南京大学
人工智能学院
SCHOOL OF ARTIFICIAL INTELLIGENCE, NANJING UNIVERSITY

# Lecture 3: Search 2

# Previously...

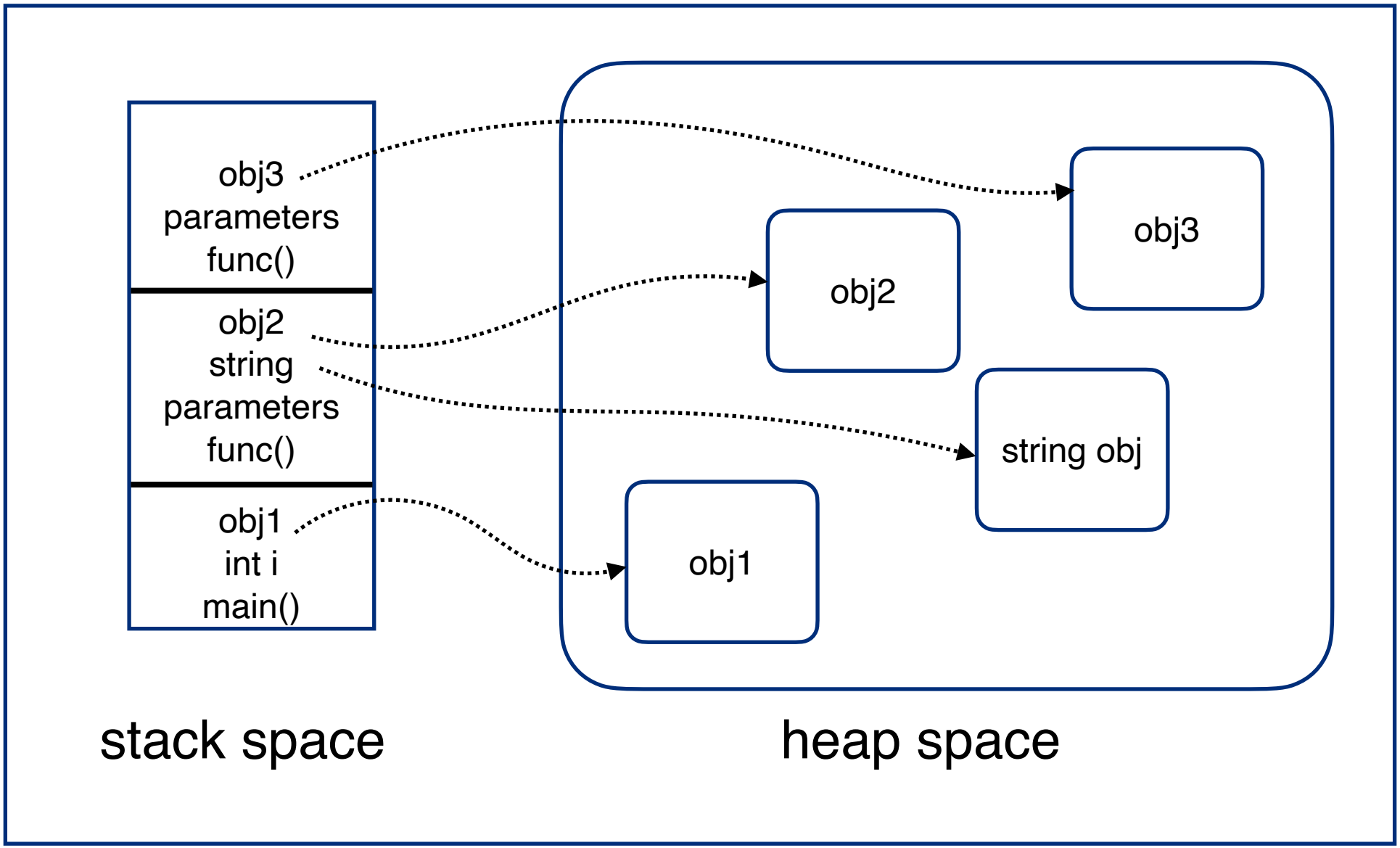
```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE(node)) then return node
        fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

*note the time of goal-test: expanding time not generating time*

---

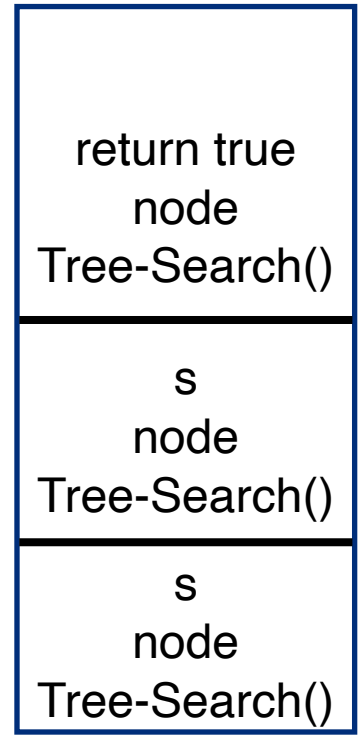
```
function EXPAND( node, problem) returns a set of nodes
    successors ← the empty set
    for each action, result in SUCCESSOR-FN(problem, STATE[node]) do
        s ← a new NODE
        PARENT-NODE[s] ← node;  ACTION[s] ← action;  STATE[s] ← result
        PATH-COST[s] ← PATH-COST[node] + STEP-COST(node, action, s)
        DEPTH[s] ← DEPTH[node] + 1
        add s to successors
    return successors
```

# Stack and heap memory space



# Deep-first search using stack

```
function Tree-Search(node)
    if node has goal then return true
    for each action, result in Successor-Fn(problem, node) do
        s <- make Node from node
        hasgoal = Tree-Search(s)
        if hasgoal then return true
    end for
return false
```




stack space

simple to code, risk of stack-overflow

# Deep-first search using heap

```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
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    loop do
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        node ← REMOVE-FRONT(fringe)
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        fringe ← INSERTALL(EXPAND(node, problem), fringe)
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    return successors
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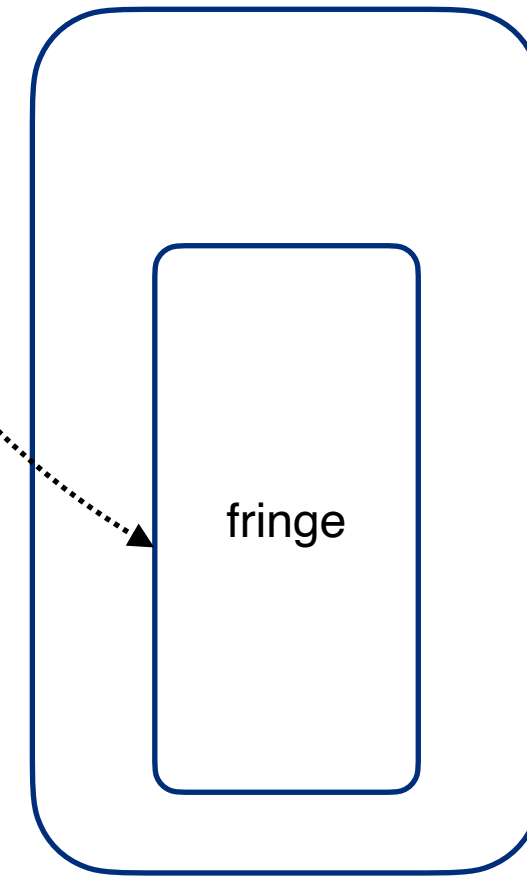


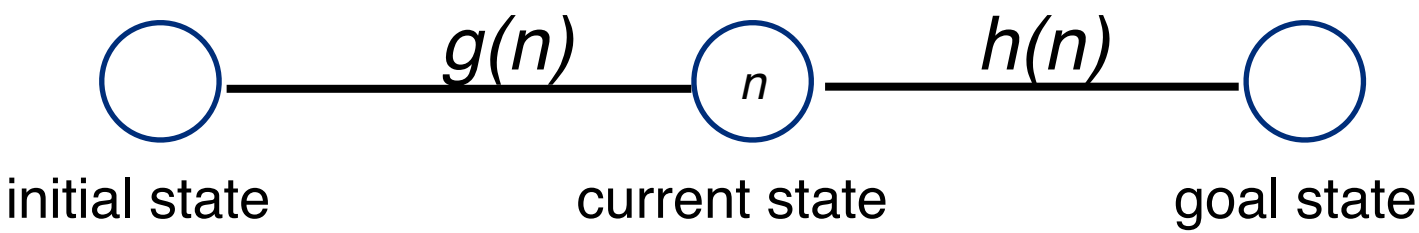

heap space

flexible memory usage

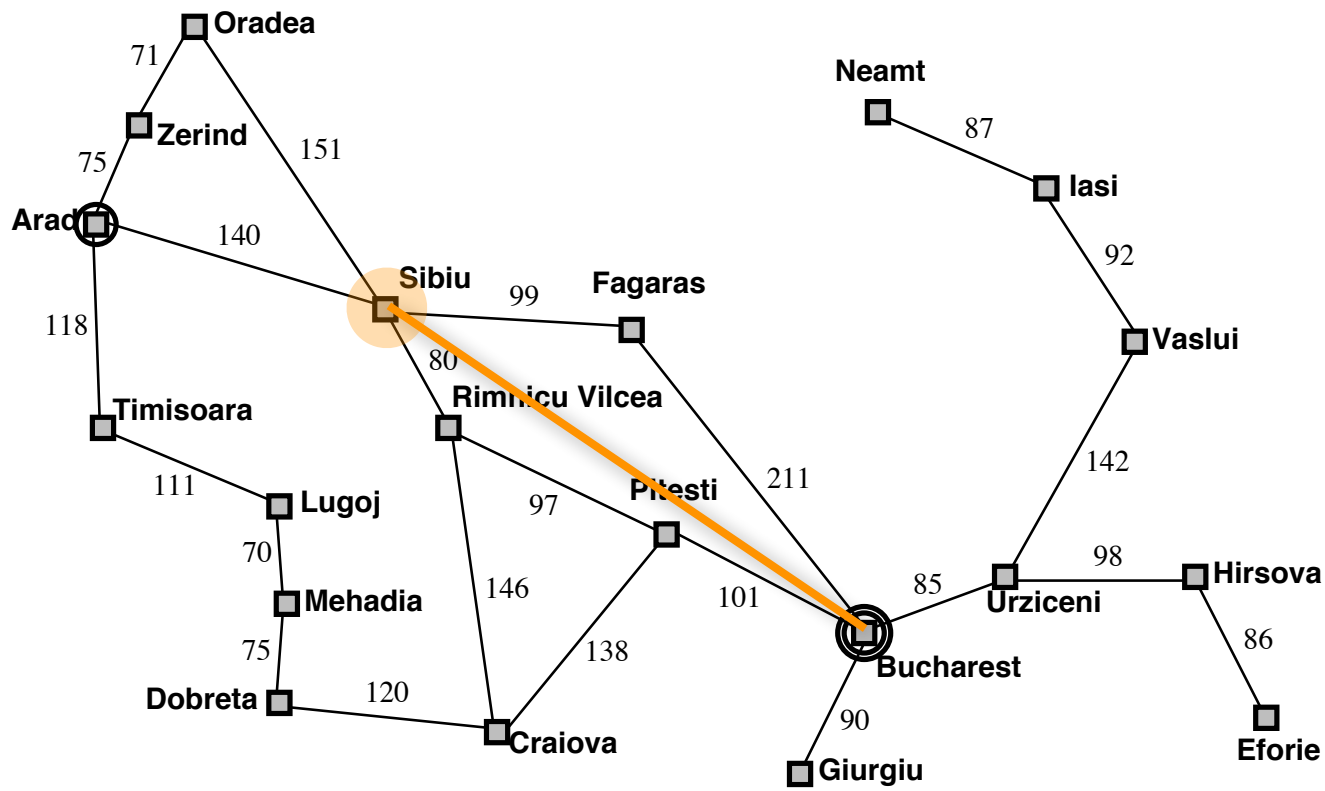
# Informed Search Strategies

best-first search: *f* but what is best?

uniform cost search: cost function *g*
**heuristic function: *h***



# Example: $h_{SLD}$




| | | | |
|---|---|---|---|
| **Arad** | 366 | **Mehadia** | 241 |
| **Bucharest** | 0 | **Neamt** | 234 |
| **Craiova** | 160 | **Oradea** | 380 |
| **Drobeta** | 242 | **Pitesti** | 100 |
| **Eforie** | 161 | **Rimnicu Vilcea** | 193 |
| **Fagaras** | 176 | **Sibiu** | 253 |
| **Giurgiu** | 77 | **Timisoara** | 329 |
| **Hirsova** | 151 | **Urziceni** | 80 |
| **Iasi** | 226 | **Vaslui** | 199 |
| **Lugoj** | 244 | **Zerind** | 374 |

**Figure 3.22** Values of $h_{SLD}$—straight-line distances to Bucharest.

# Greedy search

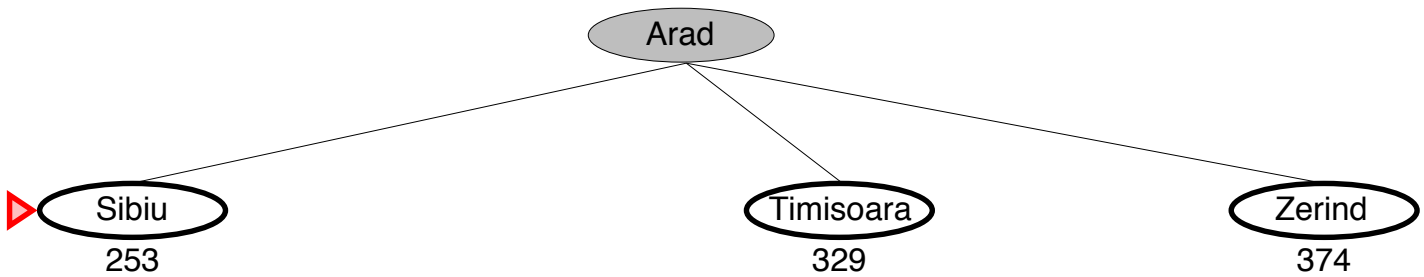
Evaluation function $h(n)$ (**h**euristic)
$=$ estimate of cost from $n$ to the closest goal

E.g., $h_{\mathrm{SLD}}(n) =$ straight-line distance from $n$ to Bucharest

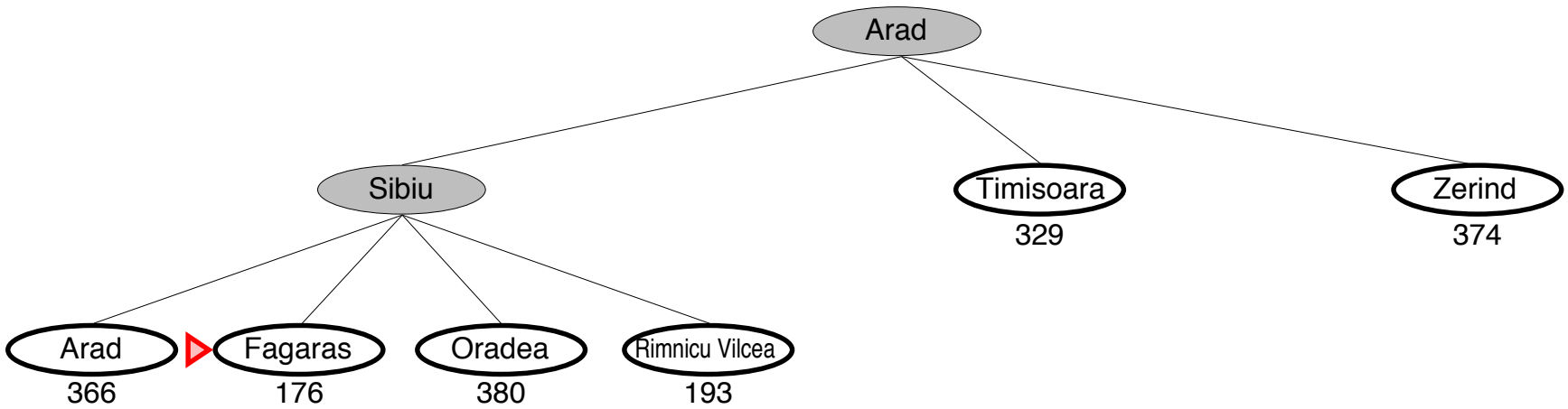
Greedy search expands the node that **appears** to be closest to goal

# Example

# Example



# Example


Arad
Sibiu
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Zerind
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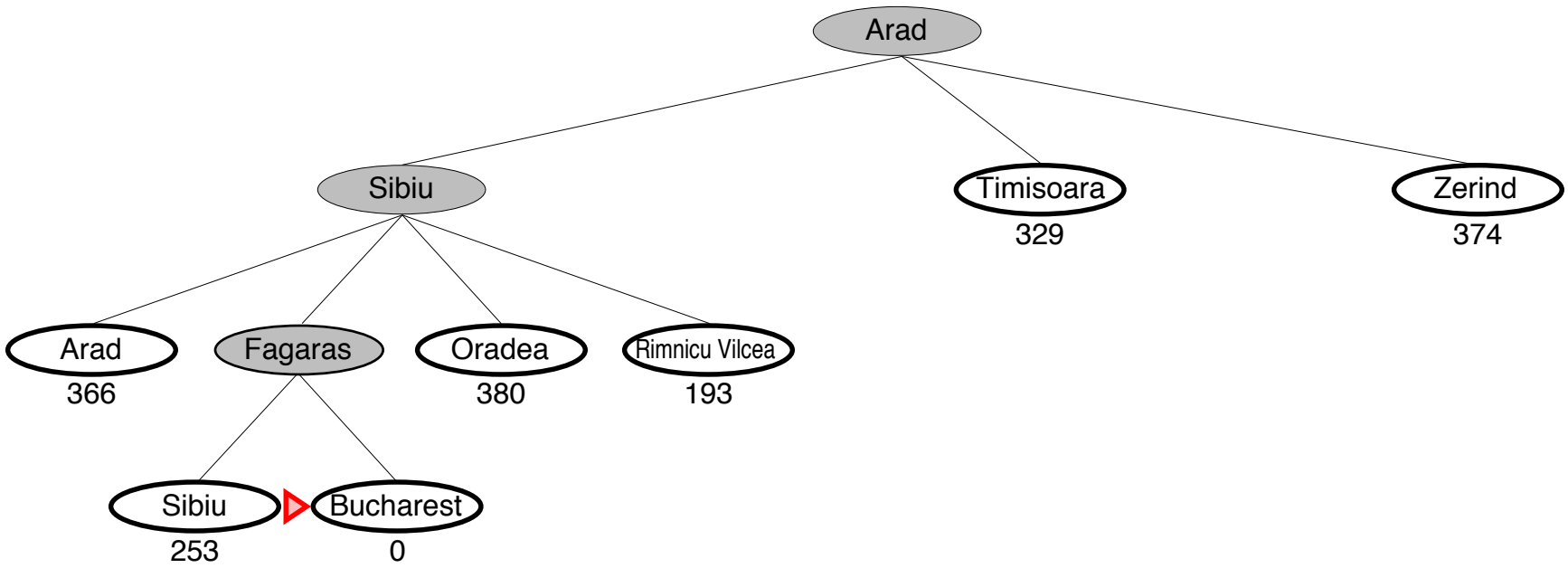
Arad
366
Fagaras
176
Oradea
380
Rimnicu Vilcea
193

# Example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374
Arad
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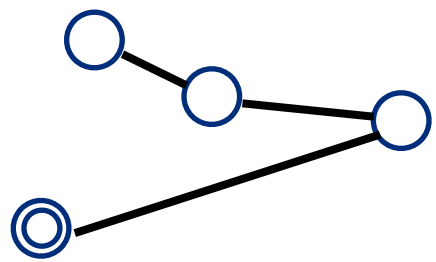
Fagaras
Oradea
380
Rimnicu Vilcea
193
Sibiu
253
Bucharest
0

# Properties

Complete?? No–can get stuck in loops, e.g.,



Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space?? $O(b^m)$—keeps all nodes in memory

Optimal?? No

# A* search

Idea: avoid expanding paths that are already expensive

Evaluation function $f(n) = g(n) + h(n)$

$g(n)$ = cost so far to reach $n$
$h(n)$ = estimated cost to goal from $n$
$f(n)$ = estimated total cost of path through $n$ to goal

A* search uses an admissible heuristic
i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the **true** cost from $n$.
(Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal $G$.)

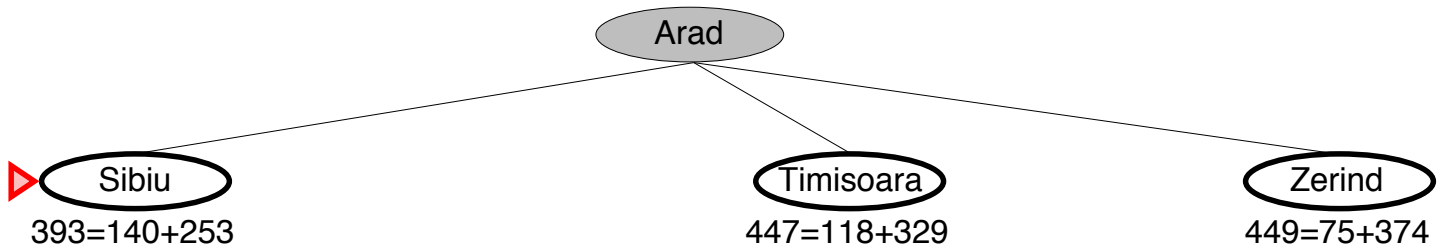
E.g., $h_{\mathrm{SLD}}(n)$ never overestimates the actual road distance

Theorem: A* search is optimal

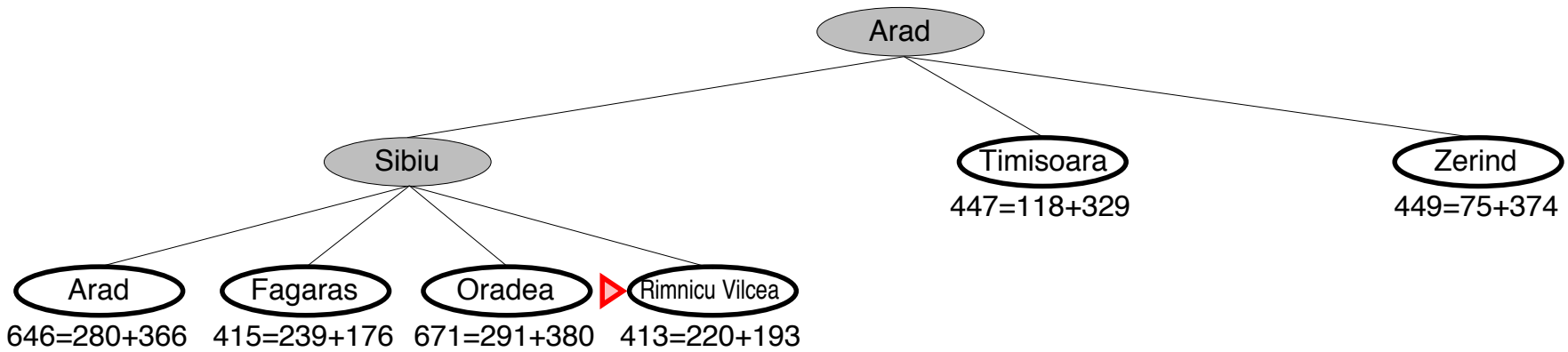
# Example

366=0+366

# Example


Arad
Sibiu
393=140+253
Timisoara
447=118+329
Zerind
449=75+374

# Example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449=75+374
Arad
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Fagaras
415=239+176
Oradea
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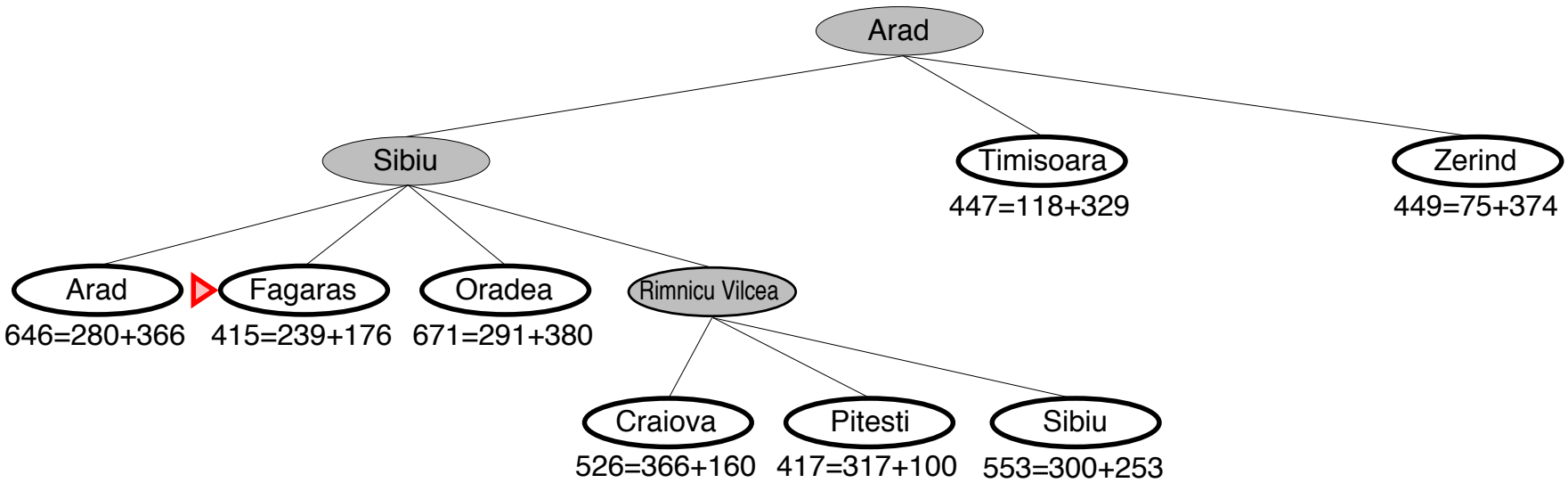
413=220+193

# Example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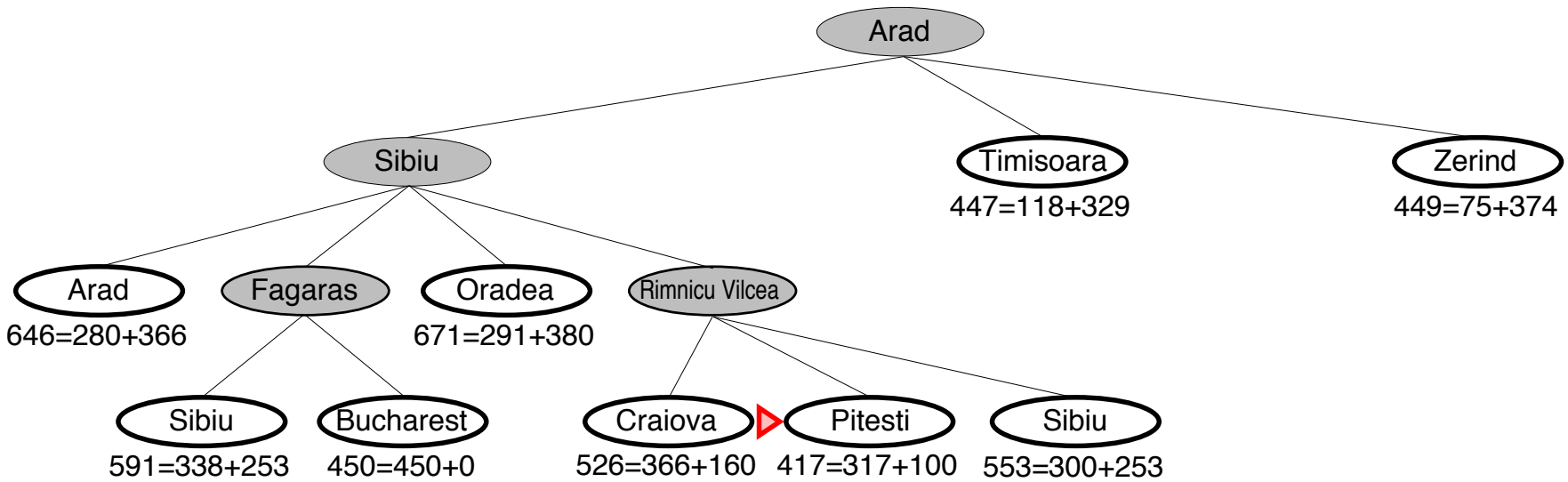
Craiova
526=366+160
Pitesti
417=317+100
Sibiu
553=300+253

# Example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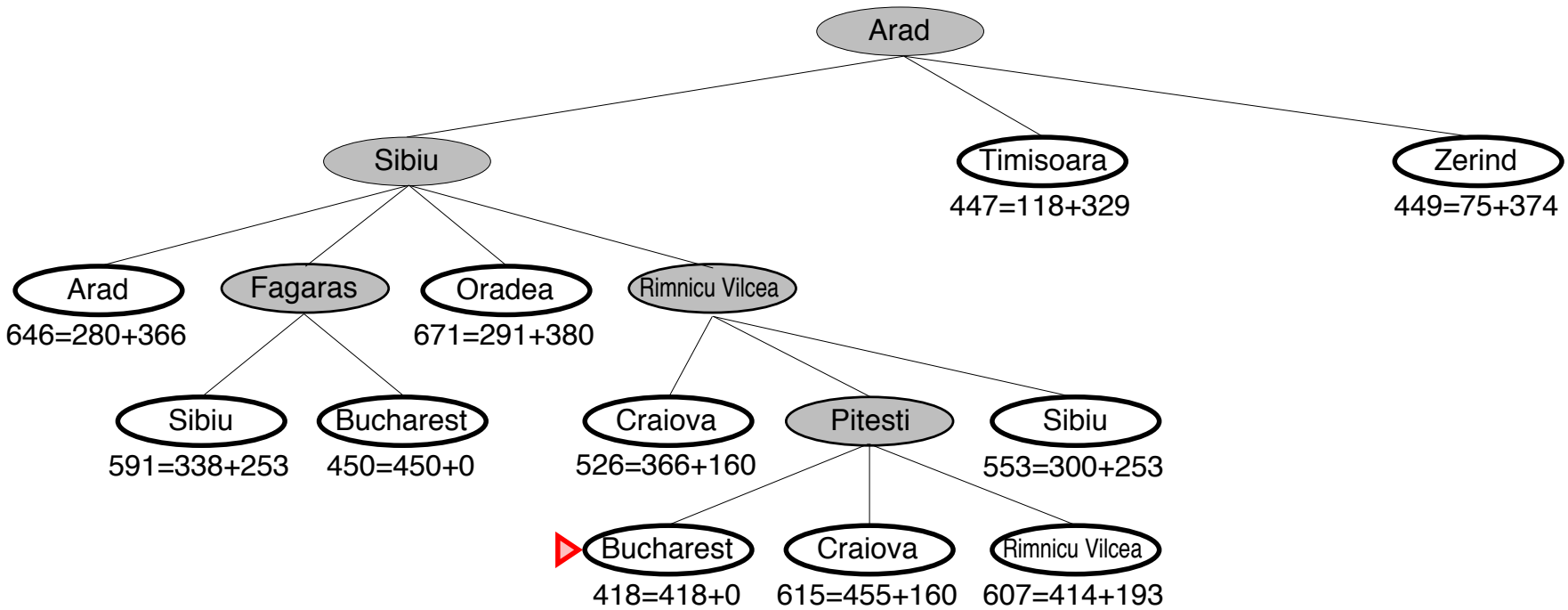
# Example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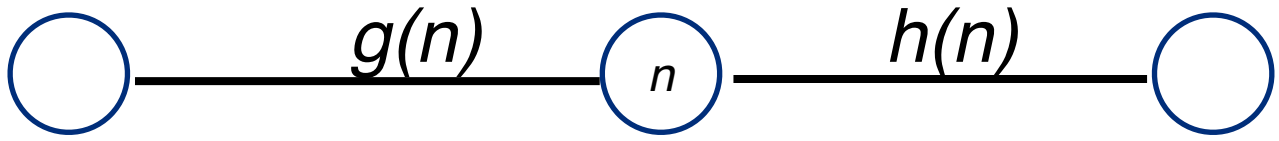
Pitesti
Sibiu
553=300+253
Bucharest
418=418+0
Craiova
615=455+160
Rimnicu Vilcea
607=414+193

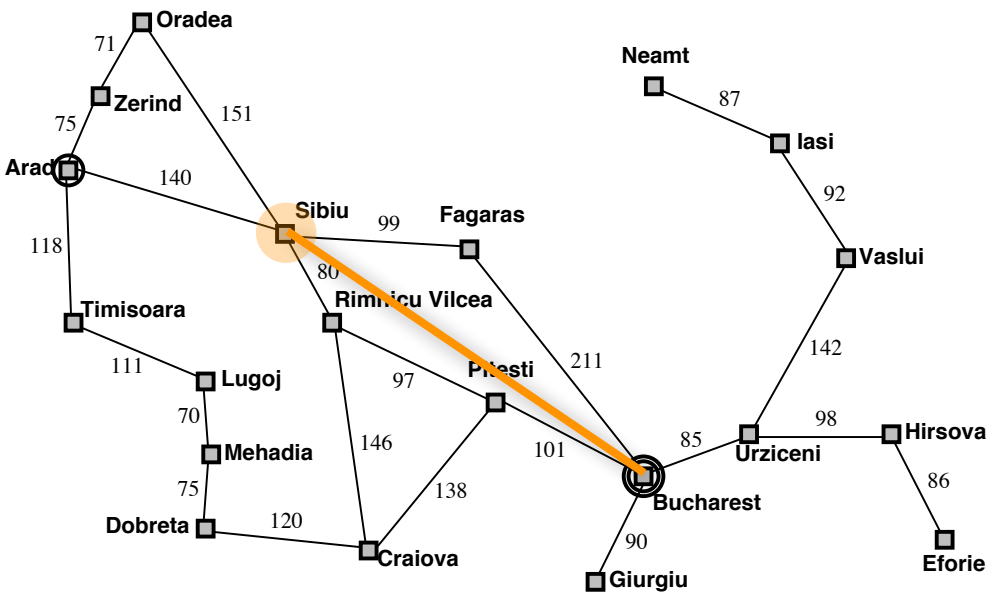
# A* is optimal: Admissible and consistency

Admissible: never over estimate the cost




initial state current state goal state

no larger than the cost of the optimal path from *n* to the goal



# A* is optimal: Admissible and consistency

A* is optimal with admissible heuristic

重点理解！

why?

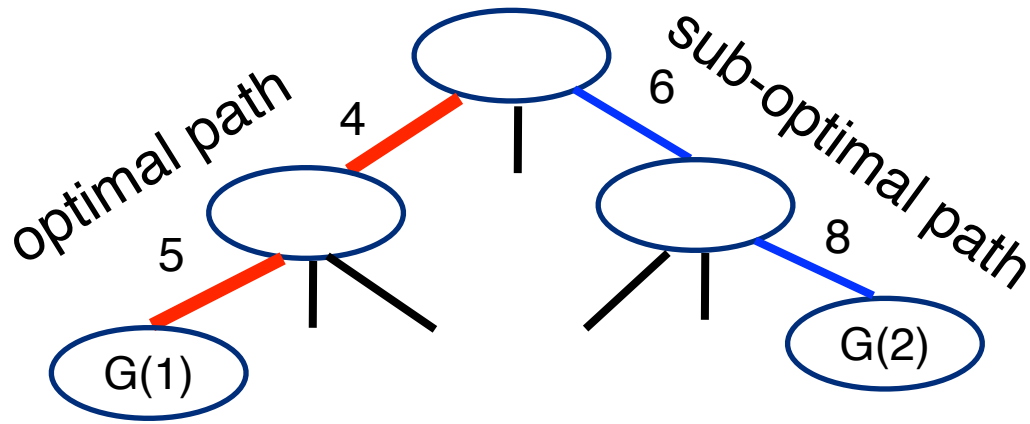
# A* is optimal: Admissible and consistency

A* is optimal with admissible heuristic 重点理解！

why? 1. when a search algorithm is optimal?

uniform-cost search is optimal, because
a) it expands node with the smallest cost
b) a state on the optimal path has smaller cost than the goal state on any sub-optimal path
c) it will never expand the goal states on sub-optimal paths before the goal state on the optimal path

*key, the goal state on the optimal path has a smaller value than that on any sub-optimal paths*



# A* is optimal: Admissible and consistency

A* is optimal with admissible heuristic 重点理解！

why? 2. when the f=g+h value of the goal state on the optimal path is smaller than that on any sub-optimal path?

# A* is optimal: Admissible and consistency

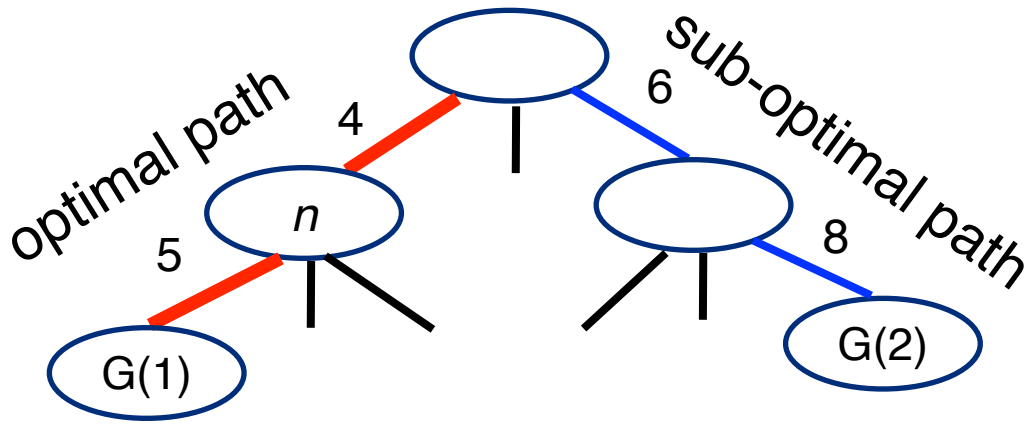
A* is optimal with admissible heuristic 重点理解！

why? 3. if h(n) <= h*(n), that is, the heuristic value is smaller than the true cost

for any node *n* on the optimal path

$$f(n) = g(n) + h(n) <= g(n) + h^*(n) = g(G(1)) <= g(G(2))$$

*so n is always expanded before the goal state on any other sub-optimal path*

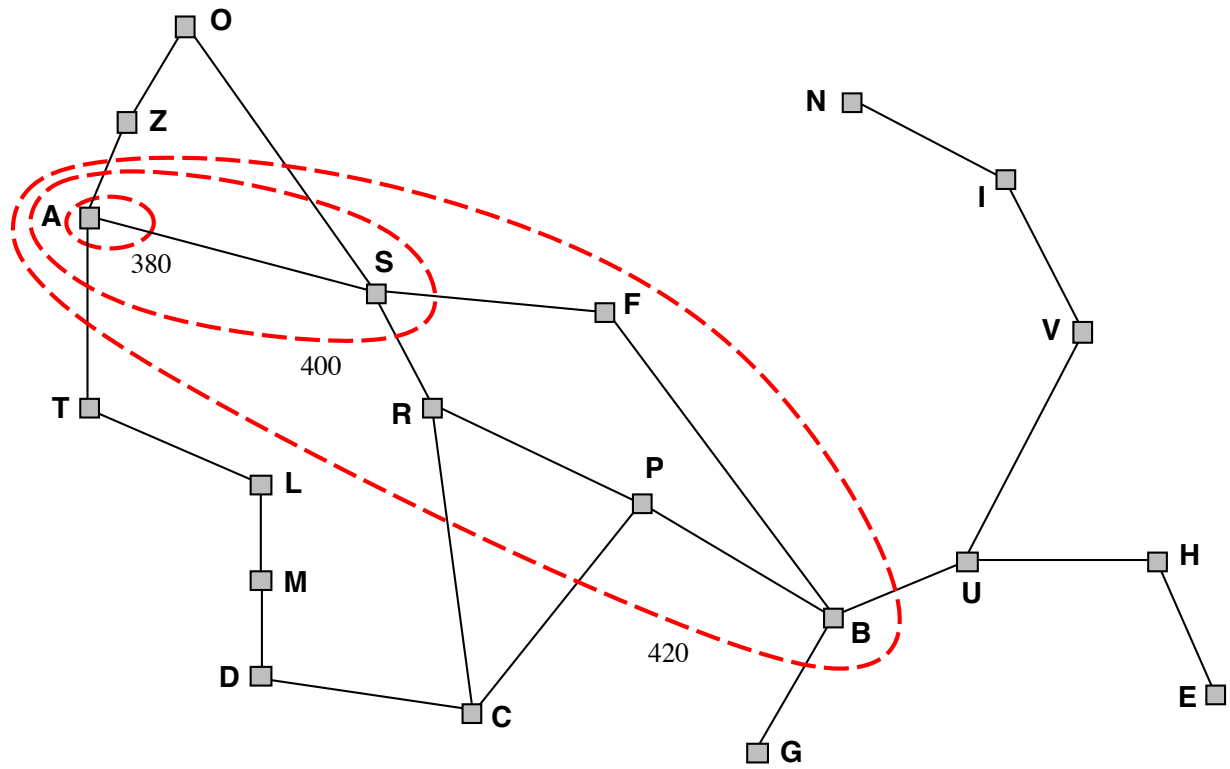


# A* is optimal: Admissible and consistency

## A* is optimal with admissible heuristic

why? Lemma: A* expands nodes in order of increasing $f$ value*

Gradually adds "$f$-contours" of nodes (cf. breadth-first adds layers)
Contour $i$ has all nodes with $f = f_i$, where $f_i < f_{i+1}$



# A* is optimal: Admissible and consistency

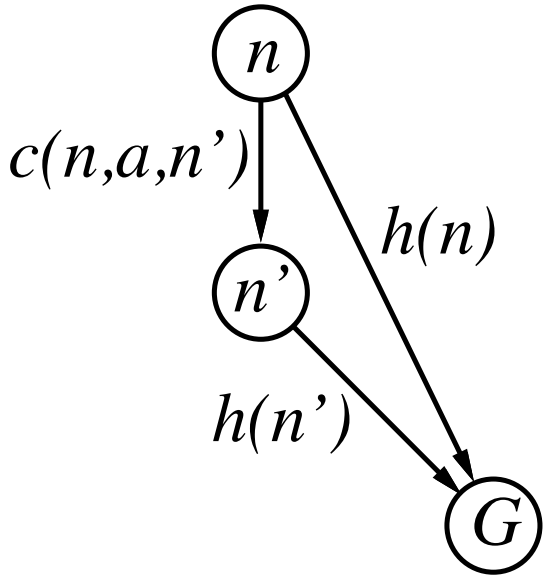
Admissible is for tree search, for graph search

A heuristic is consistent if

$$h(n) \leq c(n, a, n') + h(n')$$

If $h$ is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n, a, n') + h(n') \\ &\geq g(n) + h(n) \\ &= f(n) \end{aligned}$$



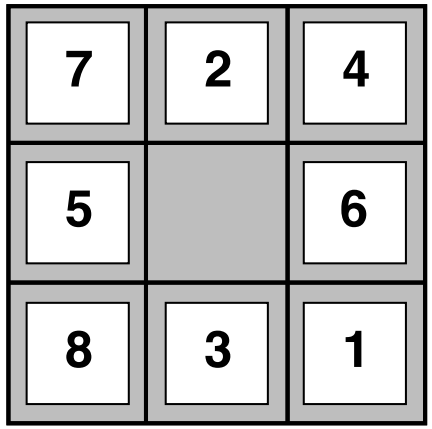

I.e., $f(n)$ is nondecreasing along any path.

Proof is similar with that of admissible

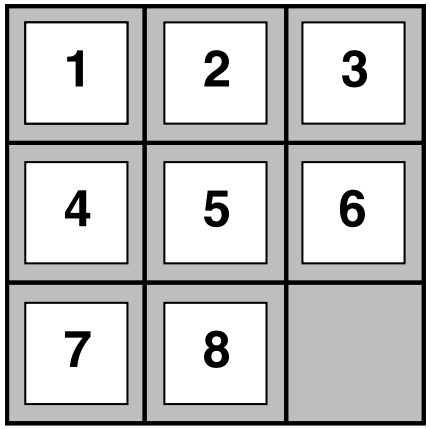
# Example

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)



Start State



Goal State

$h_1(S) =$?? 6
$h_2(S) =$?? 4+0+3+3+1+0+2+1 = 14

# Dominance

If $h_2(n) \geq h_1(n)$ for all $n$ (both admissible)
then $h_2$ dominates $h_1$ and is better for search

why?

Typical search costs:

$d = 14$ IDS = 3,473,941 nodes
A*$(h_1) = 539$ nodes
A*$(h_2) = 113$ nodes
$d = 24$ IDS $\approx$ 54,000,000,000 nodes
A*$(h_1) = 39{,}135$ nodes
A*$(h_2) = 1{,}641$ nodes

Given any admissible heuristics $h_a$, $h_b$,

$$h(n) = \max(h_a(n), h_b(n))$$

is also admissible and dominates $h_a$, $h_b$

# Admissible heuristics from relaxed problem

Admissible heuristics can be derived from the **exact**
solution cost of a **relaxed** version of the problem

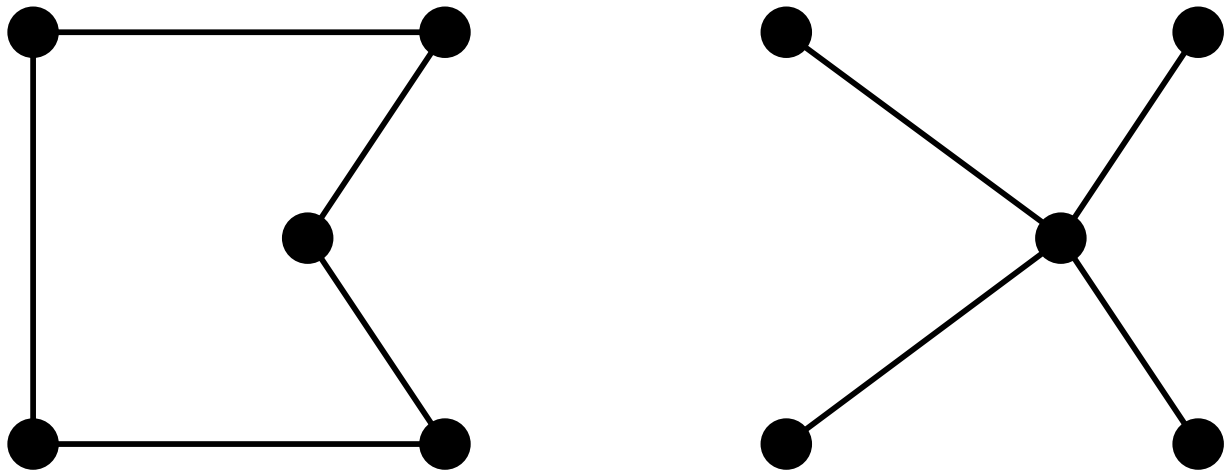
If the rules of the 8-puzzle are relaxed so that a tile can move **anywhere**,
then $h_1(n)$ gives the shortest solution

If the rules are relaxed so that a tile can move to **any adjacent square**,
then $h_2(n)$ gives the shortest solution

Key point: the optimal solution cost of a relaxed problem
is no greater than the optimal solution cost of the real problem

# Example

Well-known example: travelling salesperson problem (TSP)
Find the shortest tour visiting all cities exactly once



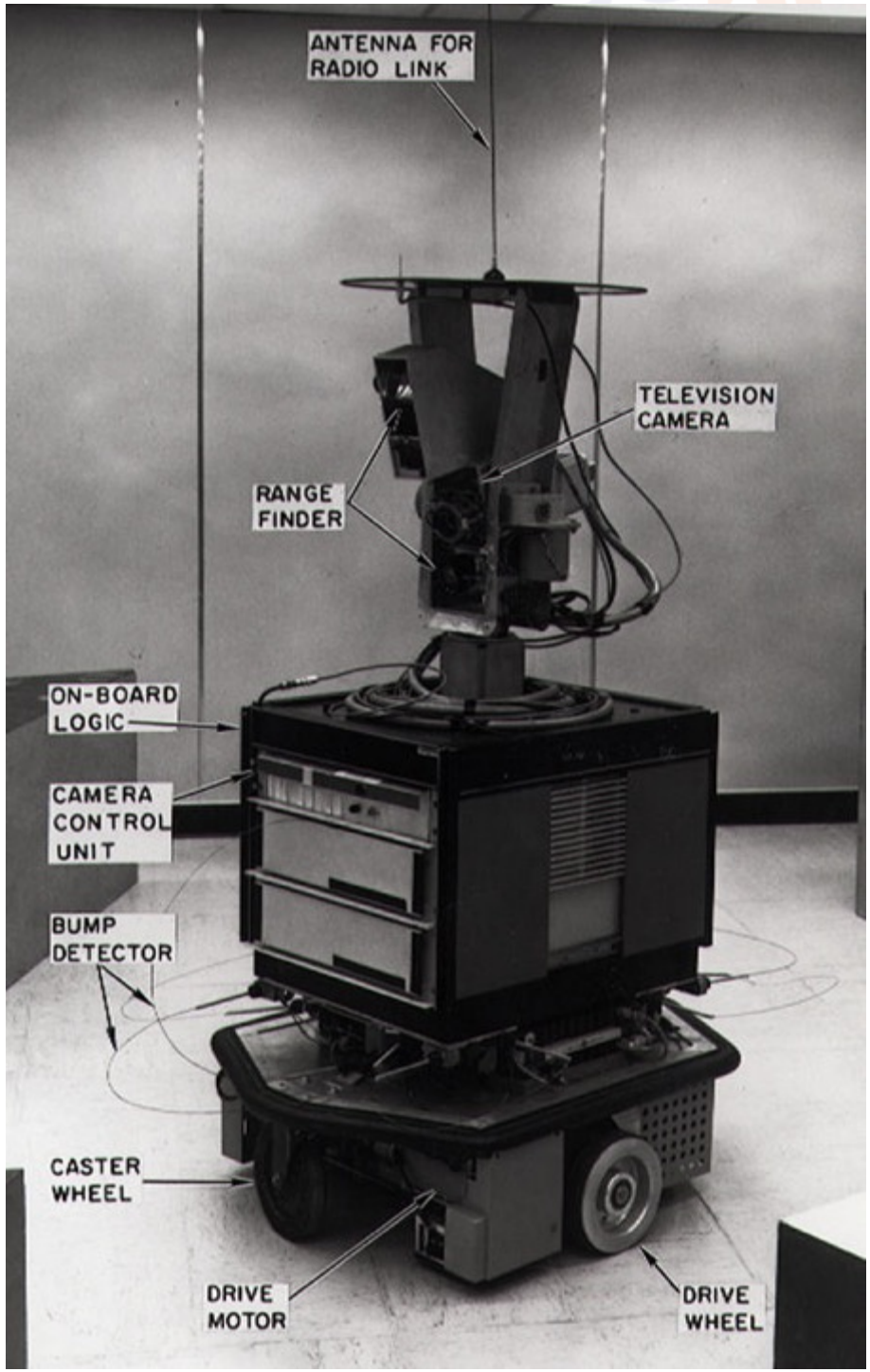
Minimum spanning tree can be computed in $O(n^2)$
and is a lower bound on the shortest (open) tour

# Where did A* come from

## Shakey 50 Years

Shakey the robot was the first general-purpose mobile robot to be able to reason about its own actions

Developed in SRI International from 1966



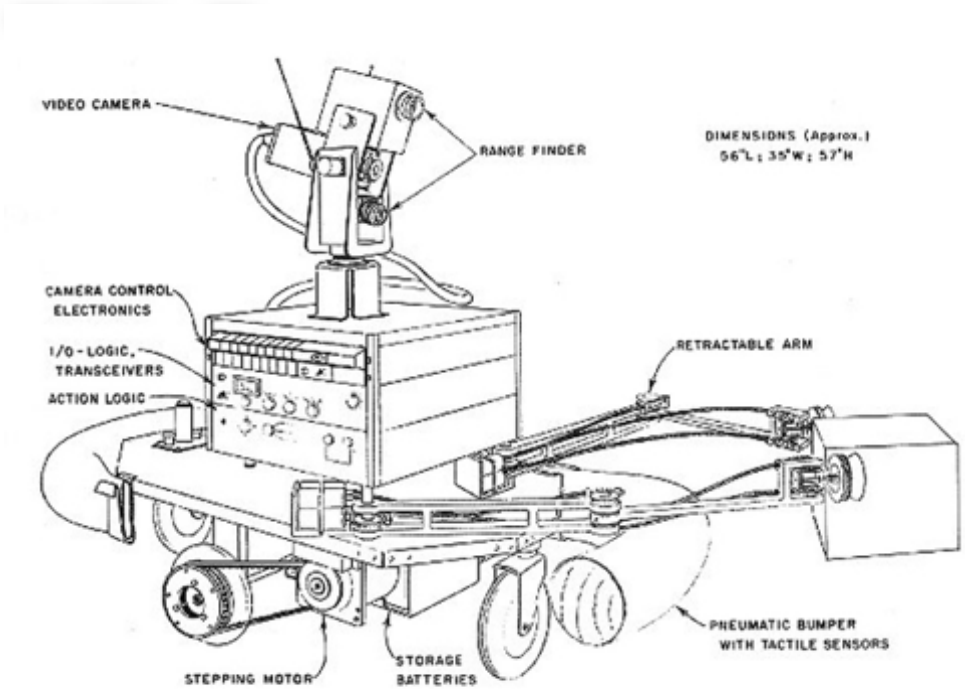
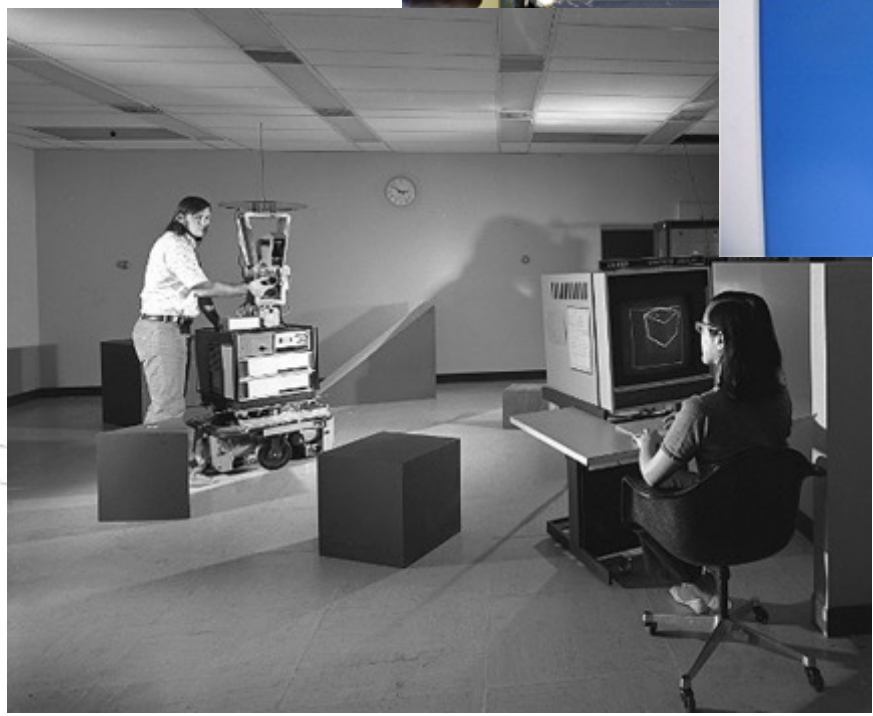
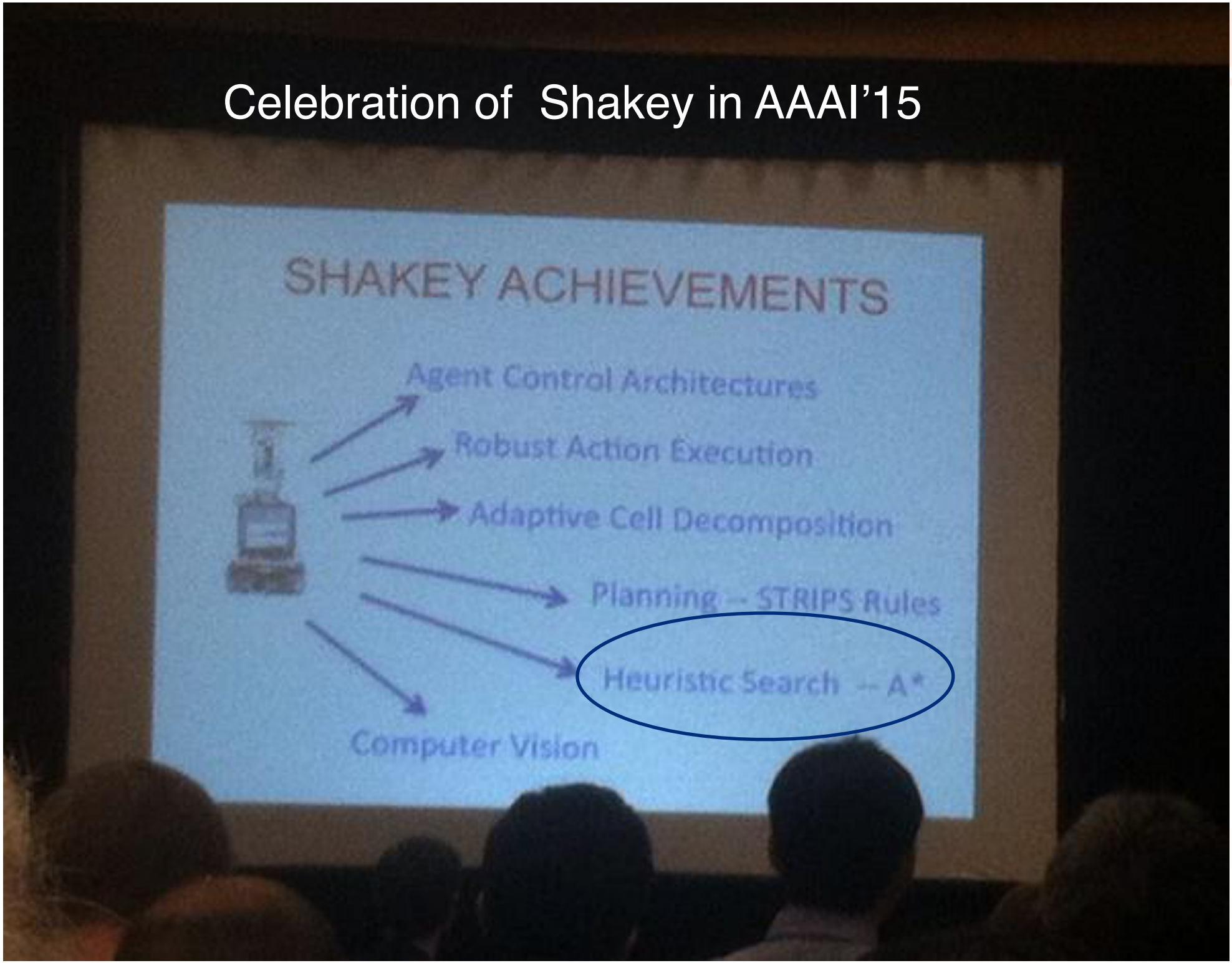


FIG. 2 AUTOMATON VEHICLE



# Celebration of Shakey in AAAI'15