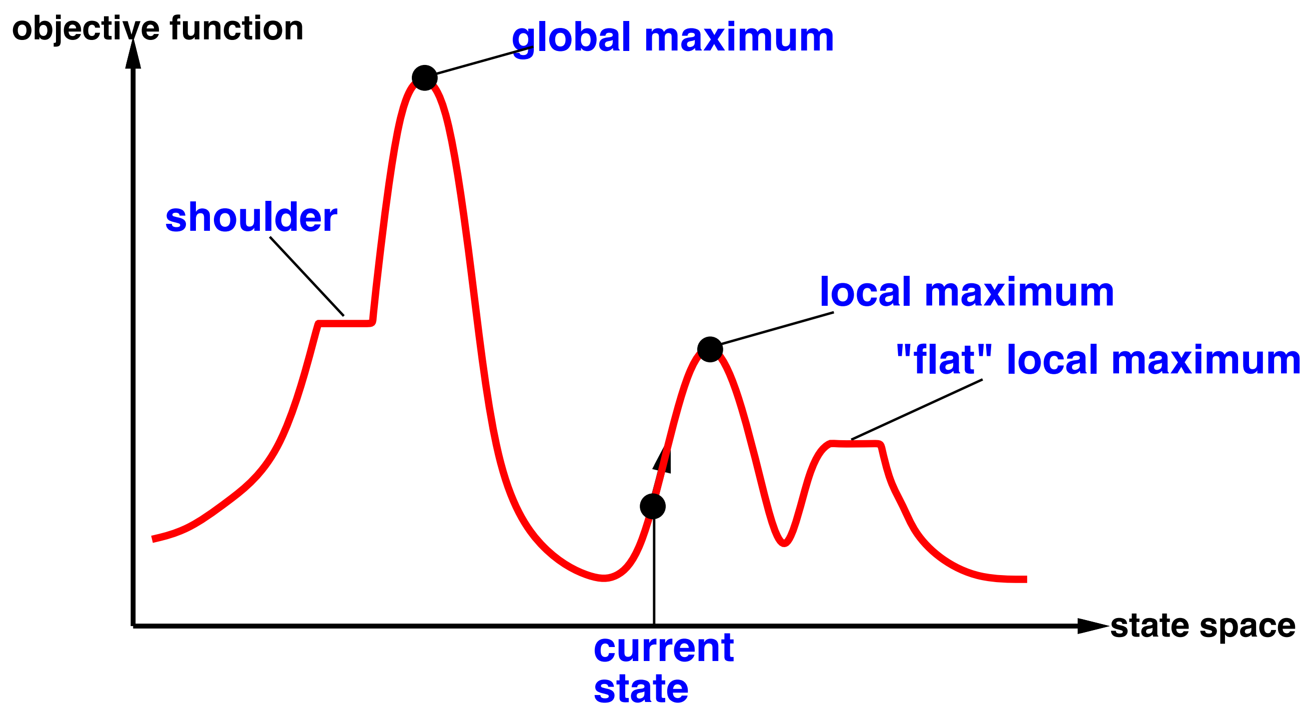
"Like climbing Everest in thick fog with amnesia"

```
function HILL-CLIMBING(problem) returns a state that is a local maximum
    inputs: problem, a problem
    local variables: current, a node
                     neighbor, a node

    current ← MAKE-NODE(INITIAL-STATE[problem])
    loop do
        neighbor ← a highest-valued successor of current
        if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
        current ← neighbor
    end
```

Hill climbing

Useful to consider state space landscape



Random-restart hill climbing overcomes local maxima—trivially complete

Random sideways moves 😊 escape from shoulders ☹ loop on flat maxima