

Artificial Intelligence, CS, Nanjing University Spring, 2018, Yang Yu

# Lecture 14: Learning 4

http://cs.nju.edu.cn/yuy/course\_ai18.ashx



How to train a dog?



# PHASE 1 DOWN



dog learns from rewards to adapt to the environment can computers do similarly?





Agent's view:  $s_0, a_0, r_1, s_1, a_2, r_2, s_2, a_3, r_3, s_3, \dots$  $\pi(s_0) \pi(s_1) \pi(s_2)$ 



all RL tasks can be defined by maximizing total reward

# **Reward examples**

#### shortest path:





- every node is a state, an action is an edge out
- reward function = the negative edge weight
- optimal policy leads to the shortest path

Difference between RL and planning?



what if we use planning/search methods to find actions that maximize total reward

Planing: find an optimal solution RL: find an optimal policy from samples

planning: shortest-path RL: shortest-path policy without knowing the graph Difference between RL and SL?

## supervised learning also learns a model ...



learning from labeled data open loop passive data



learning from delayed reward closed loop explore environment

# Reward examples general binary space problem $\max_{x \in \{0,1\}^n} f(x)$



#### solving the optimal policy is NP-hard!





## Deepmind Deep Q-learning on Atari

[Mnih *et al*. Human-level control through deep reinforcement learning. Nature, 518(7540): 529-533, 2015]







#### learning robot skills





https://www.youtube.com/watch?v=VCdxqnOfcnE

# More applications

# Search Recommendation system Stock prediction



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# every decision changes the world



## **Markov Decision Process**

#### essential mathematical model for RL

# Markov Process

(finite) state space S, transition matrix P

a process  $s_0, s_1, ...$  is Markov if has no memory  $P(s_{t+1} \mid s_t, ..., s_0) = P(s_{t+1} \mid s_t)$  discrete S -> Markov chain





 $oldsymbol{s}_{t+1} = oldsymbol{s}_t P = oldsymbol{s}_0 P^{t+1}$ 



# Markov Process

#### horizontal view





stationary distribution: s == sPsampling from a Markov process: s, c, c, r ...

# Markov Reward Process

#### introduce reward function ${\boldsymbol{R}}$



how to calculate the long-term total reward?

$$V(\text{sunny}) = E[\sum_{t=1}^{T} r_t | s_0 = \text{sunny}]$$
$$V(\text{sunny}) = E[\sum_{t=1}^{\infty} \gamma^t r_t | s_0 = \text{sunny}]$$

value function





horizontal view: consider T steps



#### recursive definition:

 $V(\text{sunny}) = P(\mathbf{s}|\mathbf{s})[R(\mathbf{s}) + V(\mathbf{s})]$  $+ P(\mathbf{c}|\mathbf{s})[R(\mathbf{c}) + V(\mathbf{c})]$  $+ P(\mathbf{r}|\mathbf{s})[R(\mathbf{r}) + V(\mathbf{r})]$ 

$$= \sum_{s} P(s|\text{sunny}) (R(s) + V(s))$$



# Markov Reward Process

horizontal view: consider discounted infinite steps





# **Markov Decision Process**

#### horizontal view







•••



#### horizontal view of the game of Go





**Markov Decision Process** 

$$\begin{split} \mathbf{MDP} < & S, A, R, P > (\text{often with } \gamma) \\ \textbf{essential model for RL} \\ \textbf{but not all of RL} \end{split}$$



# policy

stochastic

$$\pi(a|s) = P(a|s)$$

deterministic

$$\pi(s) = rg \max_{a} P(a|s)$$
  
 $|A|^{|S|}$  deterministic policies

#### tabular representation

 $\pi =$ 

S	0	0.3
	1	0.7
С	0	0.6
	1	0.4
r	0	0.1
	1	0.9



#### how to calculate the expected total reward of a policy?

similar with the Markov Reward Process

#### MRP:

$$V(s) = \sum_{s'} P(s'|s) (R(s') + V(s'))$$



#### MDP:

$$V^{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} P(s'|s, a) \left( R(s, a, s') + V^{\pi}(s') \right)$$

expectation over actions with respect to the policy



#### state value function

$$V^{\pi}(s) = E[\sum_{t=1}^{T} r_t | s]$$

state-action value function

$$Q^{\pi}(s,a) = E\left[\sum_{t=1}^{T} r_t | s, a\right] = \sum_{s'} P(s' | s, a) \left( R(s,a,s') + V^{\pi}(s') \right)$$

consequently,

$$V^{\pi}(s) = \sum_{a} \pi(a|s)Q(s,a)$$

Q-function => policy





# there exists an optimal policy $\pi^*$ $\forall \pi, \forall s, V^{\pi^*}(s) \ge V^{\pi}(s)$

#### optimal value function

$$\forall s, V^*(s) = V^{\pi^*}(s)$$
  
$$\forall s, \forall a, Q^*(s, a) = Q^{\pi^*}(s, a)$$



Bellman optimality equations

$$V^*(s) = \max_a Q^*(s, a)$$



#### from the relation between V and Q

$$Q^*(s, a) = \sum_{s'} P(s'|s, a) \left( R(s, a, s') + \gamma V^*(s') \right)$$

we have

$$Q^{*}(s,a) = \sum_{s'} P(s'|s,a) \left( R(s,a,s') + \gamma \max_{a} Q^{*}(s',a) \right)$$
$$V^{*}(s) = \max_{a} \sum_{s'} P(s'|s,a) \left( R(s,a,s') + \gamma V^{*}(s') \right)$$

the unique fixed point is the optimal value function



#### idea:

how is the current policy policy evaluation improve the current policy policy improvement

# **policy evaluation:** backward calculation $V^{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} P(s'|s, a) \left( R(s, a, s') + \gamma V^{\pi}(s') \right)$

policy improvement: from the Bellman optimality equation

$$V(s) \leftarrow \max_{a} Q^{\pi}(s, a)$$

policy improvement: from the Bellman optimality equation

$$V(s) \leftarrow \max_{a} Q^{\pi}(s, a)$$

let  $\pi'$  be derived from this update

$$V^{\pi}(s) \leq Q^{\pi}(s, \pi'(s))$$
  
=  $\sum_{s'} P(s'|s, \pi'(s))(R(s, \pi'(s), s') + \gamma V^{\pi}(s'))$   
 $\leq \sum_{s'} P(s'|s, \pi'(s))(R(s, \pi'(s), s') + \gamma Q^{\pi}(s', \pi'(s)))$   
= ...  
=  $V^{\pi'}$ 

so the policy is improved



Policy iteration algorithm:

loop until converges policy evaluation: calculate V policy improvement: choose the action greedily  $\pi_{t+1}(s) = \arg \max_{a} Q^{\pi_t}(s, a)$ 

**converges:**  $V^{\pi_{t+1}}(s) = V^{\pi_t}(s)$ 

$$Q^{\pi_{t+1}}(s,a) = \sum_{s'} P(s'|s,a) \big( R(s,a,s') + \gamma \max_{a} Q^{\pi_{t}}(s',a) \big)$$
  
recall the optimal value function about Q



embed the policy improvement in evaluation

Value iteration algorithm:

$$V_{0} = 0$$
  
for  $t=0, 1, ...$   
for all  $s \leftarrow$  synchronous v.s. asynchronous  
 $V_{t+1}(s) = \max_{a} \sum_{s'} P(s'|s, a) (R(s, a, s') + \gamma V_{t}(s))$   
end for  
break if  $||V_{t+1} - V_{t}||_{\infty}$  is small enough  
end for

recall the optimal value function about V





Dynamic programming

R. E. Bellman 1920–1984

# Complexity

needs  $\Theta(|S| \cdot |A|)$  iterations to converge on deterministic MDP

[O. Madani. Polynomial Value Iteration Algorithms for Deterministic MDPs. UAI'02]

curse of dimensionality: Go board 19x19, |S|=2.08x10<sup>170</sup>

[https://github.com/tromp/golegal]



# from MDP to reinforcement learning

MDP < S, A, R, P >

 ${\cal R}\,$  and  ${\cal P}\,$  are unknown



Methods



A: learn R and P, model-based then solve the MDP

**B:** learn policy without R or P model-free

MDP is the model

Model-free RL



explore the environment and learn policy at the same time

Monte-Carlo method

Temporal difference method


sample trajectory m times,

approximate the expectation by average

 $Q^{\pi}(s,a) = \frac{1}{m} \sum_{i=1}^{m} R(\tau_i) \quad \tau_i \text{ is sample by following } \pi \text{ after } s, a$ 

## Monte Carlo RL - evaluation+improvement

$$egin{aligned} Q_0 &= 0 \ & ext{for } i=0,\,1,\,...,\,\mathrm{m} \ & ext{generate trajectory } < s_0,\,a_0,\,r_1,\,s_1,\,...,\,s_T > \ & ext{for } t=0,\,1,\,...,\,T\text{-}1 \ & ext{R} = ext{sum of rewards from } t ext{ to } T \ & ext{$Q(s_t,a_t)=(c(s_t,a_t)Q(s_t,a_t)+R)/(c(s_t,a_t)+1)$} \ & ext{$c(s_t,a_t)++$} \ & ext{end for } \ & ext{update policy } \pi(s) = ext{arg max} \, Q(s,a) \ & ext{end for } \ & ext{improvement ?} \end{aligned}$$





problem: what if the policy takes only one path?



cannot improve the policy no exploration of the environment

needs exploration !

**Exploration methods** 

one state MDP: a.k.a. bandit model



maximize the long-term total reward

- exploration only policy: try every action in turn waste many trials
- exploitation only policy: try each action once, follow the best action forever risk of pick a bad action
   balance between exploration and exploitation

**Exploration methods** 

 $\epsilon$ -greedy:



I I

follow the best action with probability  $1\text{-}\epsilon$  choose action randomly with probability  $\epsilon$   $\epsilon$  should decrease along time

#### softmax:

probability according to action quality  $P(k) = e^{Q(k)/\theta} / \sum_{i=1}^{K} e^{Q(i)/\theta}$ 

upper confidence bound (UCB): choose by action quality + confidence I **Action-level exploration** 



 $\epsilon$ -greedy policy:

## given a policy $\pi$

$$\pi_{\epsilon}(s) = \begin{cases} \pi(s), \text{with prob. } 1 - \epsilon \\ \text{randomly chosen action, with prob. } \epsilon \end{cases}$$

ensure probability of visiting every state > 0

exploration can also be in other levels

## Monte Carlo RL

 $Q_0 = 0$ for *i*=0, 1, ..., m generate trajectory  $\langle s_0, a_0, r_1, s_1, ..., s_T \rangle$  by  $\pi_{\epsilon}$ for t=0, 1, ..., T-1 R = sum of rewards from t to T $Q(s_t, a_t) = (c(s_t, a_t)Q(s_t, a_t) + R)/(c(s_t, a_t) + 1)$  $c(s_t, a_t) + +$ end for update policy  $\pi(s) = \arg \max Q(s, a)$ end for

Monte Carlo RL - on/off-policy



this algorithm evaluates  $\pi_{\epsilon}$  ! on-policy what if we want to evaluate  $\pi$  ? off-policy

importance sampling:

$$E[f] = \int_{x} p(x)f(x)dx = \int_{x} q(x)\frac{p(x)}{q(x)}f(x)dx$$

$$\int \text{sample from } p \qquad \int \text{sample from } q$$

$$\frac{1}{m}\sum_{i=1}^{m} f(x) \qquad \frac{1}{m}\sum_{i=1}^{m}\frac{p(x)}{q(x)}f(x)$$

## Monte Carlo RL -- off-policy

$$\begin{array}{l} Q_{0} = 0 \\ \text{for } i=0, \, 1, \, ..., \, m \\ \text{generate trajectory } < s_{0}, \, a_{0}, \, r_{1}, \, s_{1}, \, ..., \, s_{T} > \, \text{by } \pi_{\epsilon} \\ \text{for } t=0, \, 1, \, ..., \, T-1 \\ \text{R} = \text{sum of rewards from } t \text{ to } T \times \prod_{i=t+1}^{T-1} \frac{\pi(x_{i}, a_{i})}{p_{i}} \\ Q(s_{t}, a_{t}) = (c(s_{t}, a_{t}) Q(s_{t}, a_{t}) + \text{R}) / (c(s_{t}, a_{t}) + 1) \\ c(s_{t}, a_{t}) + + \\ \text{end for} \\ \text{update policy } \pi(s) = \arg \max_{a} Q(s, a) \\ \frac{\text{end for}}{p_{i}} = \begin{cases} 1 - \epsilon + \epsilon / |A|, a_{i} = \pi(s_{i}), \\ \epsilon / |A|, a_{i} \neq \pi(s_{i}) \end{cases} \end{cases}$$





#### summary

Monte Carlo evaluation: approximate expectation by sample average

action-level exploration

on-policy, off-policy: importance sampling

#### Monte Carlo RL:

evaluation + action-level exploration + policy improvement (on/off-policy)

#### Incremental mean

$$Q(s_t, a_t) = (\mathrm{c}(s_t, a_t) Q(s_t, a_t) + \mathrm{R}) / (\mathrm{c}(s_t, a_t) + 1)$$

$$\mu_t = \frac{1}{t} \sum_{i=1}^t x_i = \frac{1}{t} (x_t + \sum_{i=1}^{t-1} x_i) = \frac{1}{t} (x_t + (t-1)\mu_{t-1})$$
$$= \mu_{t-1} + \frac{1}{t} (x_t - \mu_{t-1})$$

In general,  $\mu_t = \mu_{t-1} + \alpha(x_t - \mu_{t-1})$ 

#### Monte-Carlo update: $Q(s_t, a_t) \Leftarrow Q(s_t, a_t) + \alpha (R - Q(s_t, a_t))$ MC error



**Temporal-Difference Learning - evaluation** 

update policy online

learn as you go

TD Evaluation

Monte-Carlo update:  $Q(s_t, a_t) \Leftarrow Q(s_t, a_t) + \alpha (R - Q(s_t, a_t))$ TD update: MC error

 $Q(s_t, a_t)$  $\Leftarrow Q(s_t, a_t) + \alpha(\underline{r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)})$ 

TD error

## **Temporal-Difference Learning - example**

elapsed time	predicted remaining time	predicted total time
0	30	30
5	35	40
20	15	35
30	10	40
40	3	43
43	0	43
	elapsed time 0 5 20 30 40 40 43	elapsed timepredicted remaining time03053520153010403430





#### SARSA

#### On-policy TD control

```
Q_0 = 0, initial state
for i=0, 1, ...
    a = \pi_{\epsilon}(s)
    s', r = do action a
    a' = \pi_{\epsilon}(s')
   Q(s,a) += \alpha(r + \gamma Q(s',a') - Q(s,a))
    \pi(s) = \arg\max Q(s, a)
    s = s'
end for
```



## **Q-learning**

## Off-policy TD control

```
Q_0 = 0, initial state
for i=0, 1, ...
    a = \pi_{\epsilon}(s)
    s', r = do action a
    a' = \pi(s')
    Q(s, a) + = \alpha(r + \gamma Q(s', a') - Q(s, a))
    \pi(s) = \arg\max Q(s, a)
    s = s'
end for
```





![](_page_53_Picture_0.jpeg)

### RL in continuous state space

$$\begin{split} \mathsf{MDP} < &S, A, R, P > \\ &S \text{ (and } A \text{) is in } \mathbb{R}^n \end{split}$$

![](_page_53_Picture_3.jpeg)

## Value function approximation

#### tabular representation

$\pi =$		0	0.3
	S	1	0.7
	с	0	0.6
		1	0.4
	r	0	0.1
		1	0.9

very powerful representation can be all possible policies !

#### linear function approx.

modern RL

$$\hat{V}(s) = w^{\top}\phi(s)$$
$$\hat{Q}(s,a) = w^{\top}\phi(s,a)$$
$$\hat{Q}(s,a_i) = w_i^{\top}\phi(s)$$

 $\phi$  is a feature mapping w is the parameter vector may not represent all policies ! Value function approximation

![](_page_55_Picture_1.jpeg)

to approximate Q and V value function least square approximation

$$J(w) = E_{s \sim \pi} [(Q^{\pi}(s, a) - \hat{Q}(s, a))^2]$$

online environment: stochastic gradient on single sample  $\Delta w_t = \theta(Q^{\pi}(s_t, a_t) - \hat{Q}(s_t, a_t)) \nabla_w \hat{Q}(s_t, a_t)$ Recall the errors: MC update:  $Q(s_t, a_t) + = \alpha(R - Q(s_t, a_t))$ TD update:  $Q(s_t, a_t) + = \alpha(R - Q(s_t, a_t))$ TD update:  $Q(s_t, a_t) + = \alpha(r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t))$ Td update:  $Q(s_t, a_t) + = \alpha(r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t))$ 

## Value function approximation

![](_page_56_Picture_1.jpeg)

#### MC update:

$$\Delta w_t = \theta(R - \hat{Q}(s_t, a_t)) \nabla_w \hat{Q}(s_t, a_t)$$

#### TD update:

$$\Delta w_t = \theta(r_{t+1} + \gamma \hat{Q}(s_{t+1}, a_{t+1}) - \hat{Q}(s_t, a_t)) \nabla_w \hat{Q}(s_t, a_t)$$

#### eligibility traces

$$E_t = \gamma \lambda E_{t-1} + \nabla_w \hat{Q}(s_t, a_t)$$

## Q-learning with function approximation

$$w = 0, \text{ initial state}$$
  
for  $i=0, 1, ...$   
 $a = \pi_{\epsilon}(s)$   
 $s', r = \text{do action } a$   
 $a' = \pi(s')$   
 $w + = \theta(r + \gamma \hat{Q}(s, a) - \hat{Q}(s, a)) \nabla_w \hat{Q}(s_t, a_t)$   
 $\pi(s) = \arg \max_a \hat{Q}(s, a)$   
 $s = s'$   
end for

Approximation model

Linear approximation 
$$\hat{Q}(s,a) = w^{\top}\phi(s,a)$$
  
 $\nabla_w \hat{Q}(s,a) = \phi(s,a)$ 

#### coarse coding: raw features

discretization: tide with indicator features

kernelization:

$$\hat{Q}(s,a) = \sum_{i=1}^{m} w_i K((s,a),(s_i,a_i)) \ (s_i,a_i)$$
 can be randomly sampled

Approximation model

![](_page_59_Figure_1.jpeg)

Nonlinear model approximation  $\hat{Q}(s,a) = f(s,a)$ 

neural network: differentiable model

recall the TD update:

$$\Delta w_t = \theta(r_{t+1} + \gamma \hat{Q}(s_{t+1}, a_{t+1}) - \hat{Q}(s_t, a_t)) \nabla_w \hat{Q}(s_t, a_t)$$

follow the BP rule to pass the gradient

RL in continuous state space

![](_page_60_Picture_1.jpeg)

## Deep Reinforcement Learning

function approximation by deep neural networks

![](_page_60_Picture_4.jpeg)

## **Convolutional neural networks**

## a powerful neural network architecture for image analysis differentiable require a lot of samples to train

![](_page_61_Figure_2.jpeg)

## Deep Q-Network

### DQN

- using  $\varepsilon\text{-greedy}$  policy
- store 1million recent history (s,a,r,s') in replay memory D
- sample a mini-batch (32) from D
- calculate Q-learning target  $ilde{Q}$
- update CNN by minimizing the Bellman error (delayed update)

$$\sum (r + \gamma \max_{a'} \tilde{Q}(s', a') - Q_w(s, a))^2$$

## DQN on Atari

learn to play from pixels

![](_page_62_Picture_10.jpeg)

![](_page_62_Figure_11.jpeg)

![](_page_62_Picture_12.jpeg)

![](_page_63_Figure_0.jpeg)

![](_page_64_Picture_0.jpeg)

![](_page_64_Picture_1.jpeg)

## effectiveness

Game	With replay, with target Q	With replay, without target Q	Without replay, with target Q	Without replay, without target Q
Breakout	316.8	240.7	10.2	3.2
Enduro	1006.3	831.4	141.9	29.1
River Raid	7446.6	4102.8	2867.7	1453.0
Seaquest	2894.4	822.6	1003.0	275.8
Space Invaders	1088.9	826.3	373.2	302.0

![](_page_65_Picture_0.jpeg)

![](_page_65_Picture_1.jpeg)

## A combination of tree search, deep neural networks and reinforcement learning

![](_page_65_Figure_3.jpeg)

## AlphaGo

![](_page_66_Picture_1.jpeg)

## fast roll-out policy: supervised learning from human v.s. human data

Feature	# of patterns	Description
Response	1	Whether move matches one or more response pattern features
Save atari	1	Move saves stone(s) from capture
Neighbour	8	Move is 8-connected to previous move
Nakade	8192	Move matches a nakade pattern at captured stone
Response pattern	32207	Move matches 12-point diamond pattern near previous move
Non-response pattern	69338	Move matches $3 \times 3$ pattern around move
Self-atari	1	Move allows stones to be captured
Last move distance	34	Manhattan distance to previous two moves
Non-response pattern	32207	Move matches 12-point diamond pattern centred around move

## AlphaGo

# policy network: a CNN output $\pi(s,a)$ value network: a CNN output V(s)

![](_page_67_Picture_2.jpeg)

Feature	# of planes	Description
Stone colour	3	Player stone / opponent stone / empty
Ones	1	A constant plane filled with 1 $p(a s) = p(a s)$
Turns since	8	How many turns since a move was played
Liberties	8	Number of liberties (empty adjacent points)
Capture size	8	How many opponent stones would be captured
Self-atari size	8	How many of own stones would be captured
Liberties after move	8	Number of liberties after this move is played
Ladder capture	$\partial \log \mathbf{p}$ (a	Whether a move at this point is a successful ladder capture
Ladder escape	$\partial_{ m d}\sigma$	Whether a move at this point is a successful ladder escape
Sensibleness	1	Whether a move is legal and does not fill its own eyes
Zeros	1	A constant plane filled with 0
Player color	1	Whether current player is black

![](_page_68_Picture_0.jpeg)

![](_page_68_Picture_1.jpeg)

## policy network: initialization supervised learning from human v.s. human data

	Architecture				Evaluation		
Filters	Symmetries	Features	Test accu- racy %	Train accu- racy %	Raw net wins %	<i>AlphaGo</i> wins %	Forward time (ms)
128	1	48	54.6	57.0	36	53	2.8
192	1	48	55.4	58.0	50	50	4.8
256	1	48	55.9	59.1	67	55	7.1
256	2	48	56.5	59.8	67	38	13.9
256	4	48	56.9	60.2	69	14	27.6
256	8	48	57.0	60.4	69	5	55.3
192	1	4	47.6	51.4	25	15	4.8
192	1	12	54.7	57.1	30	34	4.8
192	1	20	54.7	57.2	38	40	4.8
192	8	4	49.2	53.2	24	2	36.8
192	8	12	55.7	58.3	32	3	36.8
192	8	20	55.8	58.4	42	3	36.8

#### $v_{\theta}(s) \approx v^{p}(s)$ policy network: further improvement reinforcement learning environment $p_{\sigma/\rho}$ (a | s) $p_{\sigma/\rho}$ (as s) ۵ **h** . L. s,r $p_{\sigma}(a|s)$ an old policy a randomly picked policy in previous iterations a.k.a. self-play $p_{\sigma}(a|s)$ p(a|s) $p_{\sigma}(a|s)$ reward: +1 -- win at terminate state -1 -- loss at terminate state

 $\partial \log p(a|s)$ 

## AlphaGo

![](_page_69_Picture_4.jpeg)

#### $a_t \sim p(\cdot | s_t)$

## AlphaGo

#### value network: supervised learning from RL data

![](_page_70_Figure_3.jpeg)


本地玩家

## SIMPLE TEST MAP 64X64





 $\times$ 

\_\_\_\_