Lecture 5: Search 4

http://cs.nju.edu.cn/yuy/course_ai18.ashx

Previously...

Path-based search

- Uninformed search
  - Depth-first, breadth first, uniform-cost search
- Informed search
  - Best-first, **A* search**

Adversarial search

- Alpha-Beta search

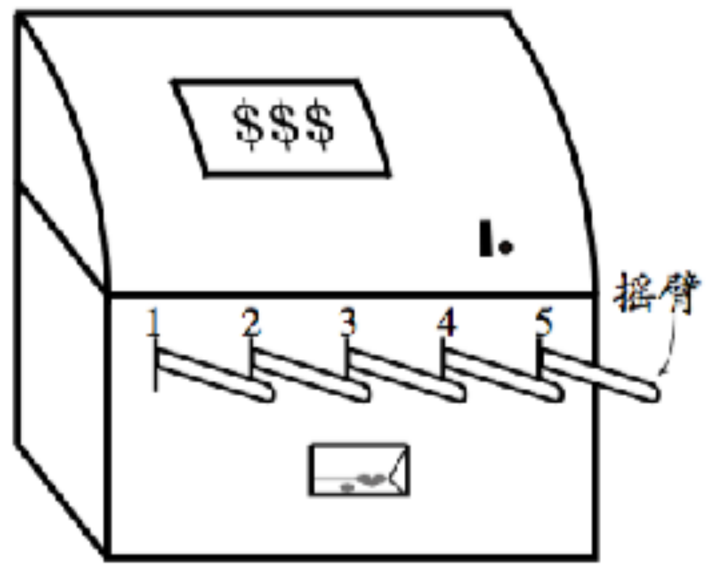
Beyond classical search

Bandit search

Tree search: Monte-Carlo Tree Search

General search:
- Gradient decent
- Metaheuristic search

Bandit



Multiple arms
Each arm has an expected reward,
but unknown, with an unknown distribution

Maximize your award in fixed trials

Simplest strategies

Two simplest strategies

Exploration-only:

for T trails and K arms, try each arm T/K times

problem? waste on suboptimal arms

Exploitation-only:

1. try each arm once
2. try the observed best arm T-K times

problem? risk of wrong best arm

ε-greedy

Balance the exploration and exploitation:

with ε probability, try a random arm
with 1-ε probability, try the best arm

ε controls the balance

输入: 摇臂数 K;
奖赏函数 R;
尝试次数 T;
探索概率 ϵ.

过程:

```
r = 0;
∀i = 1,2,...K : Q(i) = 0, count(i) = 0;
for t = 1,2,...,T do
    if rand() < ε then
        k = 从 1,2,...,K 中以均匀分布随机选取
    else
        k = arg max_i Q(i)
    end if
    v = R(k);
    r = r + v;
    Q(k) = (Q(k) × count(k) + v) / (count(k) + 1);
    count(k) = count(k) + 1;
end for
```

输出: 累积奖赏 r

Softmax

Balance the exploration and exploitation:

Choose arm with probability

$$P(k) = \frac{e^{\frac{Q(k)}{\tau}}}{\sum_{i=1}^{K} e^{\frac{Q(i)}{\tau}}}, \tag{16.4}$$

τ controls the balance

输入: 摇臂数 K;
奖赏函数 R;
尝试次数 T;
温度参数 τ.

过程:

```
r = 0;
∀i = 1, 2, ... K : Q(i) = 0, count(i) = 0;
for t = 1, 2, ..., T do
    k = 从1, 2, ..., K 中根据式(16.4)随机选取
    v = R(k);
    r = r + v;
    Q(k) = (Q(k) × count(k) + v) / (count(k) + 1);
    count(k) = count(k) + 1;
end for
```

输出: 累积奖赏 r

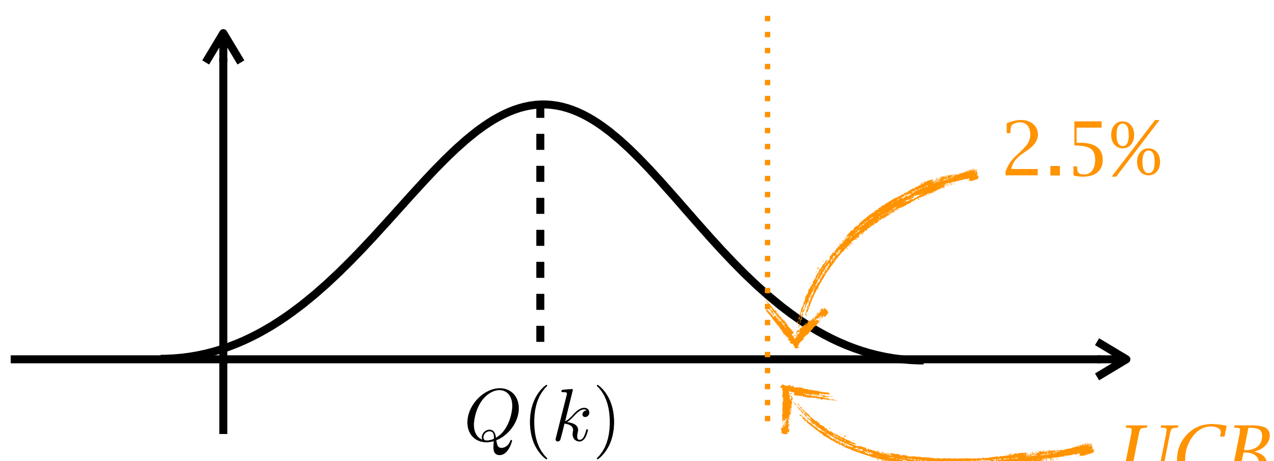
Upper-confidence bound

Balance the exploration and exploitation:

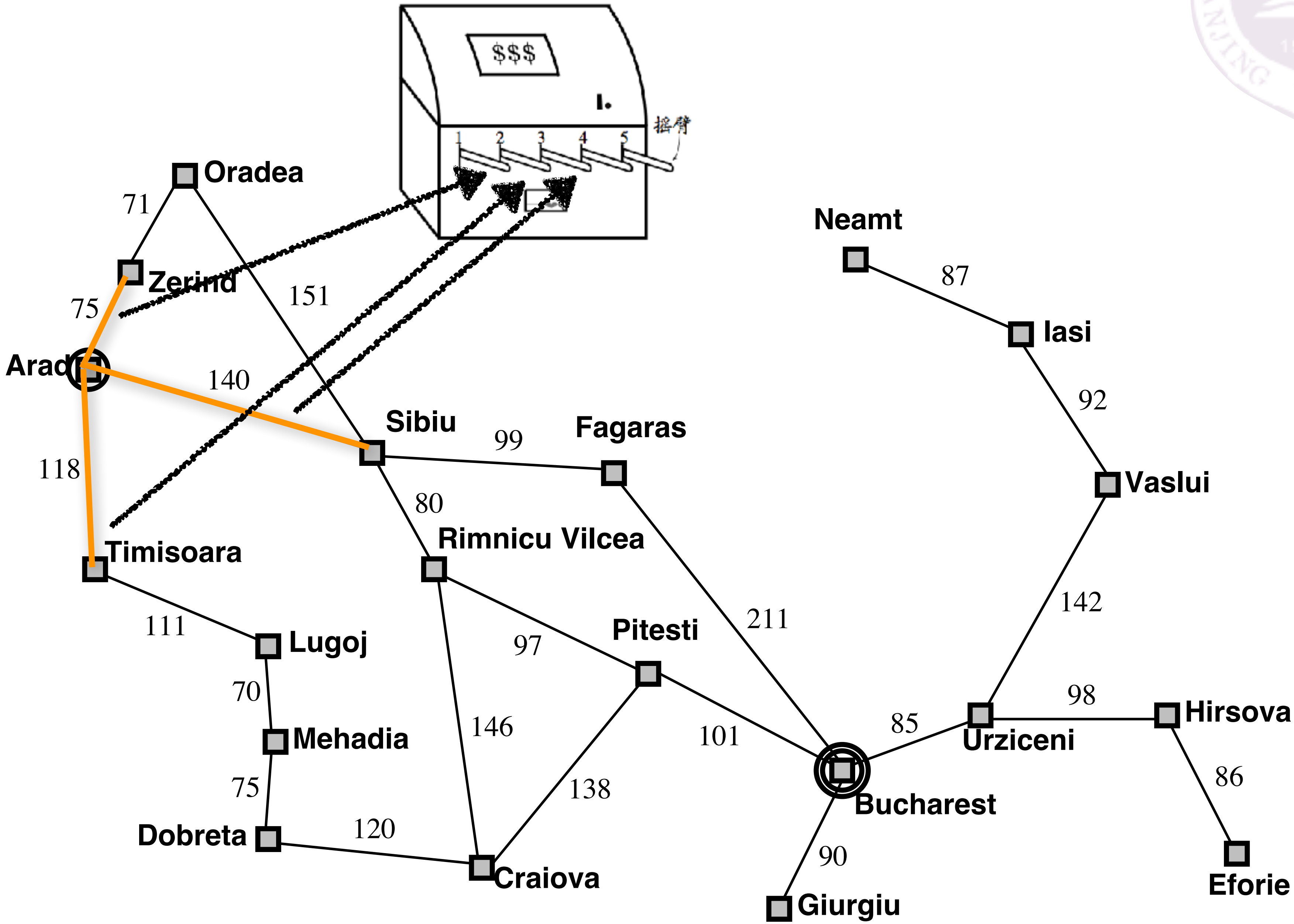
Choose arm with the largest value of

average reward + upper confidence bound

$$Q(k) + \sqrt{\frac{2\ln n}{n_k}},$$

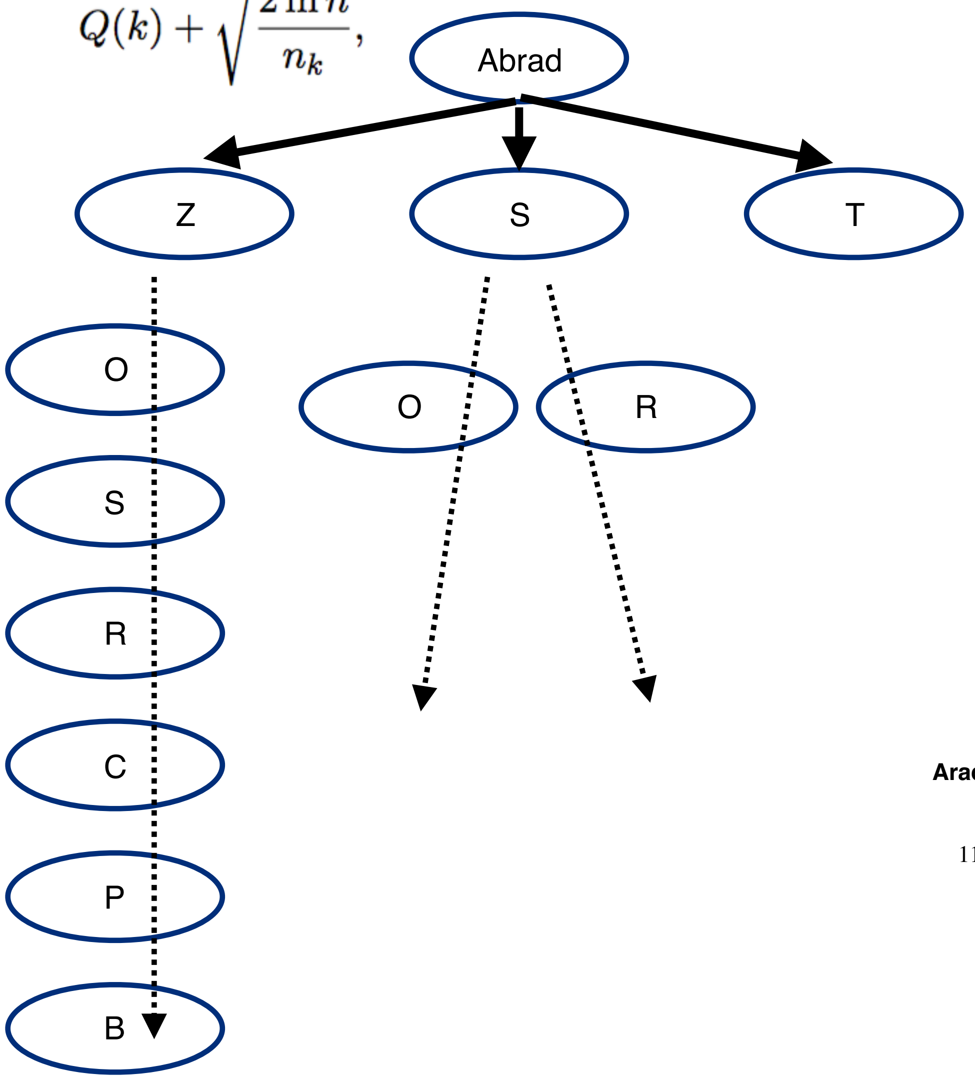


Use bandit to search



Use bandit to search

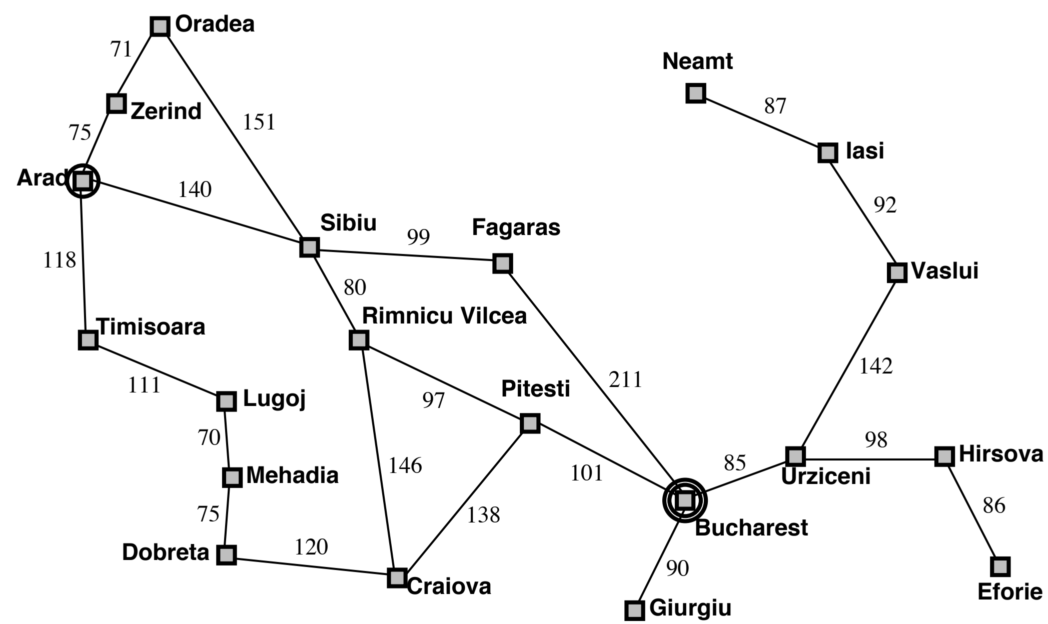
$$Q(k) + \sqrt{\frac{2 \ln n}{n_k}},$$



762 a roll-out

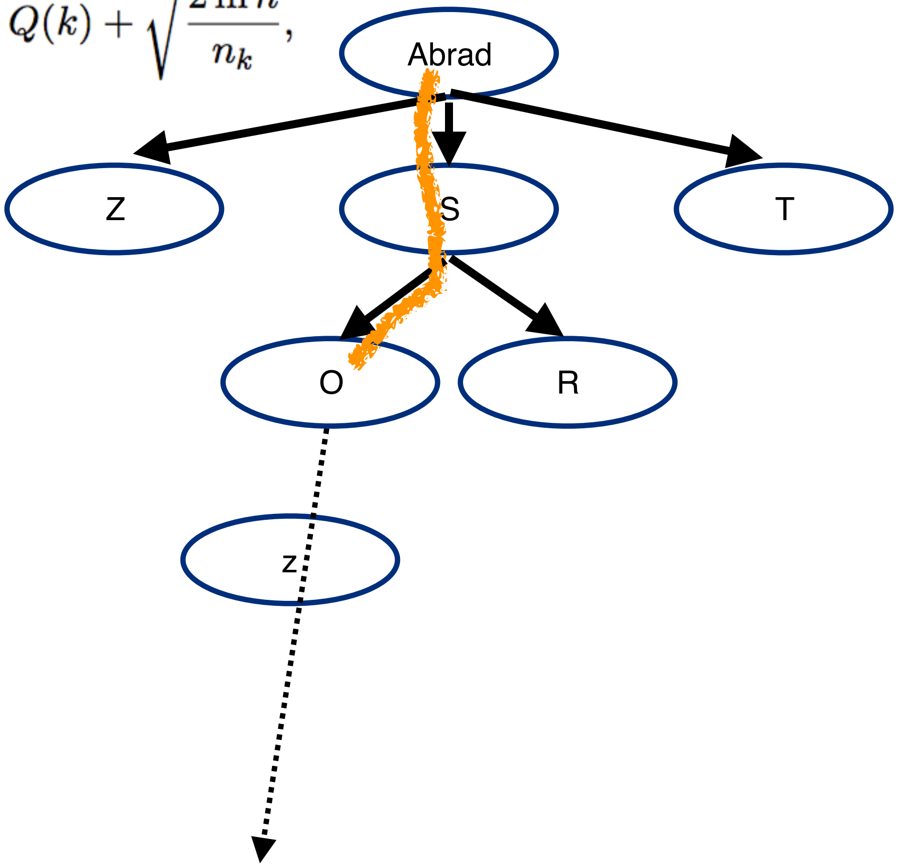
use many roll-outs to estimate the average cost of each arm

arm selection: UCB



From bandit to tree

$$Q(k) + \sqrt{\frac{2\ln n}{n_k}},$$



grow a tree

update the values along the path

Monte-Carlo Tree Search

also called Upper-Confidence Tree (UCT)

Kocsis Szepesvári, 06

Gradually grow the search tree:

- Iterate Tree-Walk
  - Building Blocks
    - Select next action

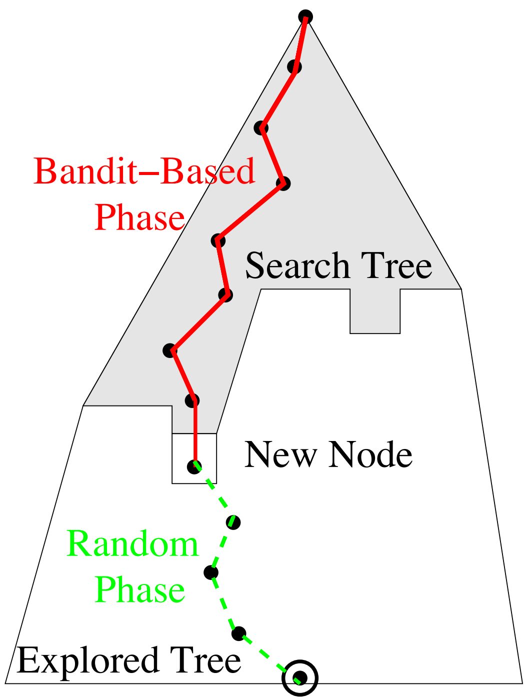
      Bandit phase
    - Add a node

      Grow a leaf of the search tree
    - Select next action bis

      Random phase, roll-out
    - Compute instant reward

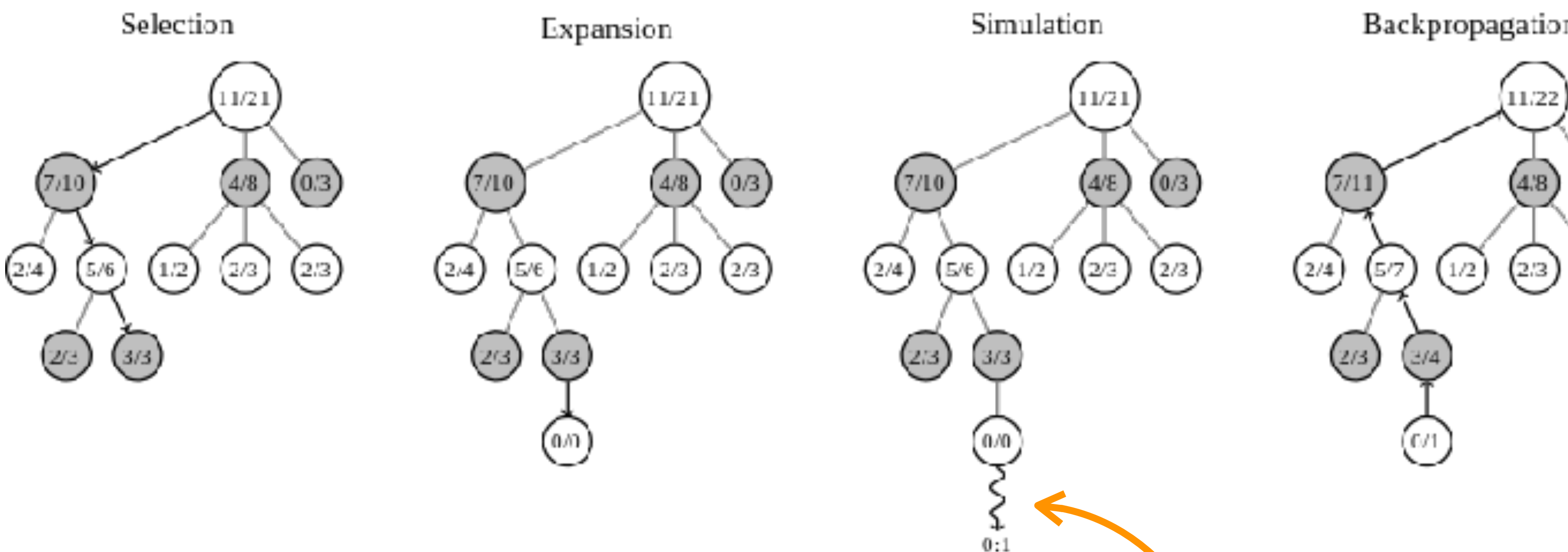
      Evaluate
    - Update information in visited nodes

      Propagate
- Returned solution:
  - Path visited most often



Monte-Carlo Tree Search

Example:



Pic from https://en.wikipedia.org/wiki/Monte_Carlo_tree_search#cite_note-Kocsis-Szepesvari-5

Monte-Carlo Tree Search

```
private TreeNode select() {
    TreeNode selected = null;
    double bestValue = Double.MIN_VALUE;
    for (TreeNode c : children) {
        double uctValue = c.totValue / (c.nVisits + epsilon) +
                    Math.sqrt(Math.log(nVisits+1) / (c.nVisits + epsilon)) +
                        r.nextDouble() * epsilon;
        // small random number to break ties randomly in unexpanded nodes
        if (uctValue > bestValue) {
            selected = c;
            bestValue = uctValue;
        }
    }
    return selected;
}
```

```
public
    st
    st
    st

    Tr
    do

    pu
        cur = cur.select();
        visited.add(cur);
    }
    cur.expand();
    TreeNode newNode = cur.select();
    visited.add(newNode);
    double value = rollOut(newNode);
    for (TreeNode node : visited) {
        // would need extra logic for n-player game
        node.updateStats(value);
    }
}
```

```
    totValue += value;
}
```

codes from http://mcts.ai/code/java.html

Monte-Carlo Tree Search

optimal? Yes, after infinite tries

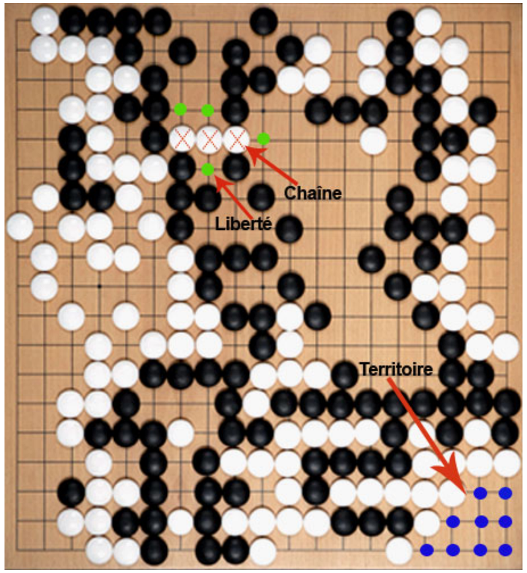
compare with alpha-beta pruning
no need of heuristic function

Monte-Carlo Tree Search

Improving random rollout

Monte-Carlo-based   Brügman 93

1. Until the goban is filled,
   add a stone (black or white in turn)
   at a uniformly selected empty position
2. Compute $r = \text{Win(black)}$
3. The outcome of the tree-walk is r



Improvements ?

- Put stones randomly in the neighborhood of a previous stone
- Put stones matching patterns   prior knowledge
- Put stones optimizing a value function   Silver et al. 07

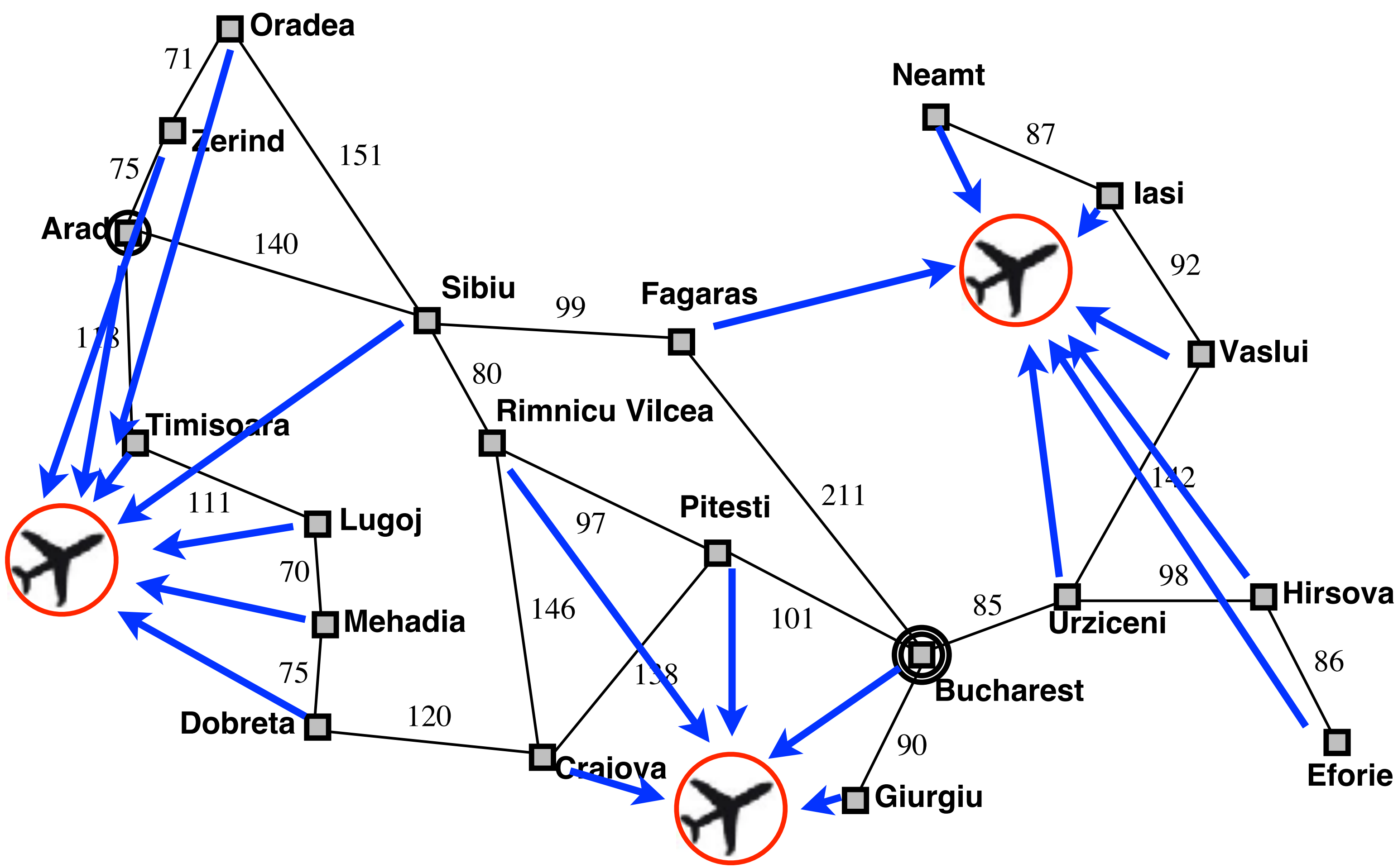
General solution space search

Greedy idea in continuous space

Suppose we want to site three airports in Romania:

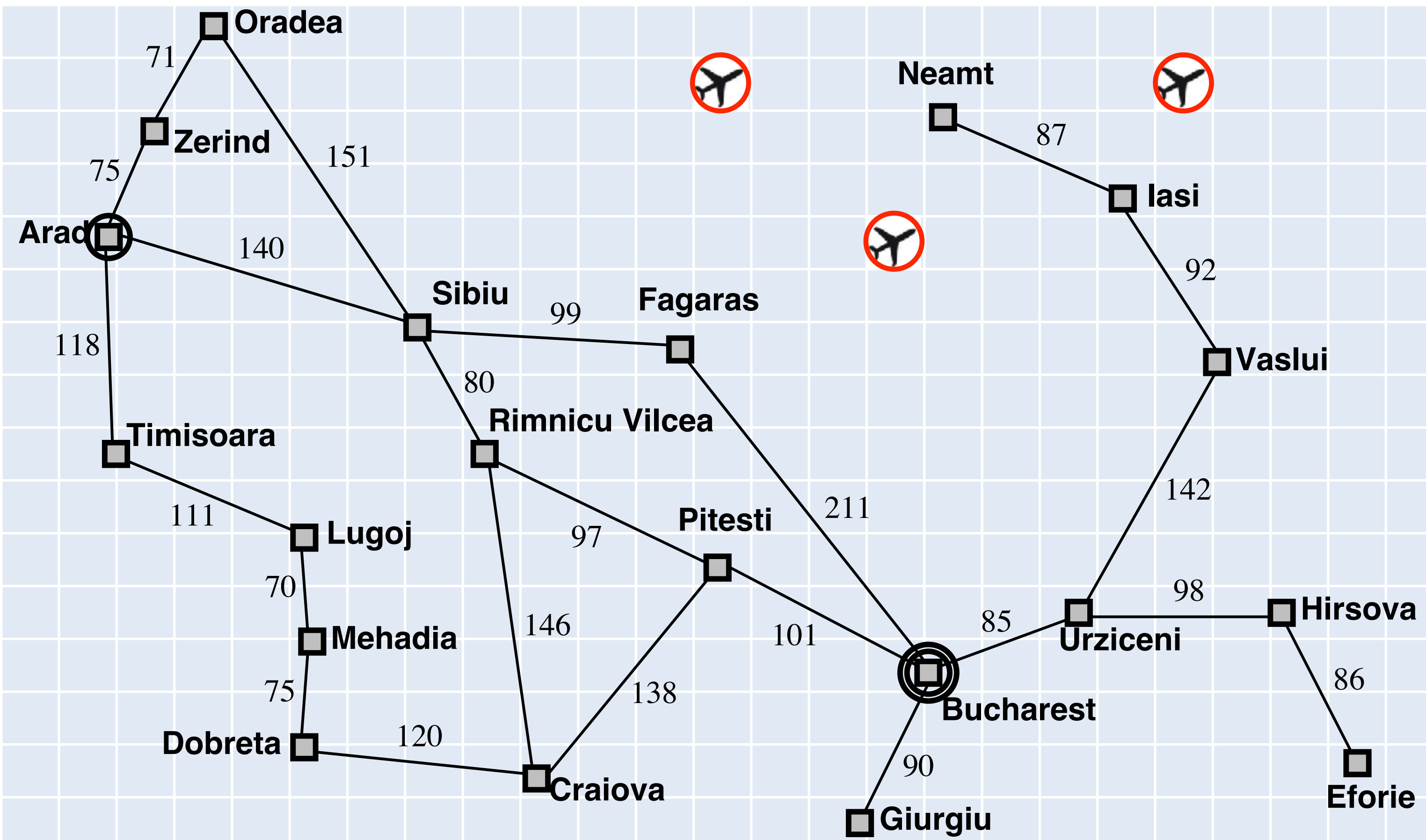
– 6-D state space defined by (x_1, y_2), (x_2, y_2), (x_3, y_3)

– objective function $f(x_1, y_2, x_2, y_2, x_3, y_3) =$
  sum of squared distances from each city to nearest airport



Greedy idea in continuous space

discretize and use hill climbing



Greedy idea in continuous space

gradient decent

– 6-D state space defined by (x_1, y_2), (x_2, y_2), (x_3, y_3)
– objective function $f(x_1, y_2, x_2, y_2, x_3, y_3) =$
  sum of squared distances from each city to nearest airport

Gradient methods compute

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3}\right)$$

to increase/reduce f, e.g., by $\mathbf{x} \leftarrow \mathbf{x} + \alpha \nabla f(\mathbf{x})$

1-order method

Greedy idea in continuous space

gradient decent

- 6-D state space defined by (x_1, y_2), (x_2, y_2), (x_3, y_3)
- objective function $f(x_1, y_2, x_2, y_2, x_3, y_3) =$
  sum of squared distances from each city to nearest airport

Sometimes can solve for $\nabla f(\mathbf{x}) = 0$ exactly (e.g., with one city).
Newton–Raphson (1664, 1690) iterates $\mathbf{x} \leftarrow \mathbf{x} - \mathbf{H}_f^{-1}(\mathbf{x})\nabla f(\mathbf{x})$
to solve $\nabla f(\mathbf{x}) = 0$, where $\mathbf{H}_{ij} = \partial^2 f / \partial x_i \partial x_j$

2-order method

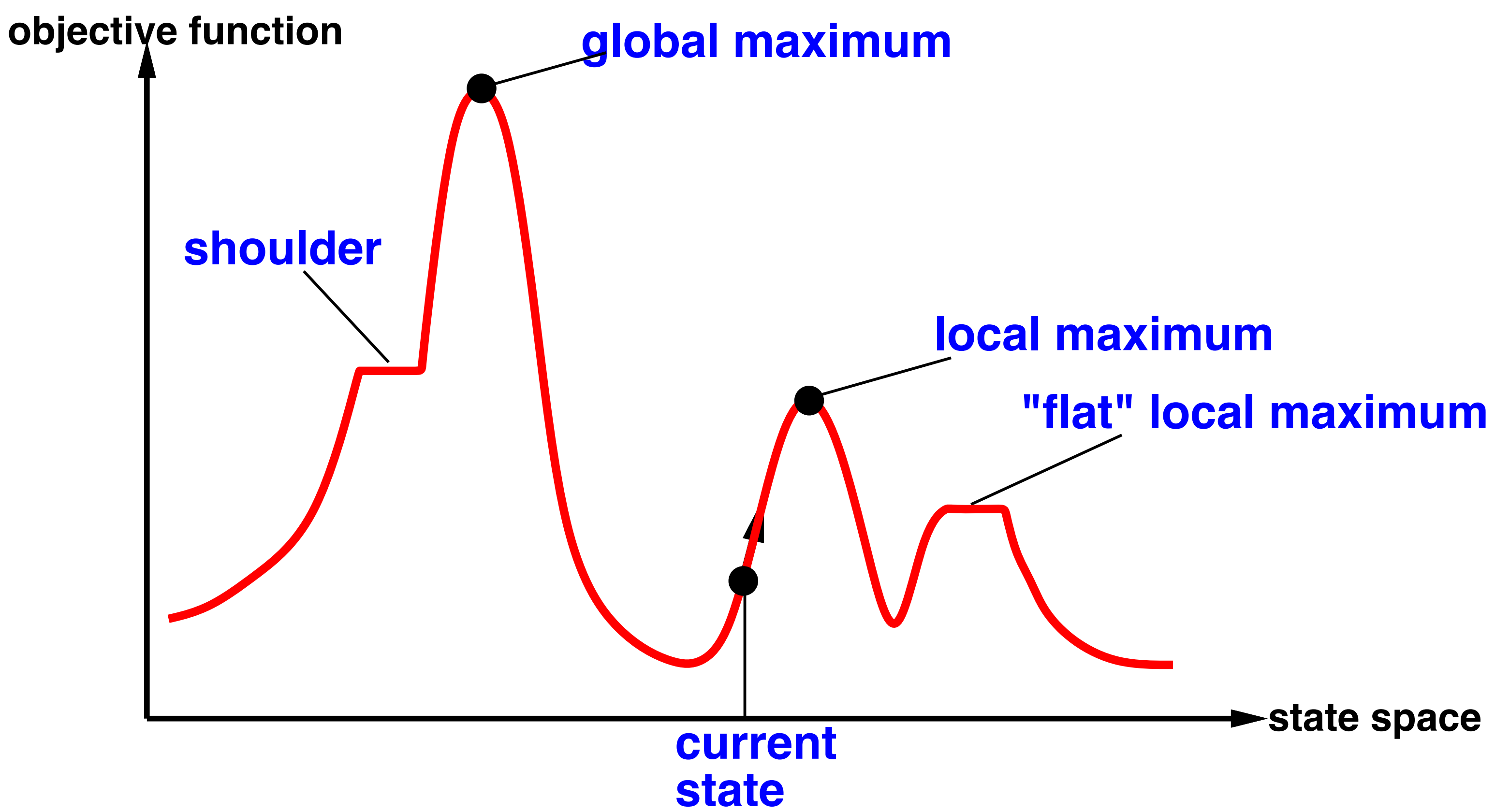
Taylor's series:

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2} f''(a) + \cdots = \sum_{i=0}^{\infty} \frac{(x-a)^i}{i!} f^{(i)}(a).$$

Greedy idea

1st and 2nd order methods may not find global optimal solutions

they work for convex functions



Meta-heuristics

“problem independent

“black-box

“zeroth-order method

...

and usually inspired from nature phenomenon

Simulated annealing

temperature from high to low

when high temperature, form the shape
when low temperature, polish the detail

Simulated annealing

Idea: escape local maxima by allowing some "bad" moves
but gradually decrease their size and frequency

function SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state
  inputs: *problem*, a problem
    schedule, a mapping from time to "temperature"
  local variables: *current*, a node
    next, a node
    T, a "temperature" controlling prob. of downward steps

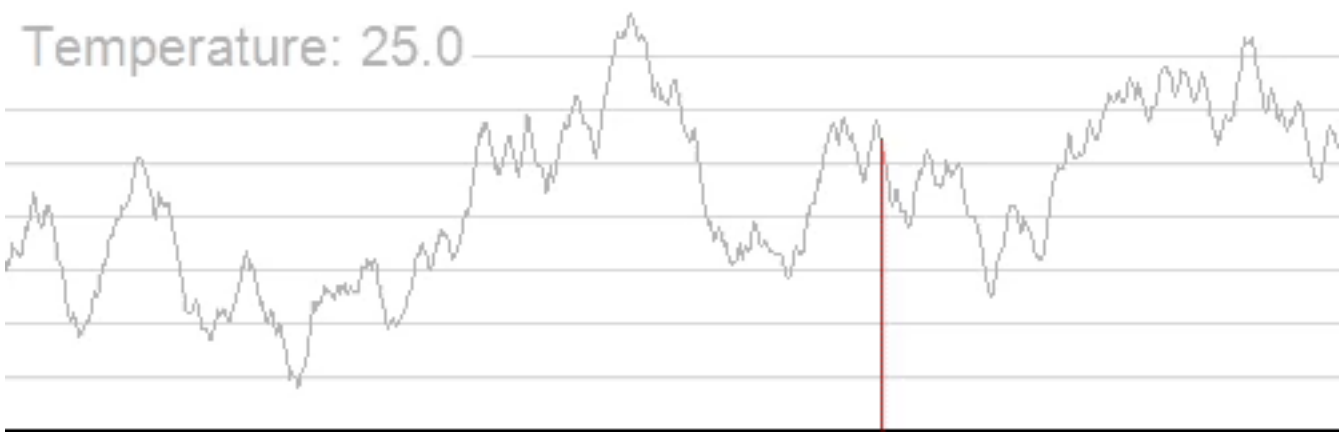
  current ← MAKE-NODE(INITIAL-STATE[*problem*])
  for t ← 1 **to** ∞ **do**
    T ← *schedule*[t]
    if $T = 0$ **then return** *current*
    next ← a randomly selected successor of *current*
    ΔE ← VALUE[*next*] – VALUE[*current*]
    if $\Delta E > 0$ **then** *current* ← *next*
    else *current* ← *next* only with probability $e^{\Delta E/T}$

the neighborhood range shrinks with T

the probability of accepting a bad solution decreases with T

Simulated annealing

a demo



graphic from http://en.wikipedia.org/wiki/Simulated_annealing

Local beam search

Idea: keep k states instead of 1; choose top k of all their successors

Not the same as k searches run in parallel!
Searches that find good states recruit other searches to join them

Problem: quite often, all k states end up on same local hill

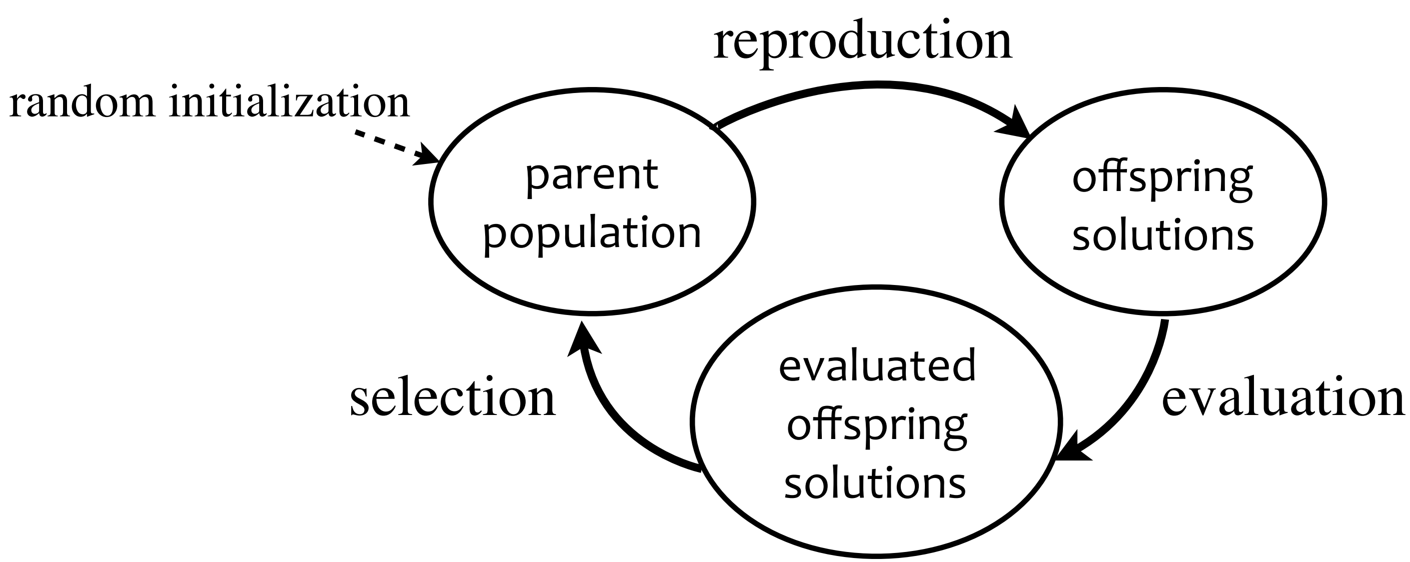
Idea: choose k successors randomly, biased towards good ones

Observe the close analogy to natural selection!

Genetic algorithm

a simulation of Darwin's evolutionary theory

over-reproduction with diversity
nature selection

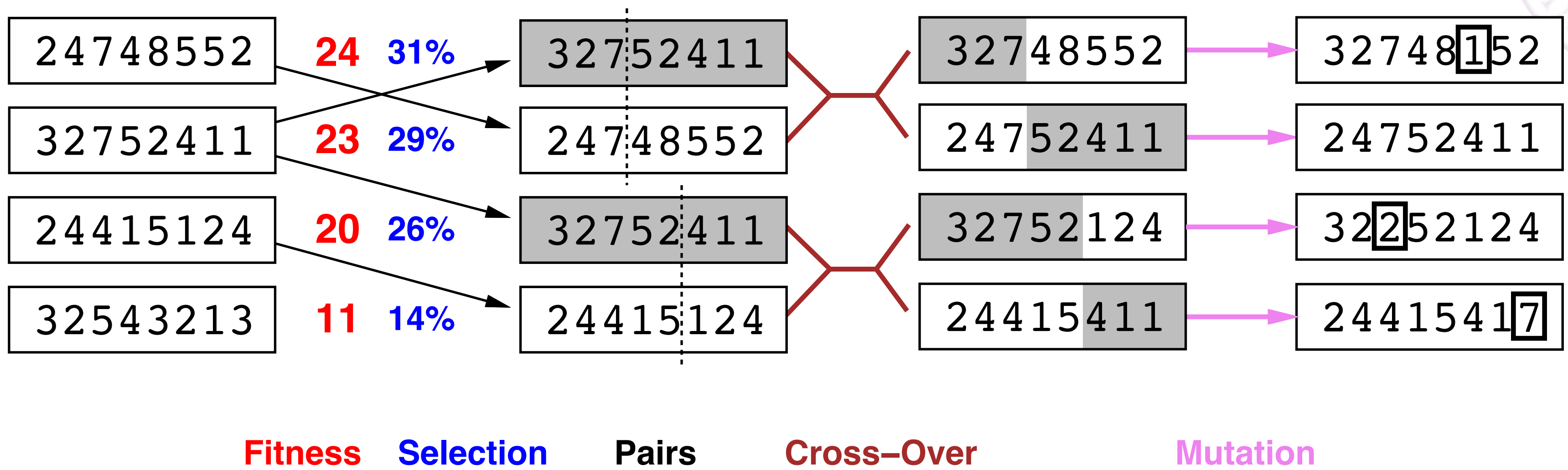


Genetic algorithm

Encode a solution as a vector,

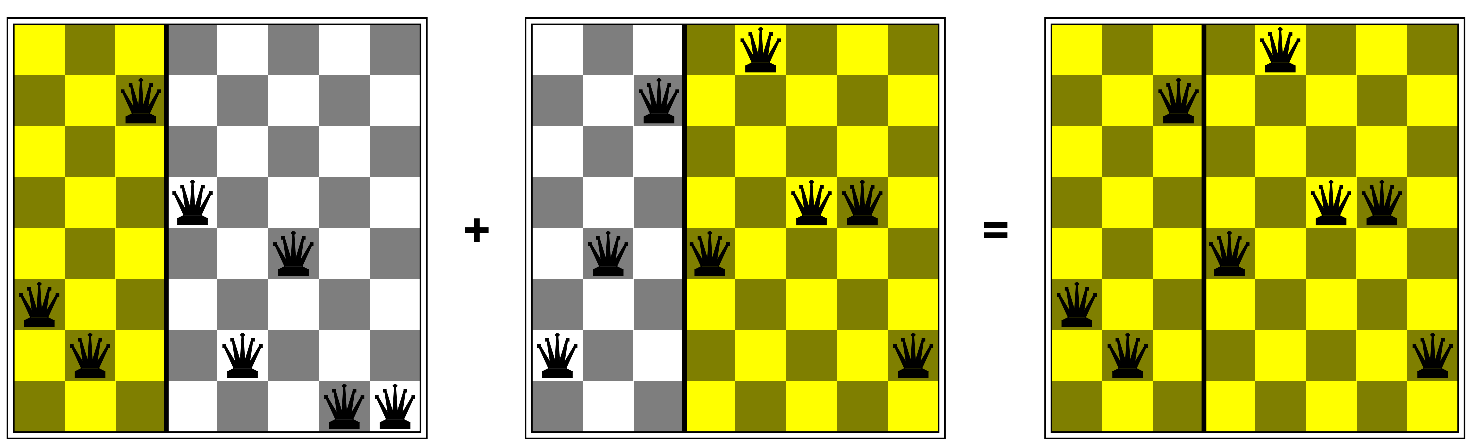
$Pop \leftarrow n$ randomly drawn solutions from $\mathcal{X}$
for t=1,2,... **do**
    $Pop^m \leftarrow \{mutate(s) \mid \forall s \in Pop\}$, the mutated solutions
    $Pop^c \leftarrow \{crossover(s_1, s_2) \mid \exists s_1, s_2 \in Pop^m\}$, the recombined solutions
    evaluate every solution in Pop^c by $f(s)(\forall s \in Pop^c)$
    $Pop^s \leftarrow$ selected solutions from Pop and Pop^c
    $Pop \leftarrow Pop^s$
    terminate if meets a stopping criterion
end for

Genetic algorithm



GAs require states encoded as strings (GPs use programs)

Crossover helps **iff substrings are meaningful components**

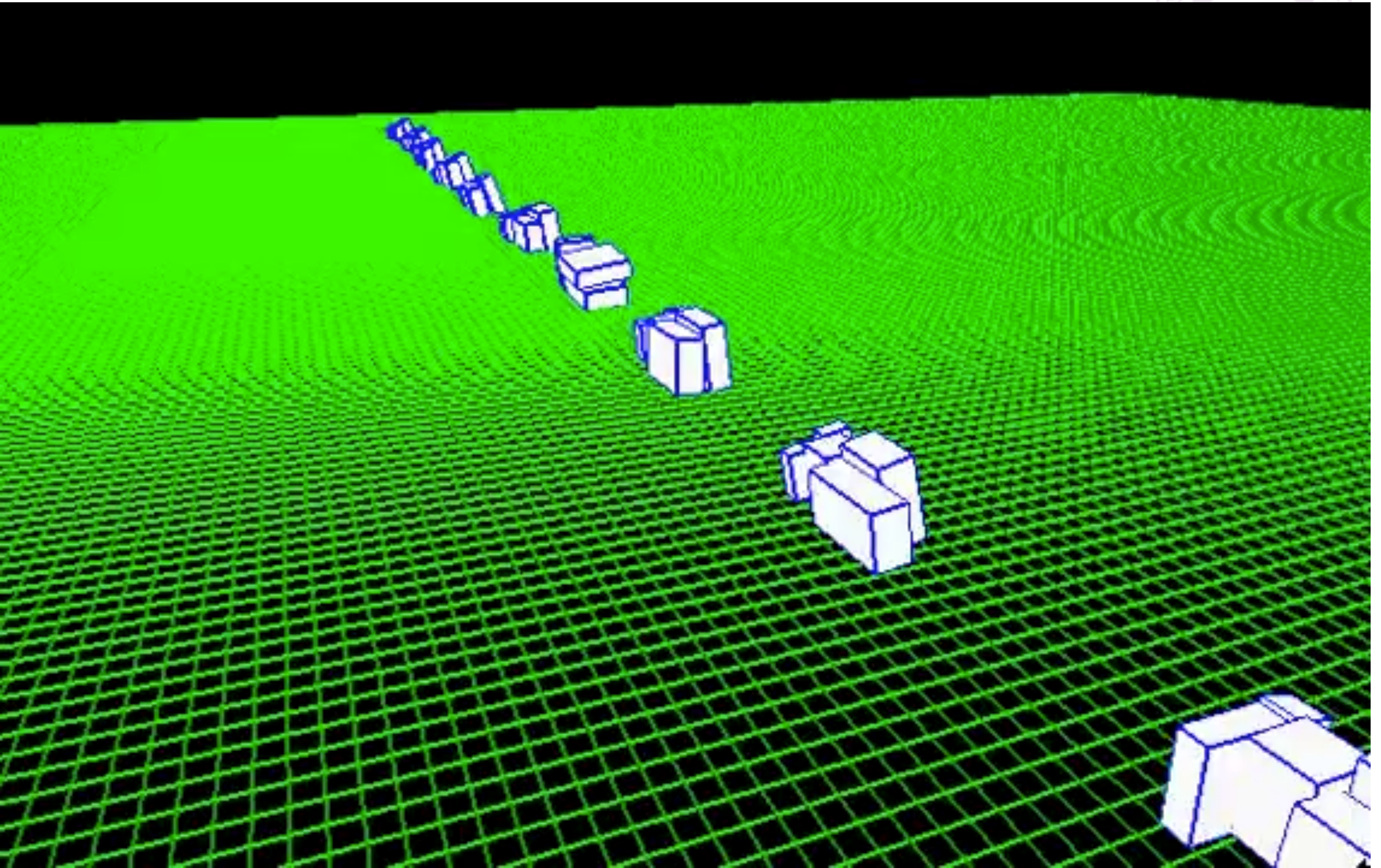


Example

Encode a solution as a vector with length n
each element of the vector can be chosen from $\{1,\ldots,V\}$
parameters: mutation probability p_m, crossover probability p_c

Pop = randomly generate n solutions from $\{1,\ldots,V\}^n$
for t=1,2, ... do
Pop^m=emptyset, Pop^c=emptyset
for $i = 1$ to n
let x be the i-th solution in *Pop*
for $j = 1$ to n: with probability p_m, change x_j by a random value from $\{1,\ldots,V\}$
add x into Pop^m
end for
for $i = 1$ to n
let x be the i-th solution in Pop^m
let x' be a randomly selected solution from Pop^m
with probability p_c, exchange a random part of x with x'
add x into Pop^c
end for
evaluate solutions in Pop^c, select the best n solutions from *Pop* and Pop^c to *Pop*
terminal if a good solution is found
end for

An evolutionary of virtual life



Properties of meta-heuristics

zeroth order

do not need differentiable functions

convergence

will find an optimal solution if $P(x^* \mid x)>0$
or $P(x \to x_1 \to \ldots \to x_k \to x^*)>0$

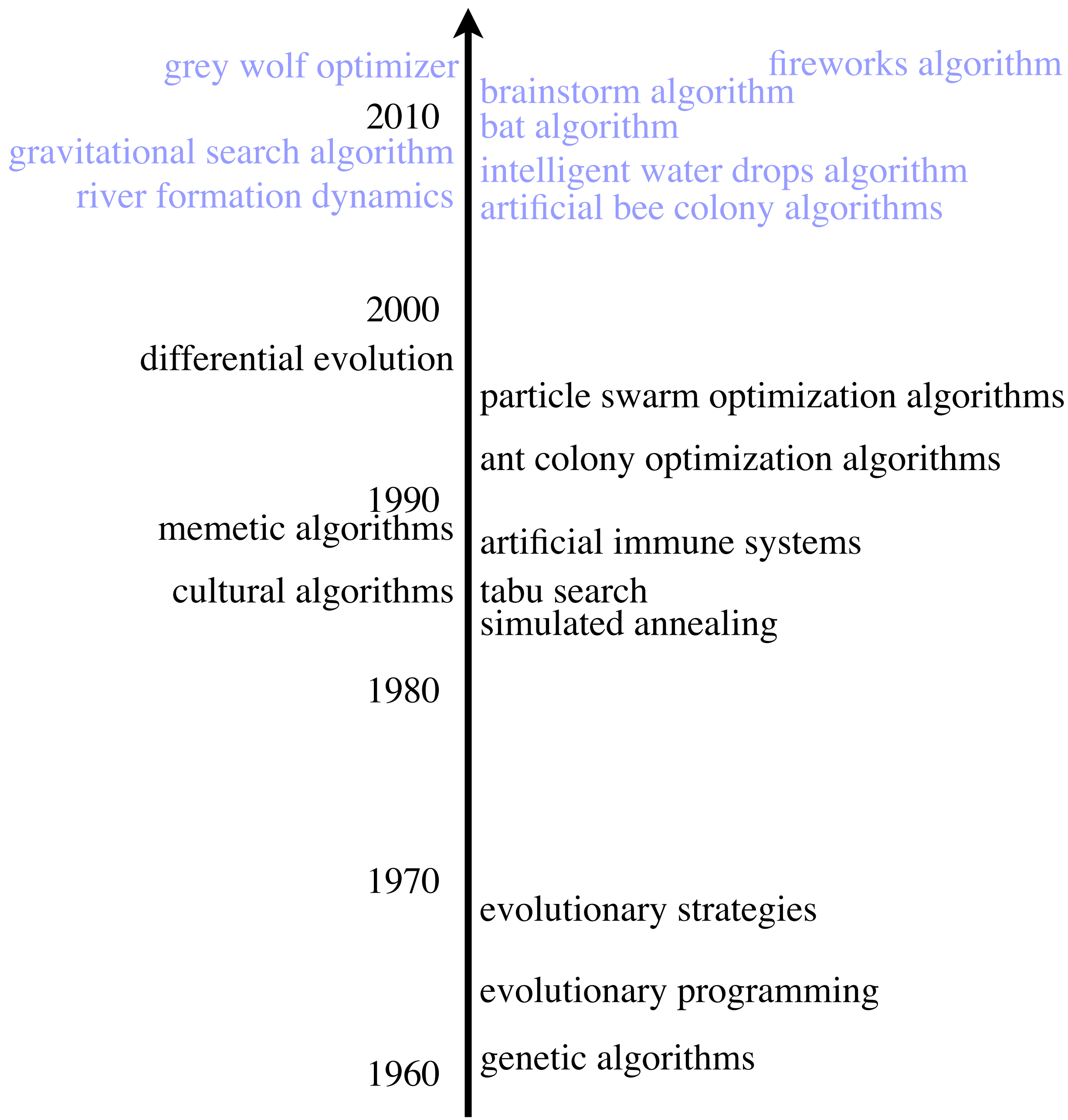
a missing link

observation → simulation

observation → **principle** → application

search from samples

Too many meta-heuristics



Example

hard to apply traditional optimization methods
but easy to test a given solution

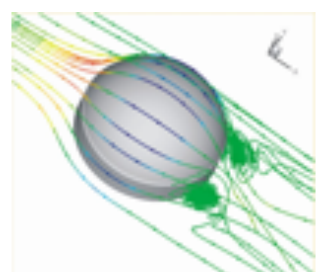
Representation:

parameterize

represented as a vector of parameters

Fitness:

x_i



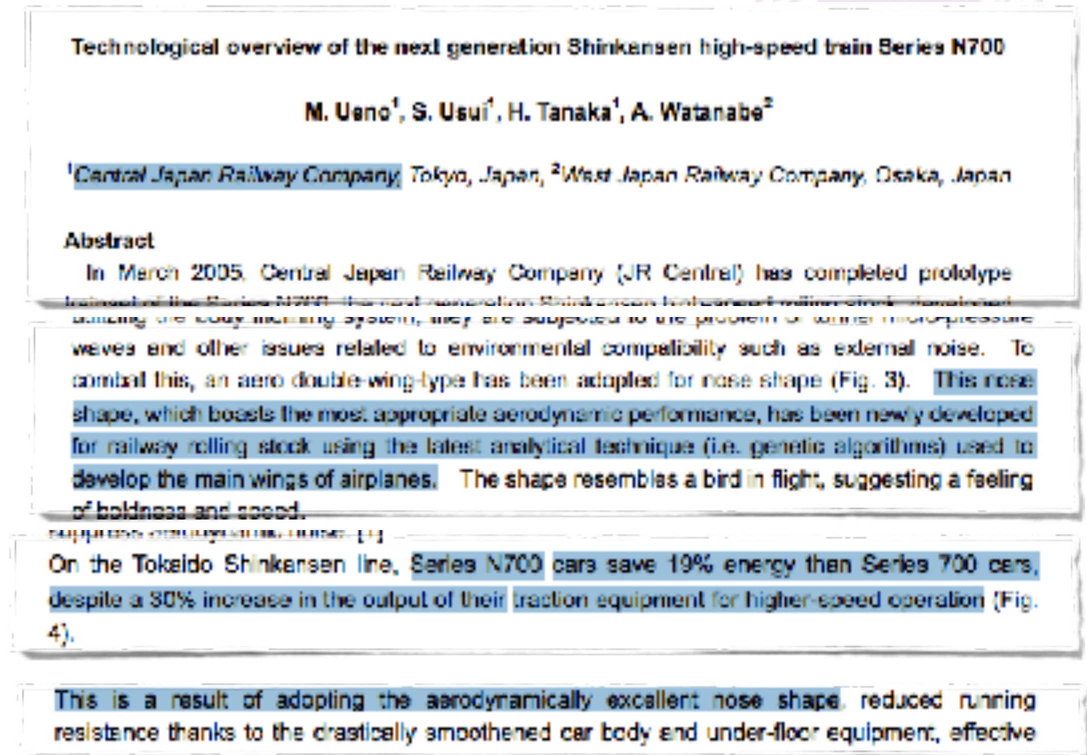
$f(x_i)$

test by simulation/experiment

Example

Series 700

Series N700



Technological overview of the next generation Shinkansen high-speed train Series N700

M. Ueno[1], S. Usui[1], H. Tanaka[1], A. Watanabe[2]

[1]Central Japan Railway Company, Tokyo, Japan, [2]West Japan Railway Company, Osaka, Japan

Abstract

In March 2005, Central Japan Railway Company (JR Central) has completed prototype

waves and other issues related to environmental compatibility such as external noise. To combat this, an aero double-wing-type has been adopted for nose shape (Fig. 3). This nose shape, which boasts the most appropriate aerodynamic performance, has been newly developed for railway rolling stock using the latest analytical technique (i.e. genetic algorithms) used to develop the main wings of airplanes. The shape resembles a bird in flight, suggesting a feeling of boldness and speed.

On the Tokaido Shinkansen line, Series N700 cars save 19% energy than Series 700 cars, despite a 30% increase in the output of their traction equipment for higher-speed operation (Fig. 4).

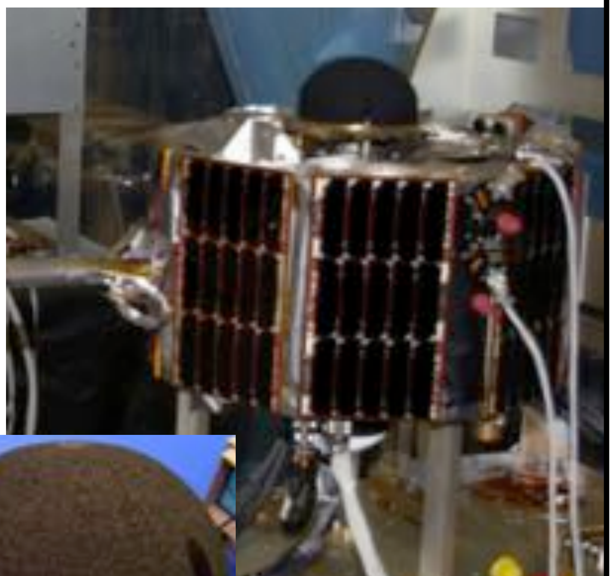
This is a result of adopting the aerodynamically excellent nose shape, reduced running resistance thanks to the drastically smoothened car body and under-floor equipment, effective

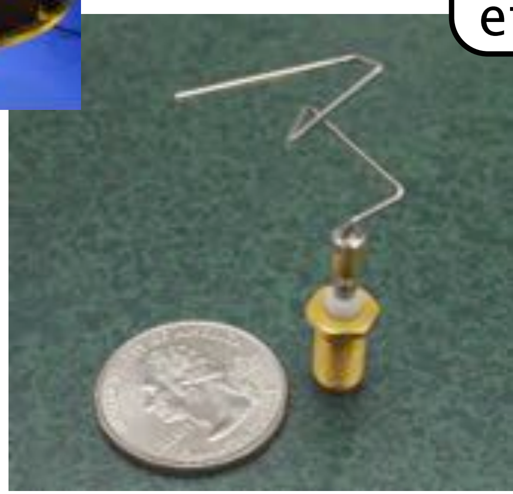
this nose ... has been newly developed ... using the latest analytical technique (i.e. **genetic algorithms**)

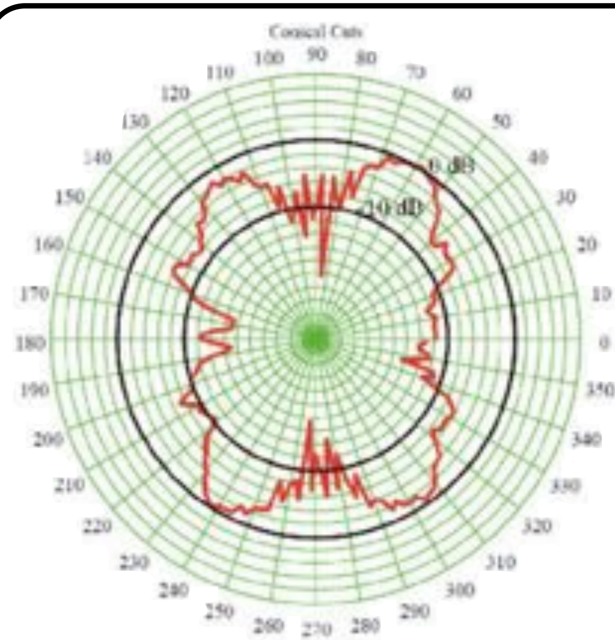
N700 cars save **19%** energy ... **30%** increase in the output... This is a result of adopting the ... nose shape

Example

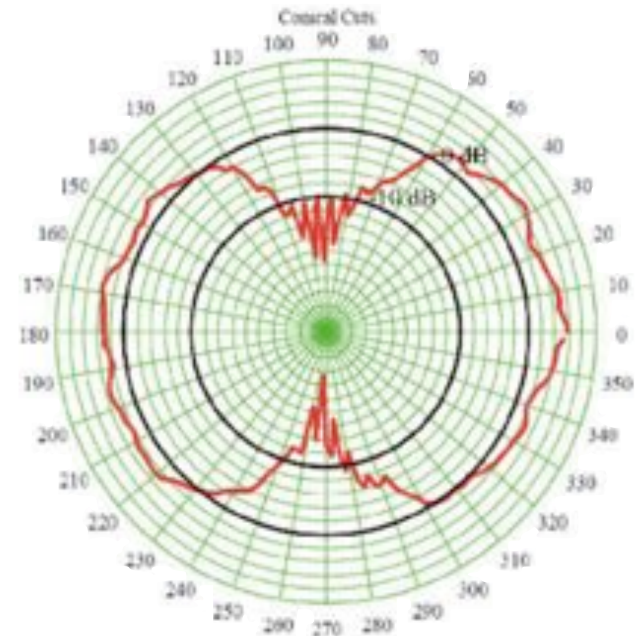
NASA ST5 satellite







QHAs(人工设计) 38% efficiency



evolved antennas resulted in 93% efficiency

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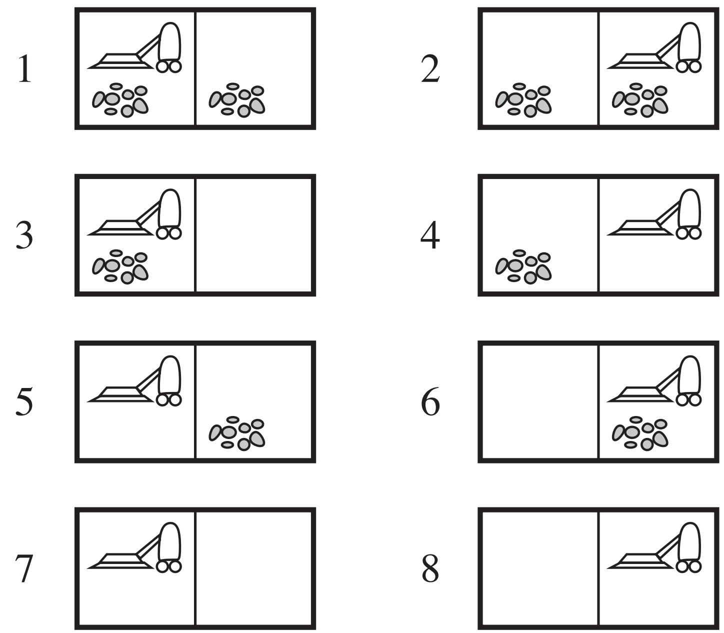
Since there are two antennas on each spacecraft, and not just one, it is important to measure the overall gain pattern with two antennas mounted on the spacecraft. For this, different combinations of the two evolved antennas and the QHA were tried on the the ST5 mock-up and measured in an anechoic chamber. With two QHAs 38% efficiency was achieved, using a QHA with an evolved antenna resulted in 80% efficiency, and using two evolved antennas resulted in 93% efficiency. Here "efficiency" means how much power is being radiated versus how much power is being eaten up in resistance, with greater efficiency resulting in a stronger signal and greater range. Figure 11

Different Environment Properties

Nondeterministic actions

In the **erratic vacuum world**, the *Suck* action works as follows:

- When applied to a dirty square the action cleans the square and sometimes cleans up dirt in an adjacent square, too.
- When applied to a clean square the action sometimes deposits dirt on the carpet.

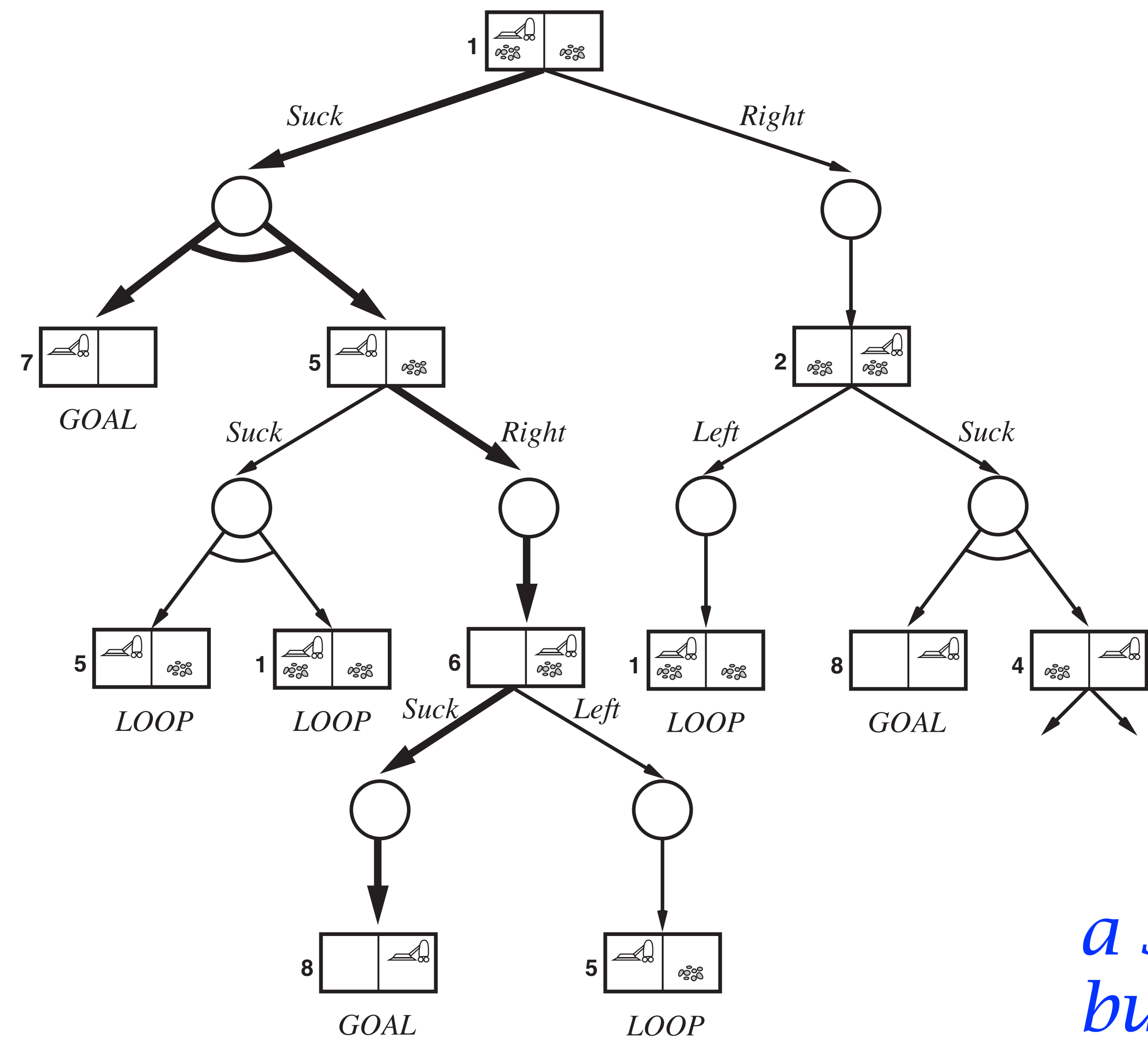


almost all real-world problems are nondeterministic

how do you solve this problem?

AND-OR tree search

OR node: different actions (as usual)
AND node: different transitions



a solution is not a path but a tree

Depth-first AND-OR tree search

function AND-OR-GRAPH-SEARCH($problem$) **returns** *a conditional plan, or failure*
  OR-SEARCH($problem$.INITIAL-STATE, $problem$, [ ])

---

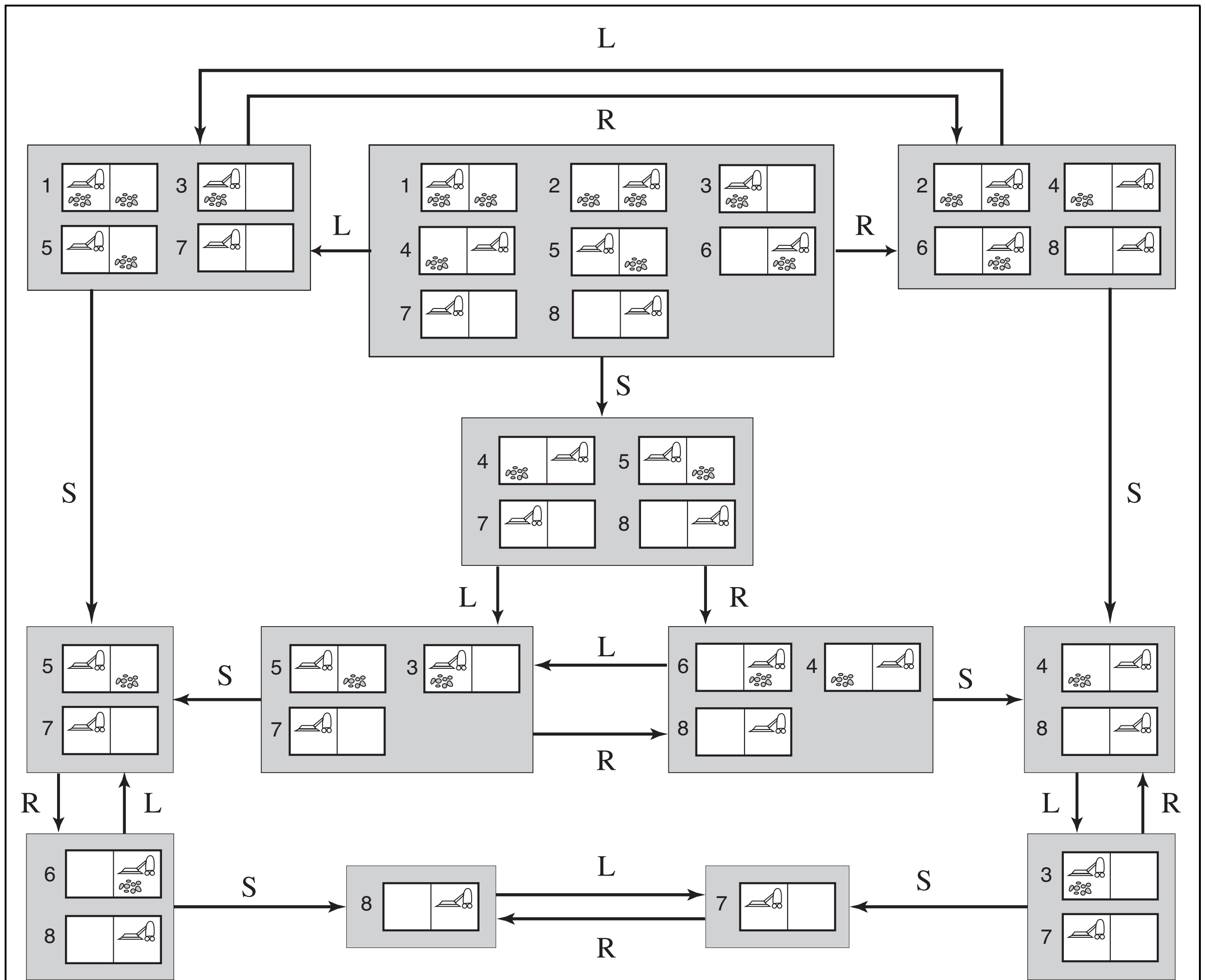
function OR-SEARCH($state, problem, path$) **returns** *a conditional plan, or failure*
  if $problem$.GOAL-TEST($state$) **then return** the empty plan
  if $state$ is on $path$ **then return** $failure$
  for each $action$ **in** $problem$.ACTIONS($state$) **do**
    $plan \leftarrow$ AND-SEARCH(RESULTS($state, action$), $problem$, $[state \mid path]$)
    if $plan \neq failure$ **then return** $[action \mid plan]$
  return $failure$

---

function AND-SEARCH($states, problem, path$) **returns** *a conditional plan, or failure*
  for each s_i **in** $states$ **do**
    $plan_i \leftarrow$ OR-SEARCH($s_i, problem, path$)
    if $plan_i = failure$ **then return** $failure$
  return [**if** s_1 **then** $plan_1$ **else if** s_2 **then** $plan_2$ **else** $\ldots$ **if** s_{n-1} **then** $plan_{n-1}$ **else** $plan_n$]

Search with no observations

search in **belief (in agent's mind)**



Constraint satisfaction problems (CSPs)

Constraint satisfaction problems (CSPs)

Standard search problem:

state is a “black box”—any old data structure
that supports goal test, eval, successor

CSP:

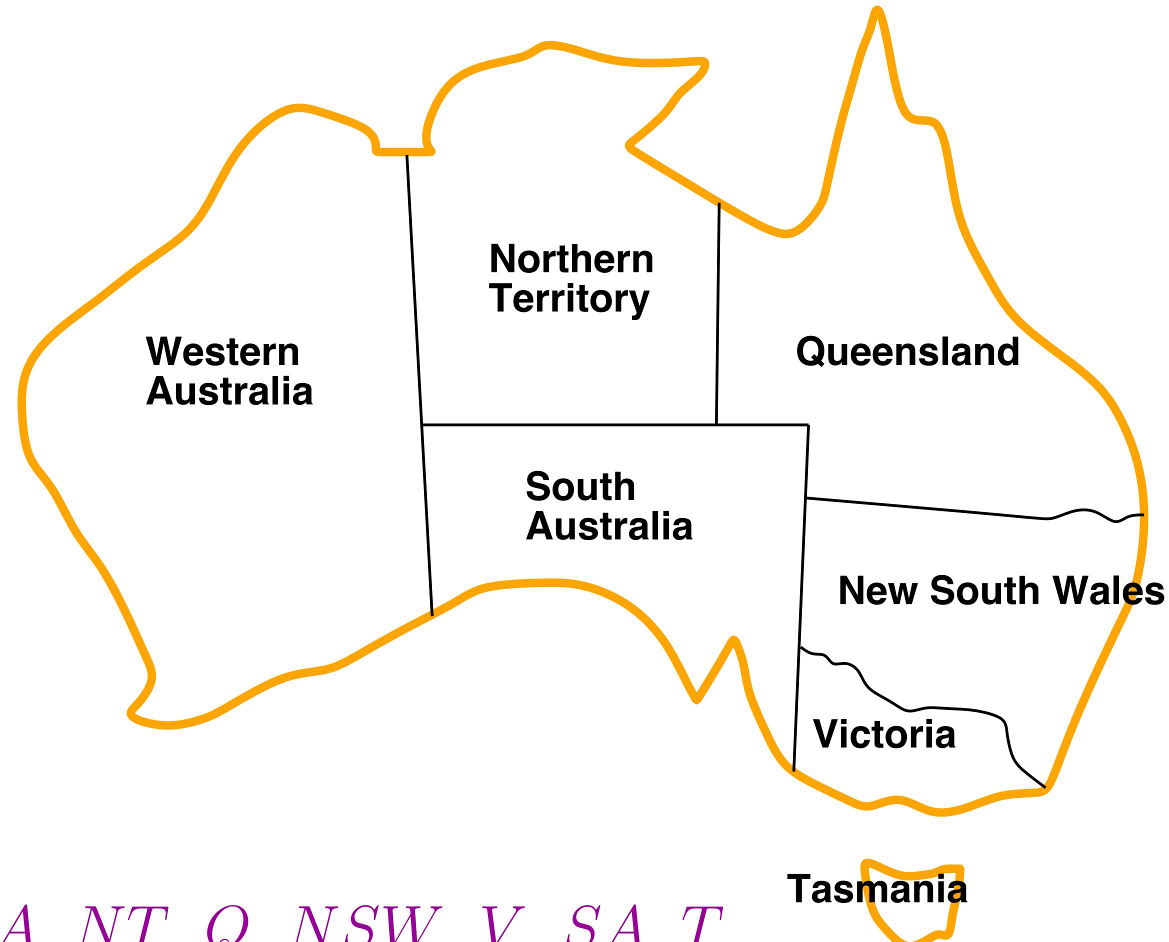
state is defined by variables X_i with values from domain D_i

goal test is a set of constraints specifying
allowable combinations of values for subsets of variables

Simple example of a **formal representation language**

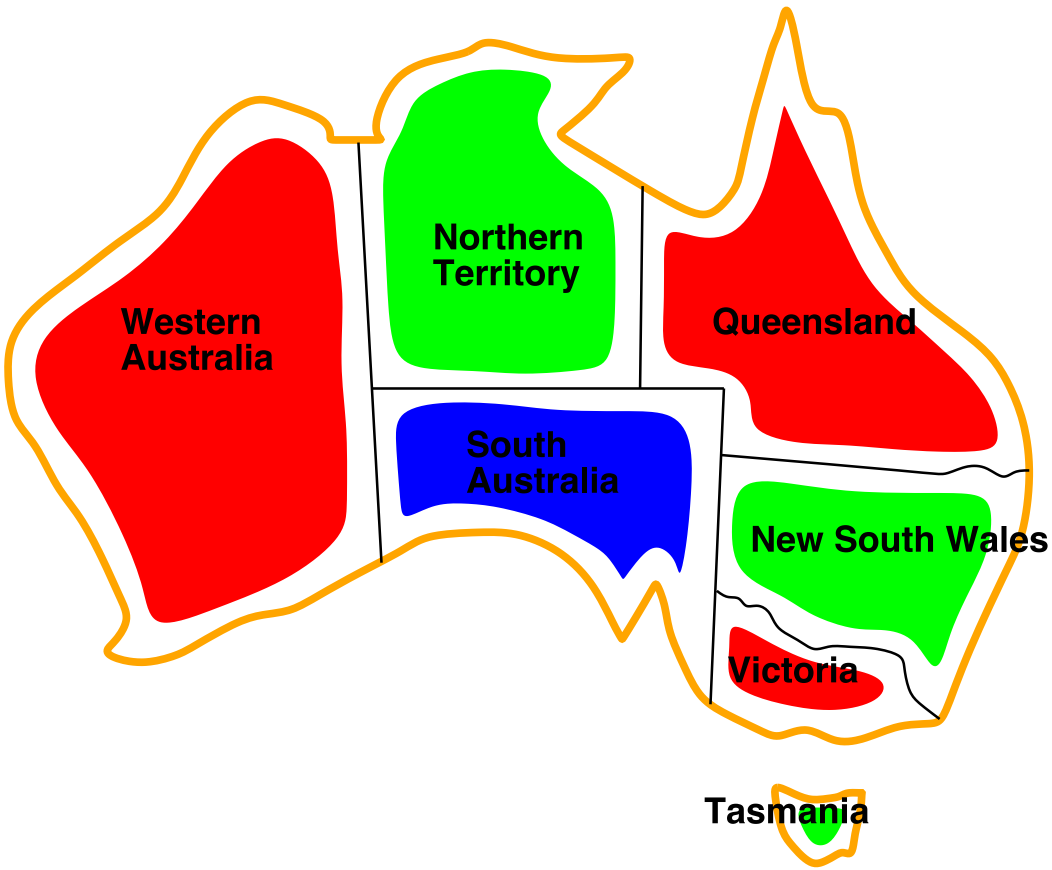
Allows useful **general-purpose** algorithms with more power than standard search algorithms

Example: Map-Coloring



Variables WA, NT, Q, NSW, V, SA, T
Domains $D_i = \{red, green, blue\}$
Constraints: adjacent regions must have different colors
e.g., $WA \neq NT$ (if the language allows this), or
$(WA, NT) \in \{(red, green), (red, blue), (green, red), (green, blue), \ldots\}$

Example: Map-Coloring



Solutions are assignments satisfying all constraints, e.g.,
$\{WA = red, NT = green, Q = red, NSW = green, V = red, SA = blue, T = green\}$

Varieties of CSPs

Discrete variables

- finite domains; size $d \;\Rightarrow\; O(d^n)$ complete assignments
  - ◇ e.g., Boolean CSPs, incl. Boolean satisfiability (NP-complete)
- infinite domains (integers, strings, etc.)
  - ◇ e.g., job scheduling, variables are start/end days for each job
  - ◇ need a constraint language, e.g., $StartJob_1 + 5 \leq StartJob_3$
  - ◇ linear constraints solvable, nonlinear undecidable

Continuous variables

- ◇ e.g., start/end times for Hubble Telescope observations
- ◇ linear constraints solvable in poly time by LP methods

Varieties of CSPs

Unary constraints involve a single variable,
e.g., $SA \neq green$

Binary constraints involve pairs of variables,
e.g., $SA \neq WA$

Higher-order constraints involve 3 or more variables,
e.g., cryptarithmetic column constraints

Preferences (soft constraints), e.g., red is better than $green$
often representable by a cost for each variable assignment
$\rightarrow$ constrained optimization problems

Real-world CSPs

Assignment problems
    e.g., who teaches what class

Timetabling problems
    e.g., which class is offered when and where?

Hardware configuration

Spreadsheets

Transportation scheduling

Factory scheduling

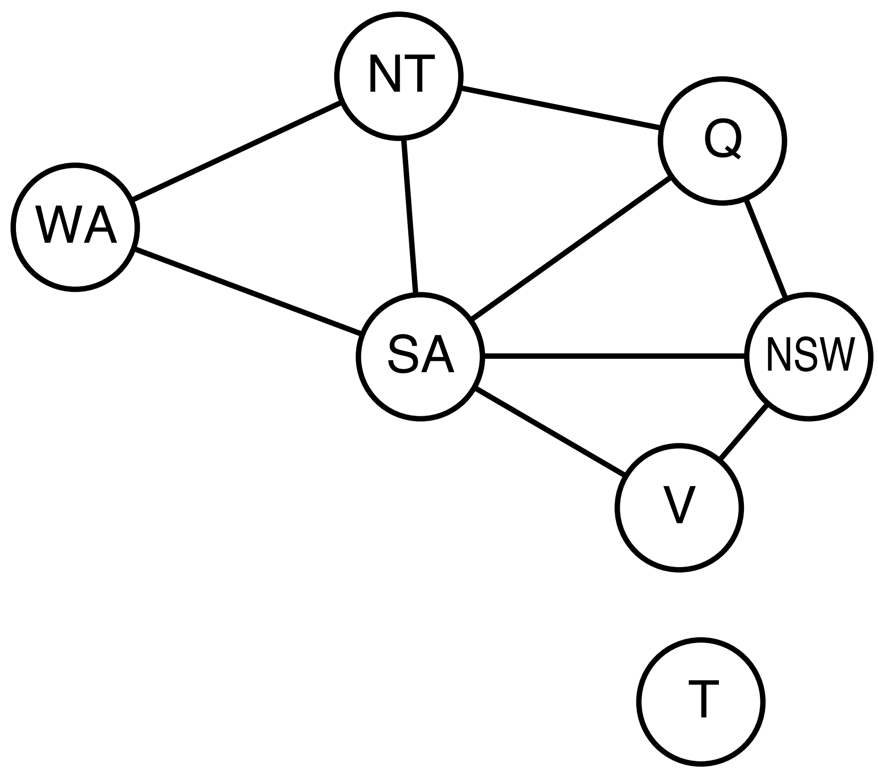
Floorplanning

Notice that many real-world problems involve real-valued variables

Constraint graph

Binary CSP: each constraint relates at most two variables

Constraint graph: nodes are variables, arcs show constraints



General-purpose CSP algorithms use the graph structure
to speed up search. E.g., Tasmania is an independent subproblem!

Convert higher-order to binary

A higher-order constraint can be converted to binary constraints with a *hidden-variable*

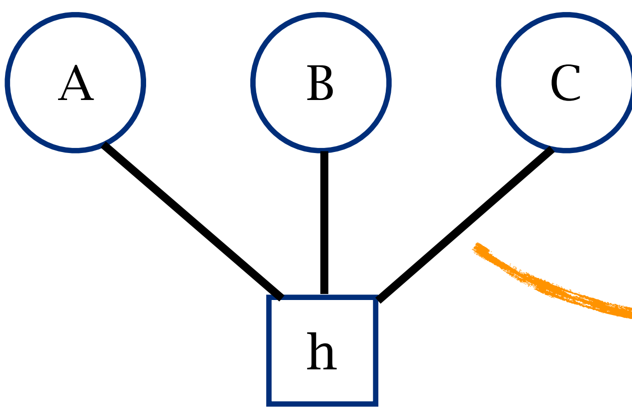
variable: A, B, C   domain: {1,2,3}   constraint: A+B=C

all possible assignments: (A,B,C) = (1,1,2), (1,2,3), (2,1,3)

hidden-variable: h with domain: {1,2,3}

(each value corresponds to an assignment)

the constraint graph:

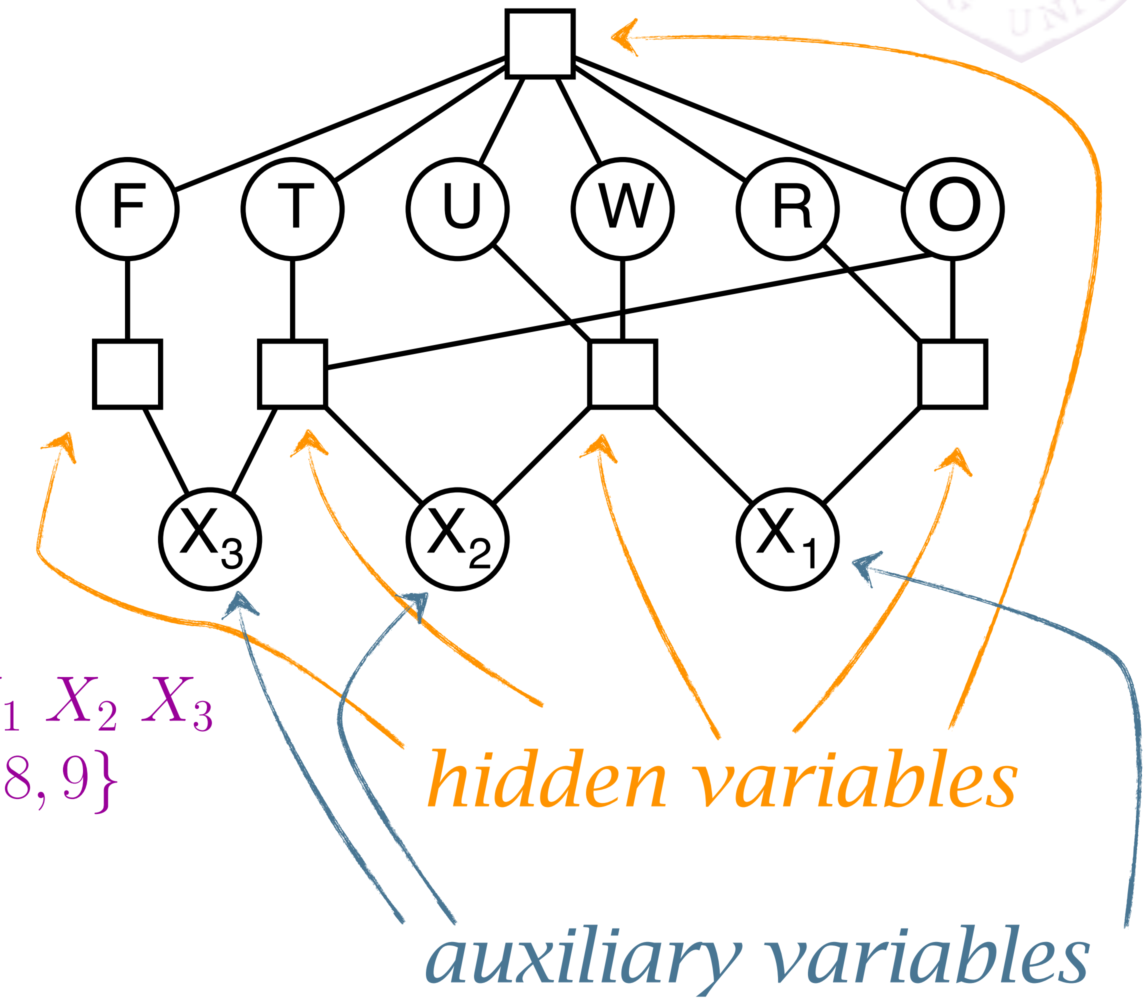


constraint:
h=1, C=2
h=2, C=3
h=3, C=3

from the definition of h

Example: Cryptarithmetic

$$\begin{array}{ccccc} & & T & W & O \\ + & & T & W & O \\ \hline & F & O & U & R \end{array}$$



Variables: $F\ T\ U\ W\ R\ O\ X_1\ X_2\ X_3$

Domains: $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Constraints

alldiff(F, T, U, W, R, O)

$O + O = R + 10 \cdot X_1$, etc.

Standard search formulation (incremental)

Let's start with the straightforward, dumb approach, then fix it

States are defined by the values assigned so far

◇ Initial state: the empty assignment, $\{ \}$

◇ Successor function: assign a value to an unassigned variable
that does not conflict with current assignment.
$\Rightarrow$ fail if no legal assignments (not fixable!)

◇ Goal test: the current assignment is complete

1) This is the same for all CSPs! 😈
2) Every solution appears at depth n with n variables
$\Rightarrow$ use depth-first search
3) Path is irrelevant, so can also use complete-state formulation
4) $b = (n - \ell)d$ at depth ℓ, hence $n!d^n$ leaves!!!! ☹

Backtracking search

Variable assignments are commutative, i.e.,

[$WA = red$ then $NT = green$] same as [$NT = green$ then $WA = red$]

Only need to consider assignments to a single variable at each node

$\Rightarrow$ $b = d$ and there are d^n leaves

Depth-first search for CSPs with single-variable assignments
is called backtracking search

Backtracking search is the basic uninformed algorithm for CSPs

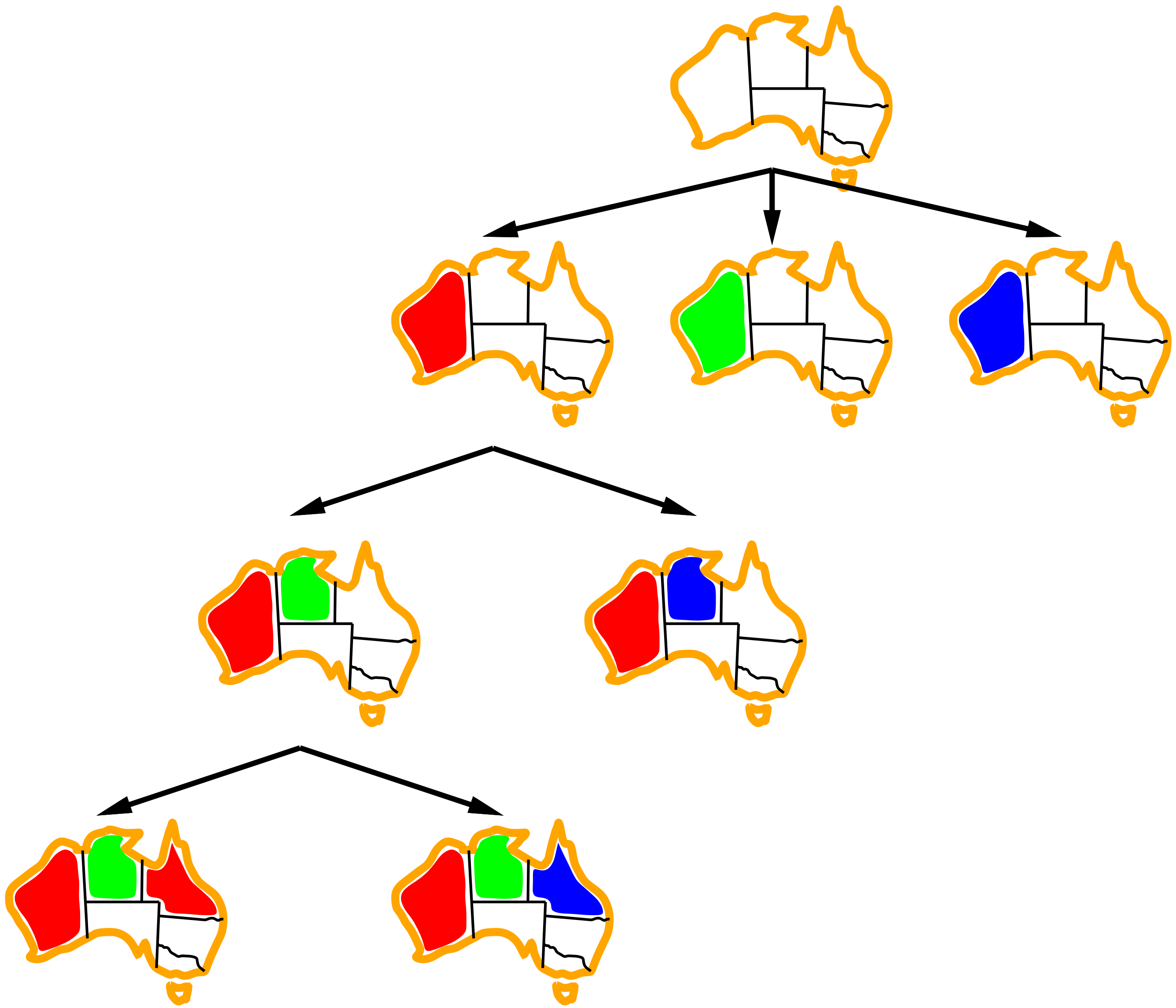
Can solve n-queens for $n \approx 25$

Backtracking search

```
function BACKTRACKING-SEARCH(csp) returns solution/failure
    return RECURSIVE-BACKTRACKING({ }, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
    if assignment is complete then return assignment
    var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
    for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
        if value is consistent with assignment given CONSTRAINTS[csp] then
            add {var = value} to assignment
            result ← RECURSIVE-BACKTRACKING(assignment, csp)
            if result ≠ failure then return result
            remove {var = value} from assignment
    return failure
```

Backtracking search example



Improving backtracking efficiency

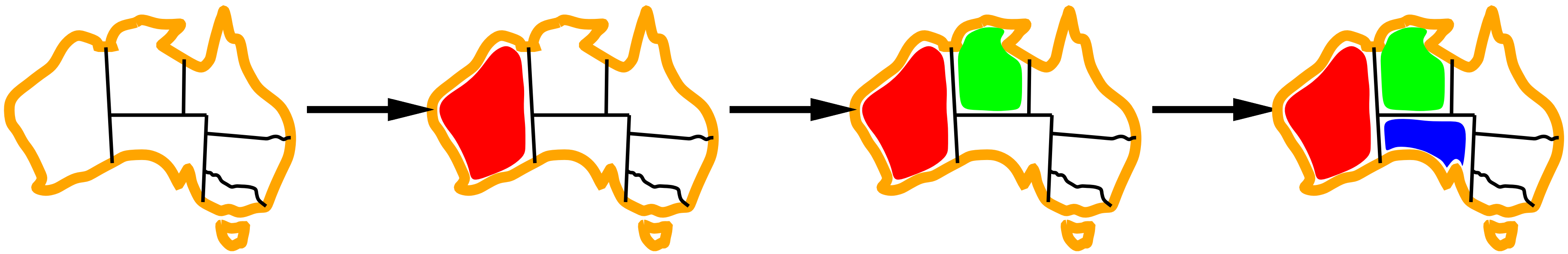
backtracking is uninformed
make it more informed

General-purpose methods can give huge gains in speed:

1. Which variable should be assigned next?
2. In what order should its values be tried?
3. Can we detect inevitable failure early?
4. Can we take advantage of problem structure?

Minimum remaining values

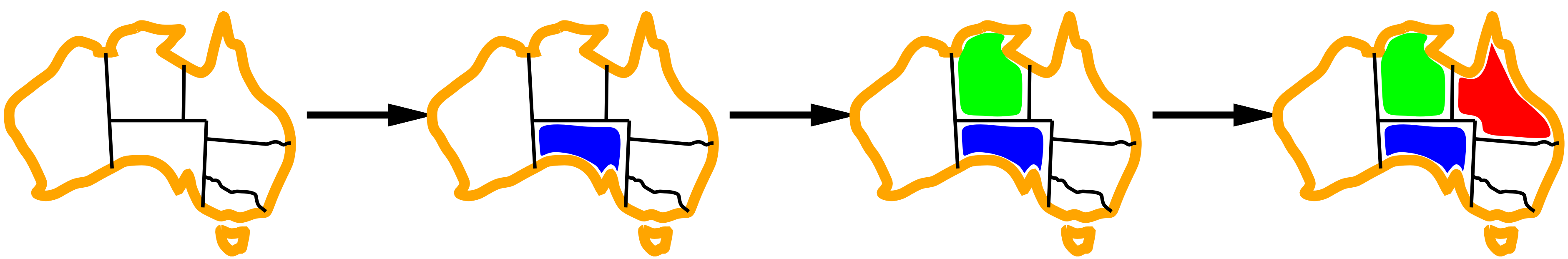
Minimum remaining values (MRV):
choose the variable with the fewest legal values



Degree heuristic

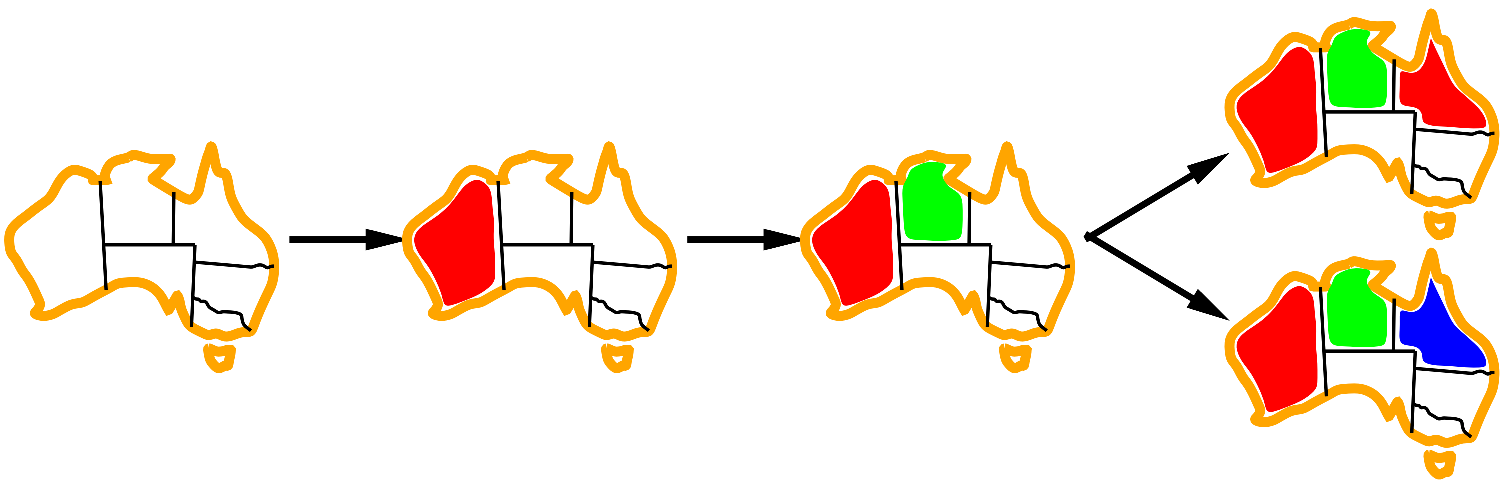
Tie-breaker among MRV variables

Degree heuristic:
choose the variable with the most constraints on remaining variables



Least constraining value

Given a variable, choose the least constraining value:
the one that rules out the fewest values in the remaining variables

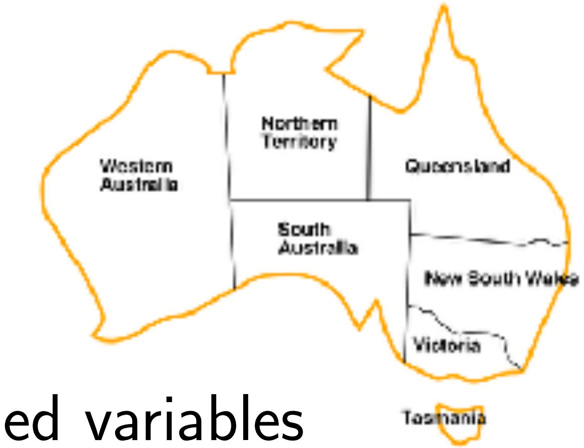


Allows 1 value for SA

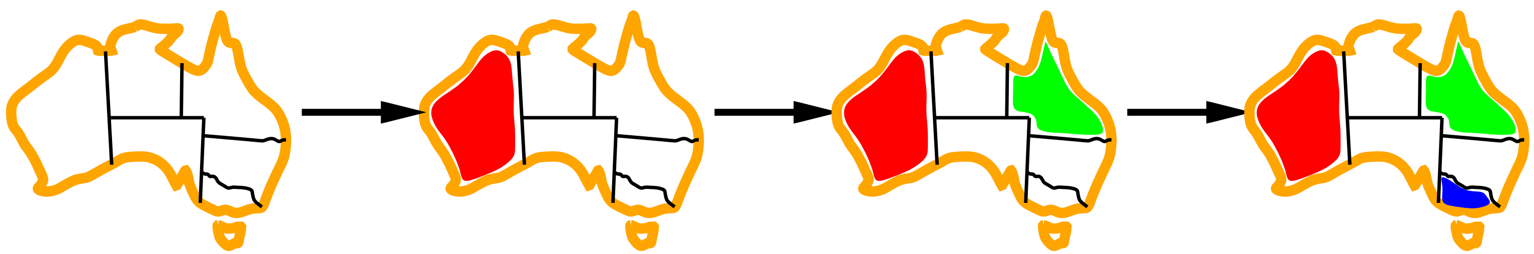
Allows 0 values for SA

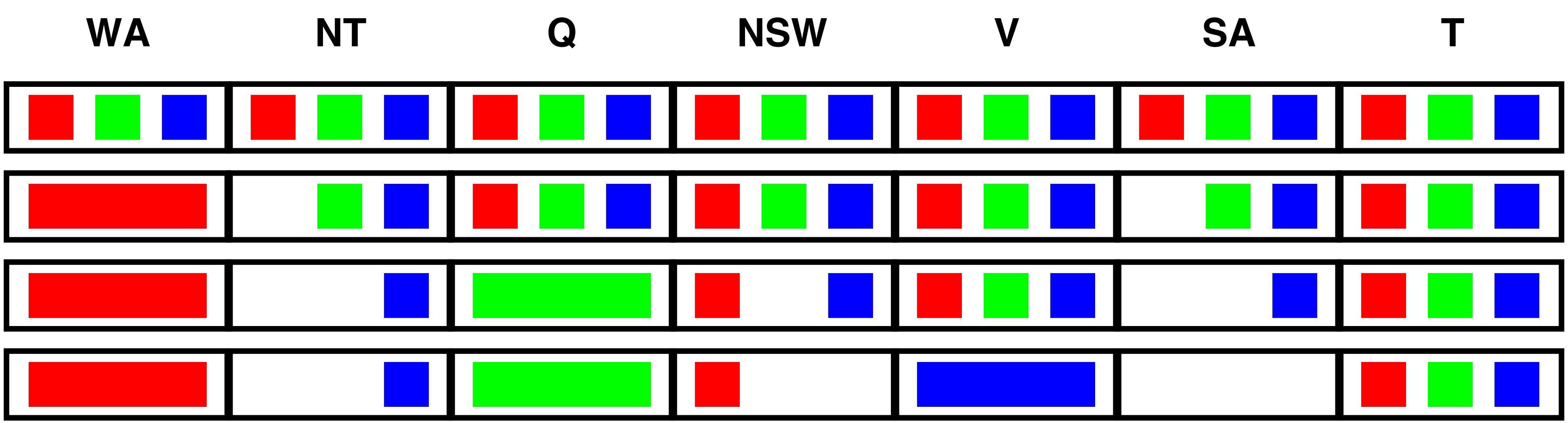
Combining these heuristics makes 1000 queens feasible

Forward checking



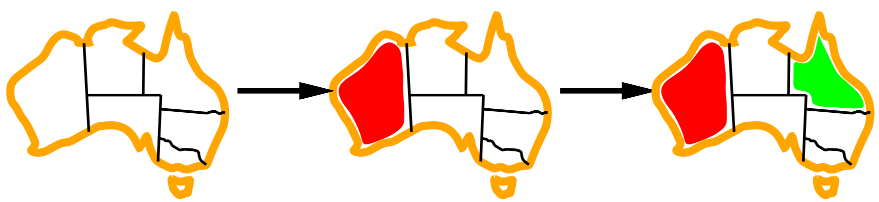
Idea: Keep track of remaining legal values for unassigned variables
Terminate search when any variable has no legal values





Constraint propagation

Forward checking propagates information from assigned to unassigned variables, but doesn't provide early detection for all failures:



NT and SA cannot both be blue!

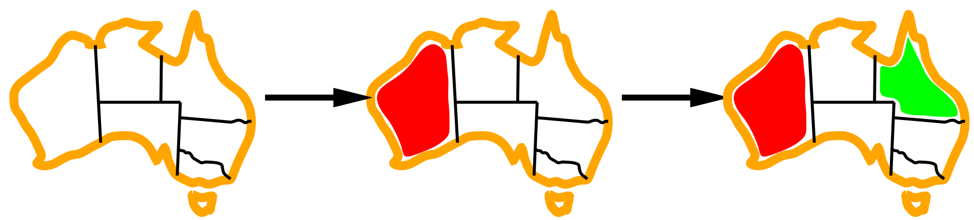
Constraint propagation repeatedly enforces constraints locally

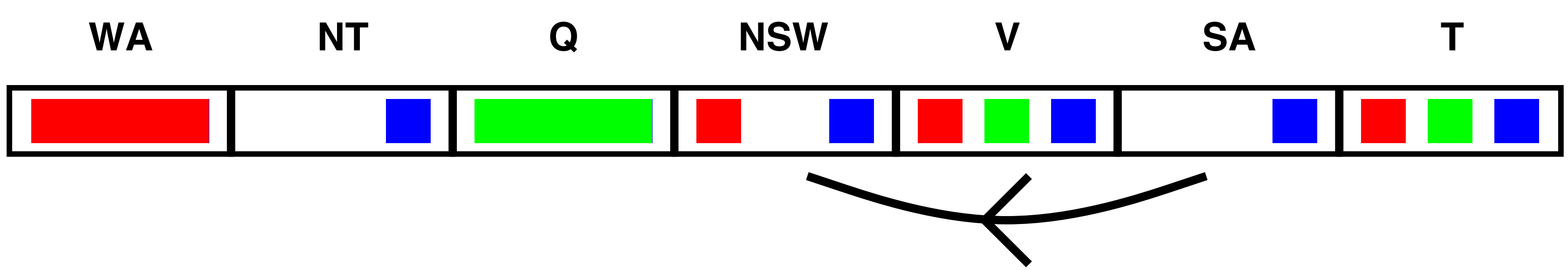
Arc consistency

Simplest form of propagation makes each arc consistent

$X \rightarrow Y$ is consistent iff

for **every** value x of X there is **some** allowed y





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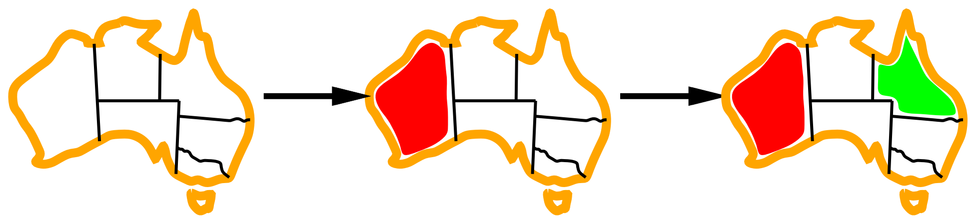

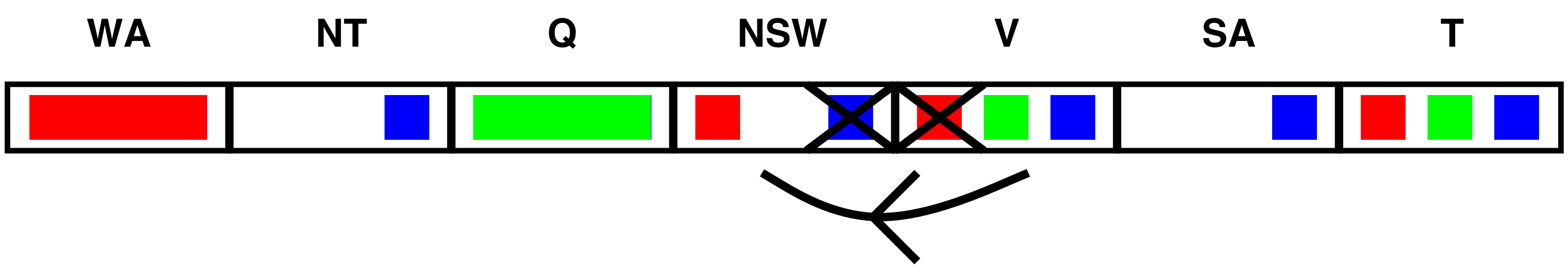

If X loses a value, neighbors of X need to be rechecked

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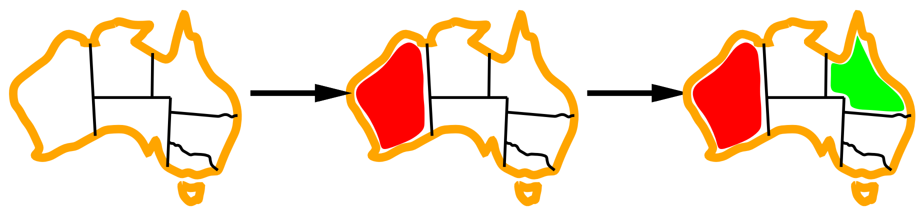
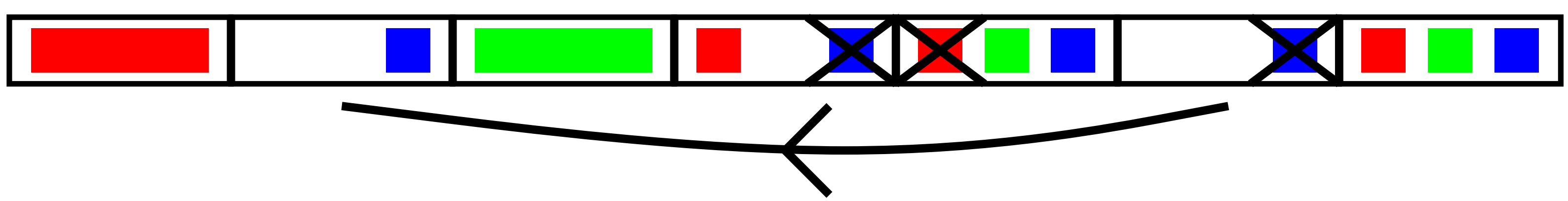




If X loses a value, neighbors of X need to be rechecked

Arc consistency detects failure earlier than forward checking

Can be run as a preprocessor or after each assignment

Arc consistency

function AC-3(*csp*) **returns** the CSP, possibly with reduced domains
  inputs: *csp*, a binary CSP with variables $\{X_1,\ X_2,\ \ldots,\ X_n\}$
  local variables: *queue*, a queue of arcs, initially all the arcs in *csp*

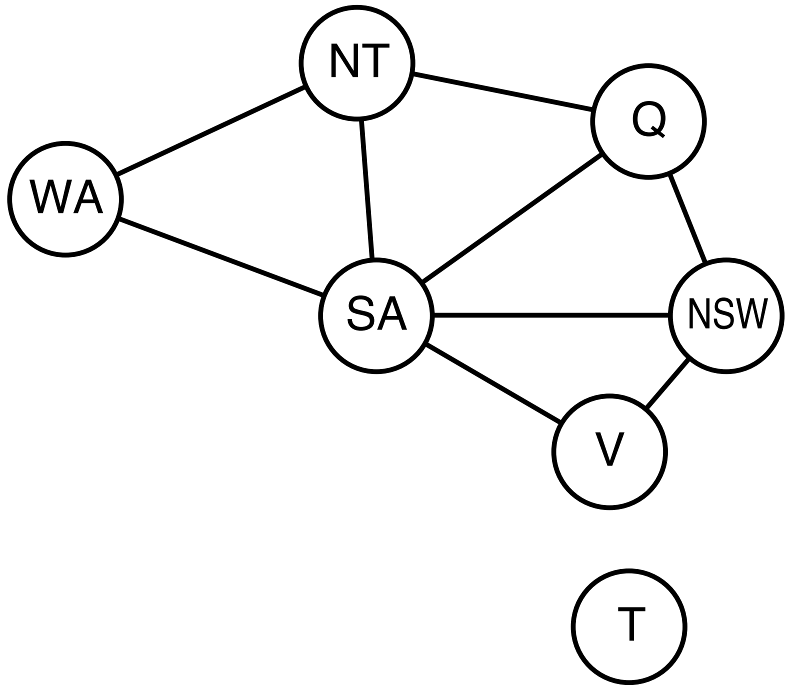
  while *queue* is not empty **do**
    $(X_i,\ X_j) \leftarrow$ REMOVE-FIRST(*queue*)
    if REMOVE-INCONSISTENT-VALUES($X_i,\ X_j$) **then**
      for each X_k **in** NEIGHBORS[X_i] **do**
        add $(X_k,\ X_i)$ to *queue*

---

function REMOVE-INCONSISTENT-VALUES($X_i,\ X_j$) **returns** true iff succeeds
  removed $\leftarrow$ *false*
  for each x **in** DOMAIN[X_i] **do**
    if no value y in DOMAIN[X_j] allows (x,y) to satisfy the constraint $X_i \leftrightarrow X_j$
      then delete x from DOMAIN[X_i]; *removed* $\leftarrow$ *true*
  return *removed*

$O(n^2d^3)$, can be reduced to $O(n^2d^2)$ (but detecting **all** is NP-hard)

Problem Structure



Tasmania and mainland are independent subproblems

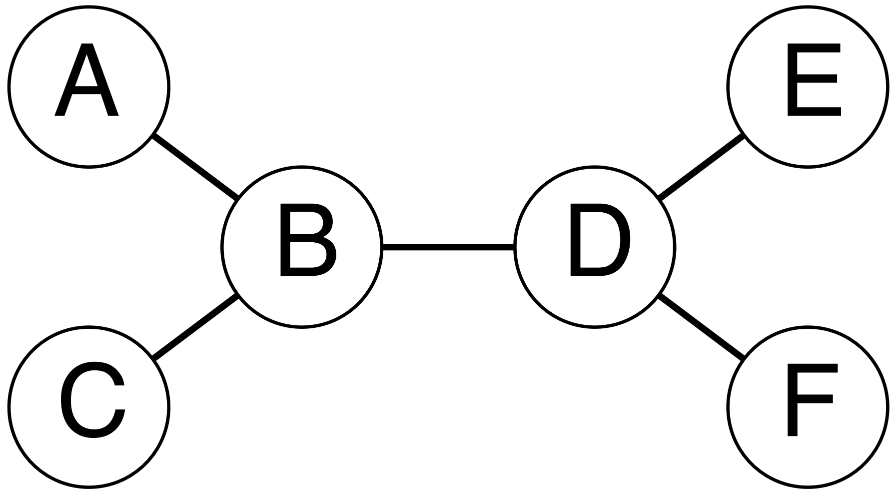
Identifiable as connected components of constraint graph

Suppose each subproblem has c variables out of n total

Worst-case solution cost is $n/c \cdot d^c$, **linear** in n

E.g., $n = 80$, $d = 2$, $c = 20$
2^{80} = 4 billion years at 10 million nodes/sec
$4 \cdot 2^{20}$ = 0.4 seconds at 10 million nodes/sec

Tree-structured CSPs



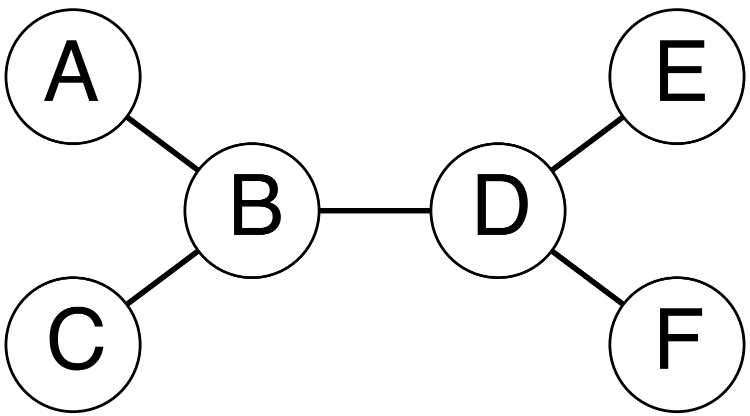
Theorem: if the constraint graph has no loops, the CSP can be solved in $O(n\,d^2)$ time

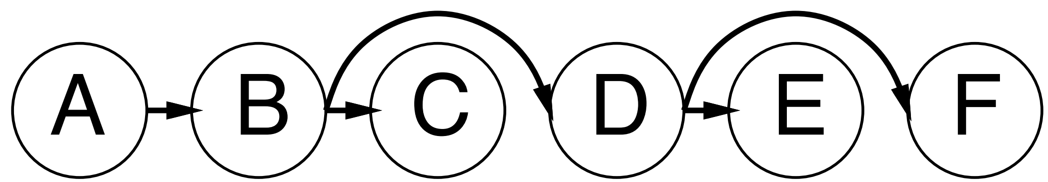
Compare to general CSPs, where worst-case time is $O(d^n)$

This property also applies to logical and probabilistic reasoning:
an important example of the relation between syntactic restrictions and the complexity of reasoning.

Algorithm for tree-structured CSPs

1. Choose a variable as root, order variables from root to leaves such that every node's parent precedes it in the ordering



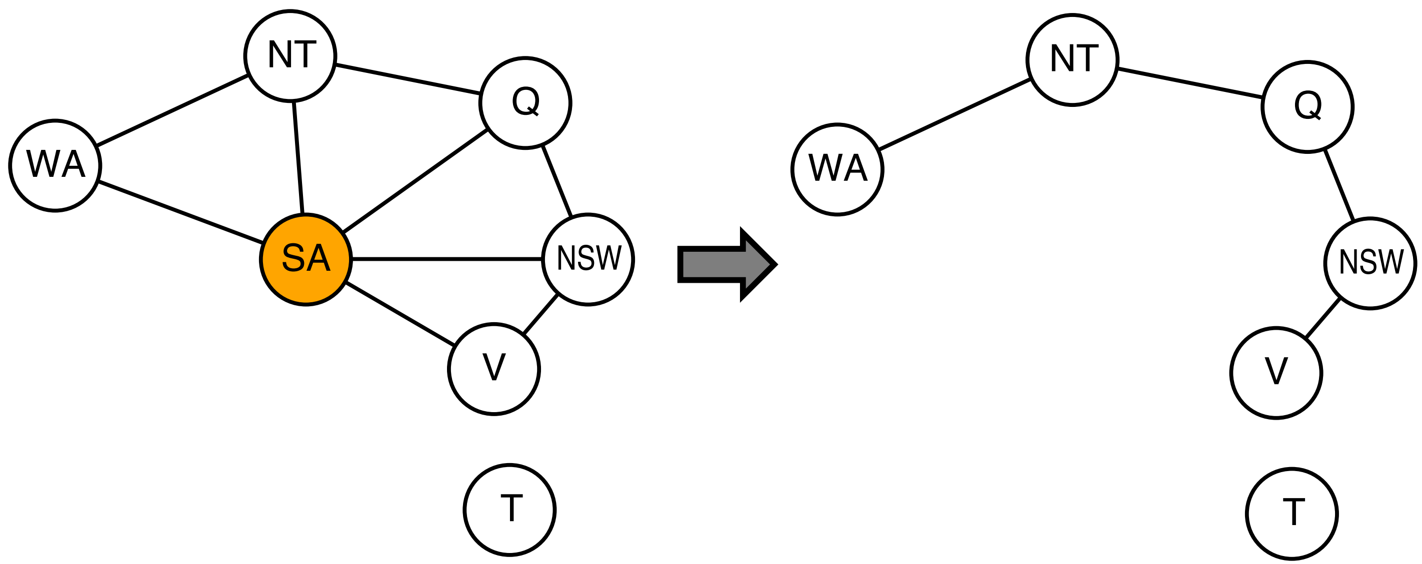


2. For j from n down to 2, apply $\textsc{RemoveInconsistent}(Parent(X_j), X_j)$

3. For j from 1 to n, assign X_j consistently with $Parent(X_j)$

Nearly tree-structured CSPs

Conditioning: instantiate a variable, prune its neighbors' domains



Cutset conditioning: instantiate (in all ways) a set of variables such that the remaining constraint graph is a tree

Cutset size c $\Rightarrow$ runtime $O(d^c \cdot (n-c)d^2)$, very fast for small c

Iterative algorithms for CSPs

Hill-climbing, simulated annealing typically work with "complete" states, i.e., all variables assigned

To apply to CSPs:
  allow states with unsatisfied constraints
  operators **reassign** variable values

Variable selection: randomly select any conflicted variable

Value selection by min-conflicts heuristic:
  choose value that violates the fewest constraints
  i.e., hillclimb with $h(n)$ = total number of violated constraints

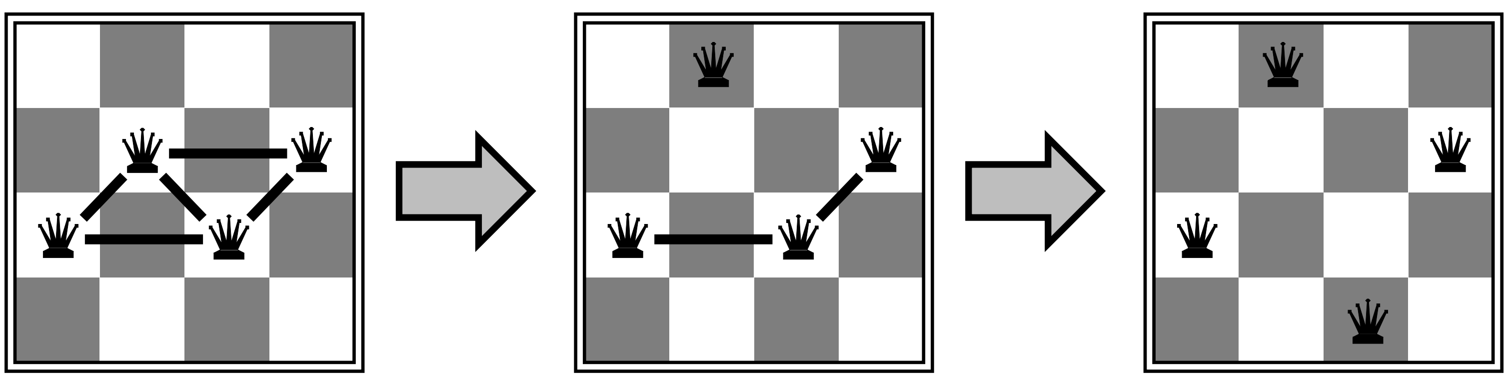
Example: 4-Queens

States: 4 queens in 4 columns ($4^4 = 256$ states)

Operators: move queen in column

Goal test: no attacks

Evaluation: $h(n)$ = number of attacks



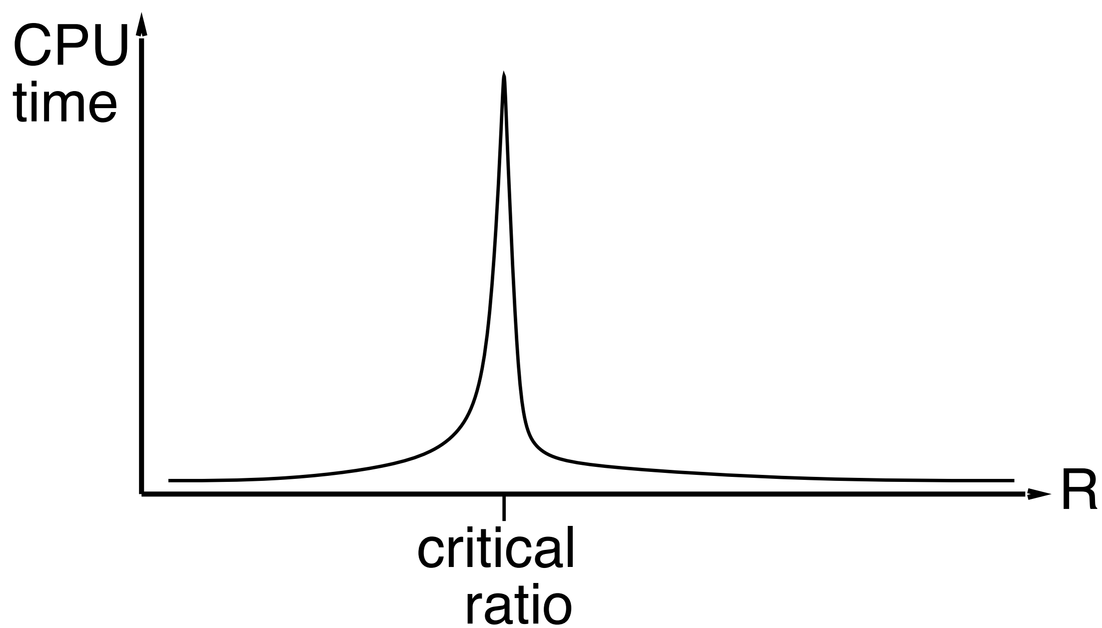
h = 5 h = 2 h = 0

Performance of min-conflicts

Given random initial state, can solve n-queens in almost constant time for arbitrary n with high probability (e.g., $n = 10{,}000{,}000$)

The same appears to be true for any randomly-generated CSP **except** in a narrow range of the ratio

$$R = \frac{\text{number of constraints}}{\text{number of variables}}$$



Summary

CSPs are a special kind of problem:
states defined by values of a fixed set of variables
goal test defined by constraints on variable values

Backtracking = depth-first search with one variable assigned per node

Variable ordering and value selection heuristics help significantly

Forward checking prevents assignments that guarantee later failure

Constraint propagation (e.g., arc consistency) does additional work to constrain values and detect inconsistencies

The CSP representation allows analysis of problem structure

Tree-structured CSPs can be solved in linear time

Iterative min-conflicts is usually effective in practice