NJUAI 南京大学 人工智能学院
SCHOOL OF ARTIFICIAL INTELLIGENCE, NANJING UNIVERSITY

01 信号的时域分析

如何刻画信号关于时间的变化

本章概要

1. 信号的分类：重点关注信号的周期性，功率特性

2. 信号的运算：信号的尺度变换、翻转、时移 ...

3. 典型的信号：
一般信号和奇异信号

4. 信号的分解：
如何有效地“表示”信号

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信号的分类 （周期性）

- **周期信号**
  - 连续周期信号：$\forall t \in \mathbb{R}$，存在正数T，使得$x(t+T)=x(t)$
  - 离散周期信号：$\forall n \in N$，存在**正整数**N，使得$x[n+N]=x[n]$
  - 满足上述条件的**最小的**正T、正N称为信号的**基波**周期（fundamental period）。

- 判断下列信号是否为周期信号，若是，确定其周期
  - (1) $x_1(t) = \sin 2t + \cos 3t$
  - (2) $x_2(t) = \cos 2t + \sin \pi t$

周期信号$x(t)$，$y(t)$的周期为T_1和T_2，若周期之比T_1/T_2为**有理数**，则其和信号$x(t)+y(t)$仍然是周期信号，其周期为T_1和T_2的**最小公倍数**。

信号的周期性

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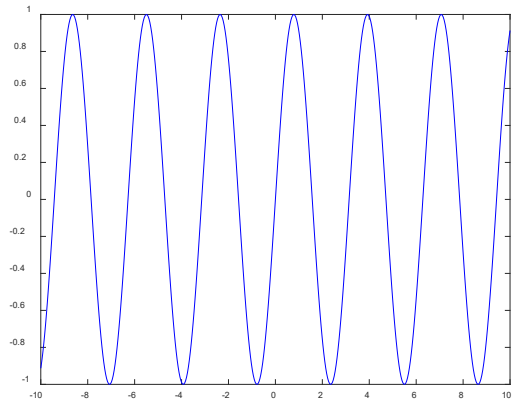
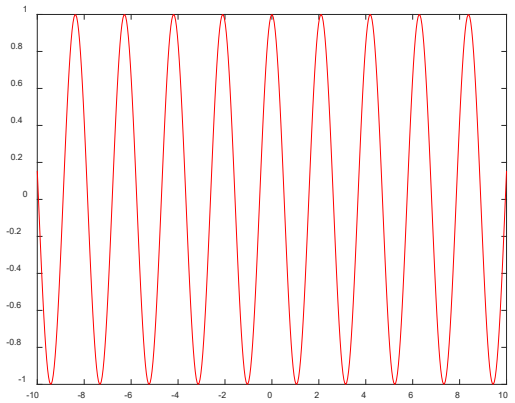
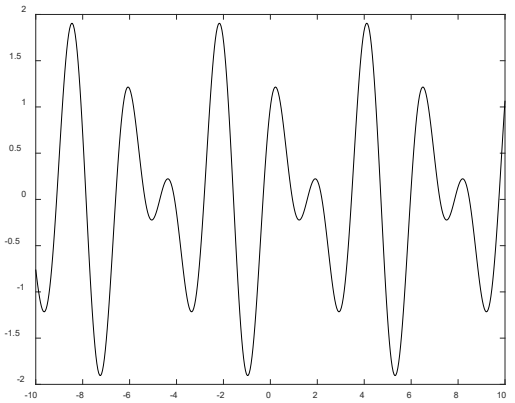
- $x_1(t) = \sin 2t + \cos 3t$
  - $\sin 2t$ 周期为$T_1 = \frac{2\pi}{2} = \pi$；　$\cos 3t$周期为$T_2 = \frac{2\pi}{3}$
  - 由于$\frac{T_1}{T_2} = \frac{3}{2}$为有理数，故$x_1(t)$为周期信号，其周期为$2\pi$



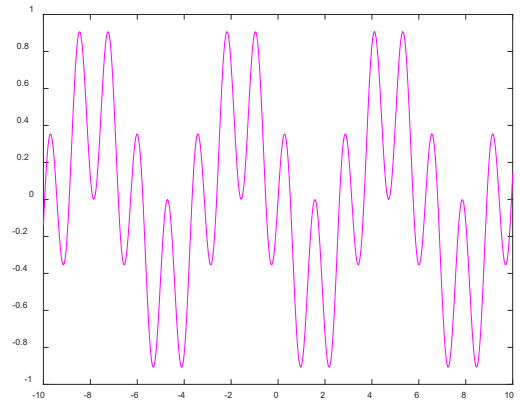
$\sin 2t$



$\cos 3t$



$\sin 2t + \cos 3t$

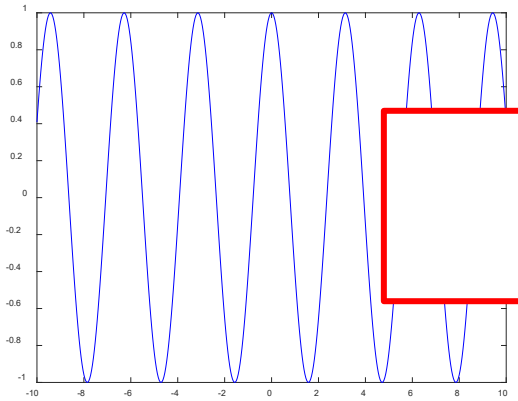


$\sin 2t * \cos 3t$

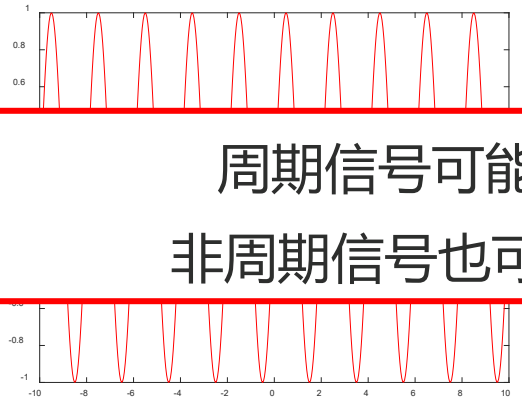
信号的周期性

周期信号$x(t)$，$y(t)$的周期为T_1和T_2，若周期之比T_1/T_2为**有理数**，则其和信号$x(t)+y(t)$仍然是周期信号，其周期为T_1和T_2的**最小公倍数。**

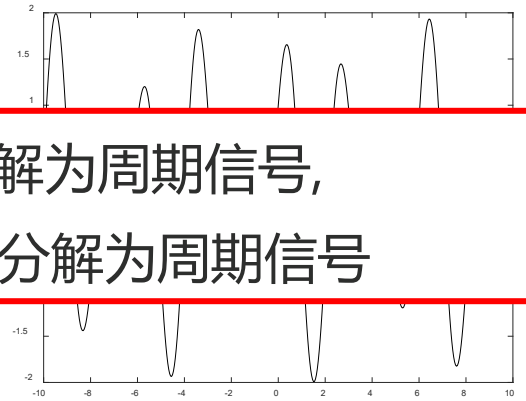
- $x_2(t) = \cos 2t + \sin \pi t$
  - $\cos 2t$ 和$\sin \pi t$的周期分别为$T_1 = \pi$，$T_2 = 2$
  - 由于T_1/T_2为无理数，故$x_2(t)$为非周期信号



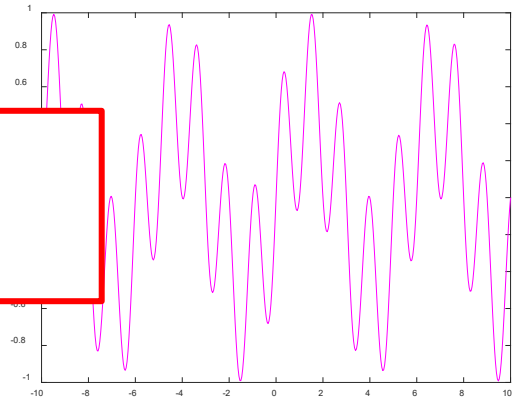
$\cos 2t$



$\sin \pi t$



$\cos 2t + \sin \pi t$



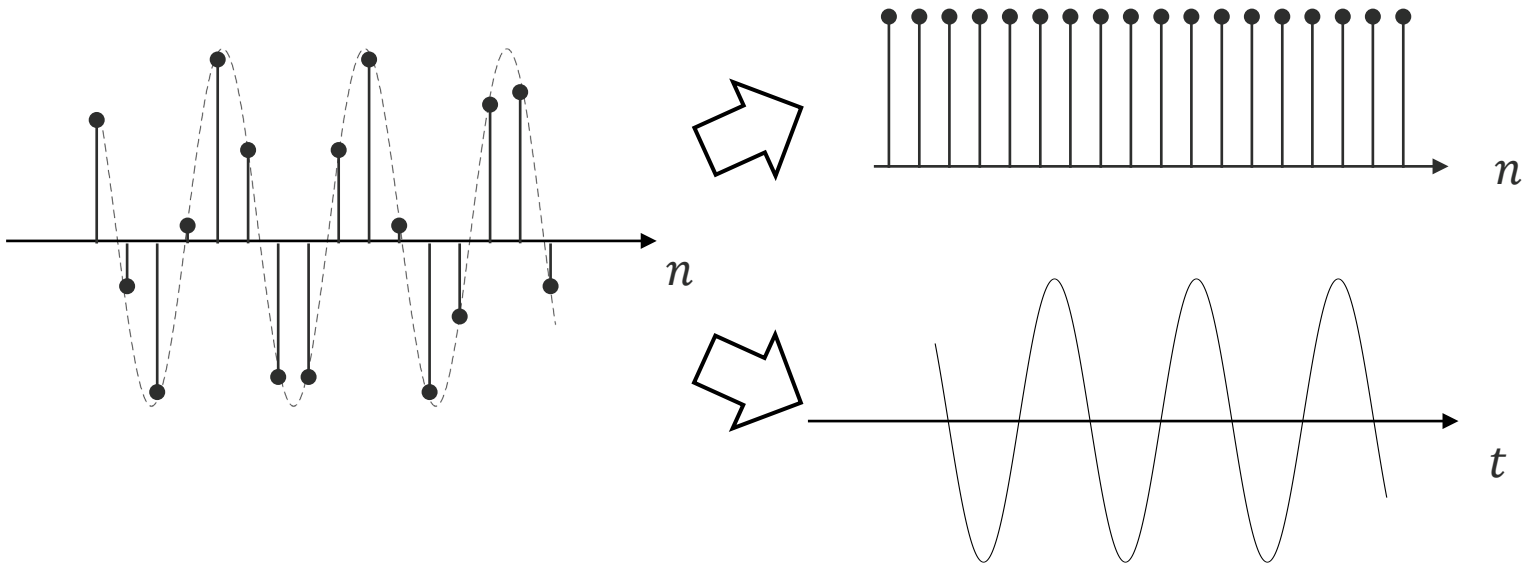
$\cos 2t * \sin \pi t$

周期信号可能分解为周期信号，

非周期信号也可能分解为周期信号

离散信号的周期性

- 离散信号可视为**连续**（包络线）信号和**周期脉冲**的乘积



- 离散信号的周期性和连续信号以及周期脉冲信号的周期有关

连续正弦信号一定是周期信号，而正弦序列不一定是周期序列。

两连续周期信号之和不一定是周期信号，而两周期序列之和一定是周期序列。

信号的分类

- **能量信号与功率信号**

	连续信号	离散信号
能量	$W = \lim\limits_{T\to\infty}\int_{-T}^{T}\|x(t)\|^2\mathrm{d}t$	$W = \lim\limits_{N\to\infty}\sum_{-N}^{N}\|x[n]\|^2$
功率	$P = \lim\limits_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\|x(t)\|^2\mathrm{d}t$	$P = \lim\limits_{N\to\infty}\frac{1}{2N+1}\sum_{-N}^{N}\|x[n]\|^2$

- 能量信号：$0 < W < \infty$，$P = 0$。
- 功率信号：$W \to \infty$，$0 < P < \infty$。
- 直流信号与周期信号都是功率信号

信号的分类

- **能量信号与功率信号**

	连续信号	离散信号
能量	$W=\lim\limits_{T\to\infty}\int_{-T}^{T}\lvert x(t)\rvert^2\mathrm{d}t$	$W=\lim\limits_{N\to\infty}\sum\limits_{-N}^{N}\lvert x[n]\rvert^2$
功率	$P=\lim\limits_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\lvert x(t)\rvert^2\mathrm{d}t$	$P=\lim\limits_{N\to\infty}\frac{1}{2N+1}\sum\limits_{-N}^{N}\lvert x[n]\rvert^2$

信号$x(t)$可以是一个既非功率信号，又非能量信号。但一个信号不可能同时既是功率信号，又是能量信号。

周期信号都是功率信号；非周期信号可能是能量信号（$t\to\infty, x(t)=0$）也可能是功率信号（$t\to\infty, x(t)\neq 0$）。

信号的分类

- **因果**信号
  - 若当 $t < 0$ 时 $f(t) = 0$, 当 $t > 0$ 时 $x(t) \neq 0$的信号,称为因果信号。
  - 非因果信号指的是在时间零点之前有非零值。

- 一维信号与多维信号
  - 信号可以表示为一个或多个变量的函数，称为一维或多维函数。

- 连续信号和离散信号

- 确定信号和随机信号

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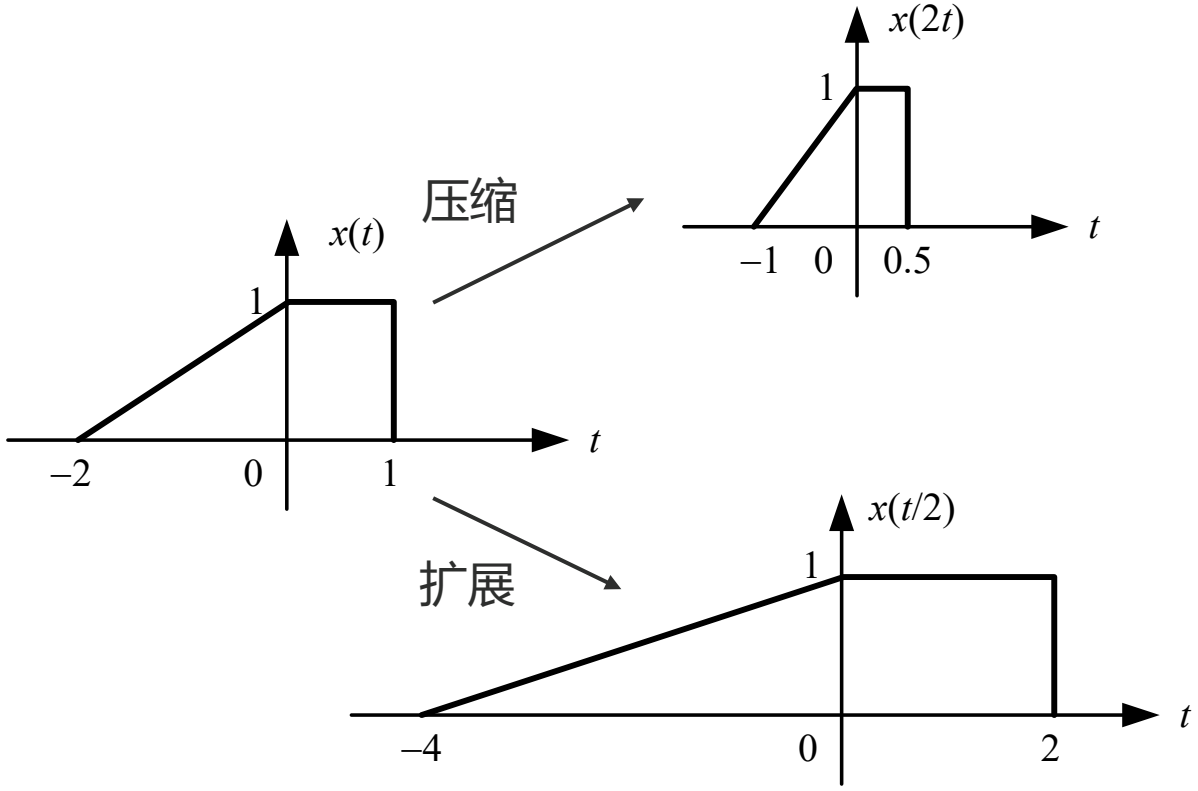
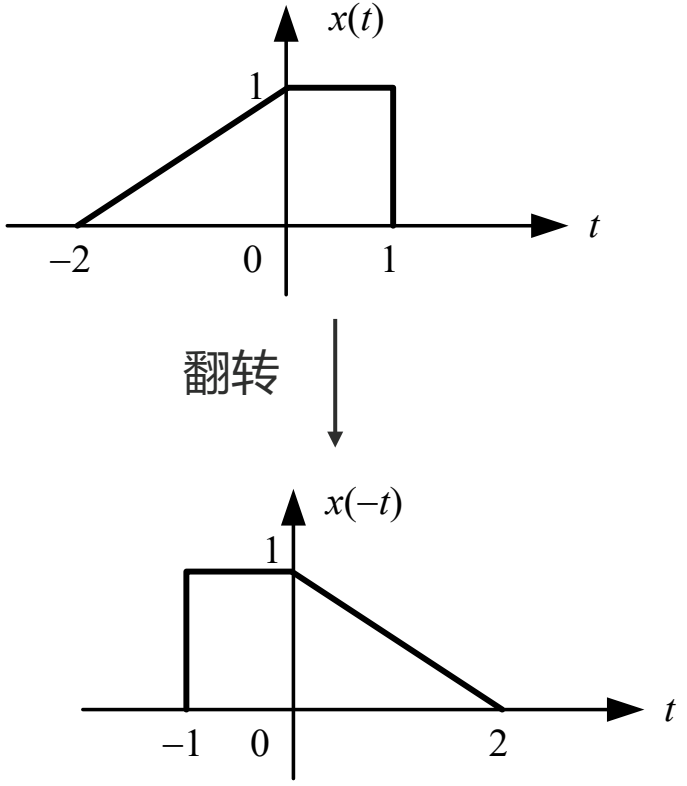
信号的尺度变换

- 尺度变换：$x(t) \rightarrow x(at)\ a > 0$
  - 若$0 < a < 1$，则$x(at)$是$x(t)$的扩展
  - 若$a > 1$，　　则$x(at)$是$x(t)$的压缩

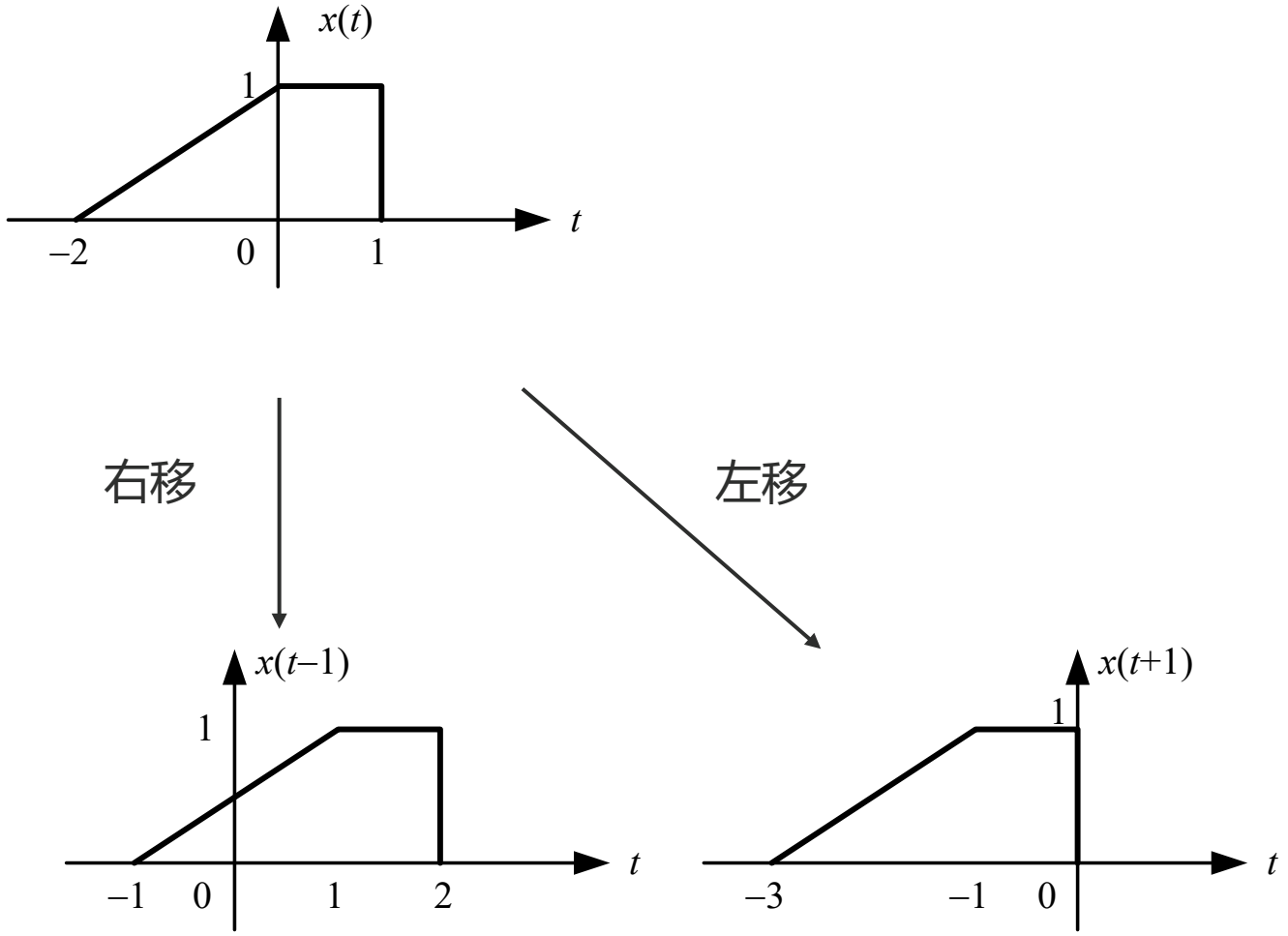


- 信号的翻转：$x(t) \rightarrow x(-t)$
  - 将 $x(t)$ 以纵轴为中心作180°翻转



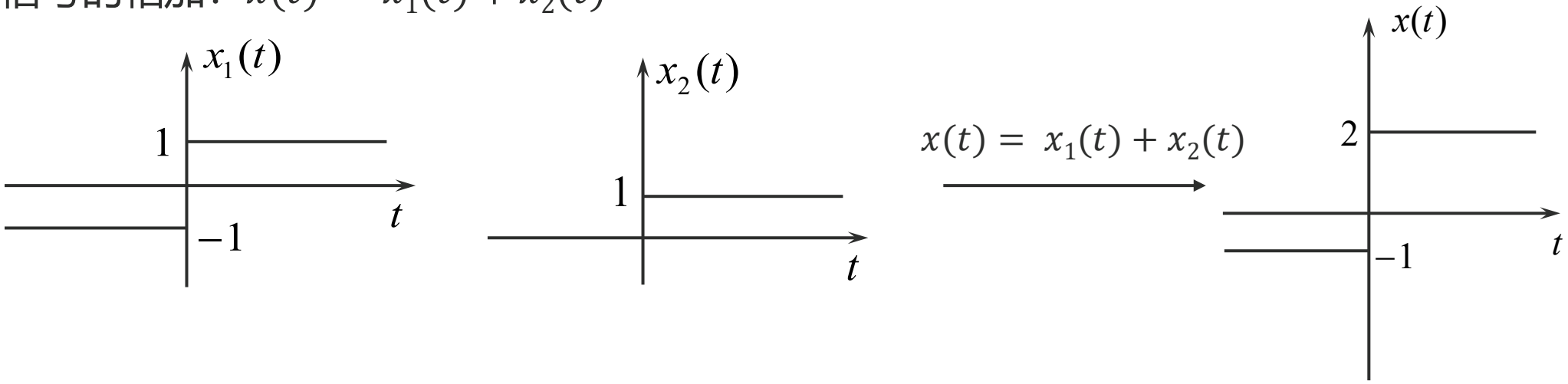
信号的时移

- 信号的时移：$x(t) \rightarrow x(t-t_0)$
  - $x(t-t_0)$表示信号**右**移t_0单位
  - $x(t+t_0)$表示信号**左**移t_0单位

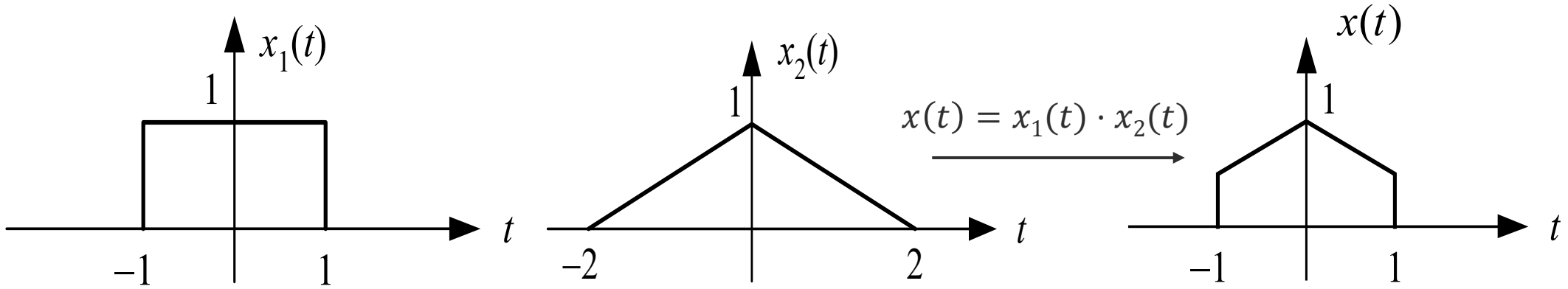


信号的加和乘

- 信号的相加：$x(t) = x_1(t) + x_2(t)$

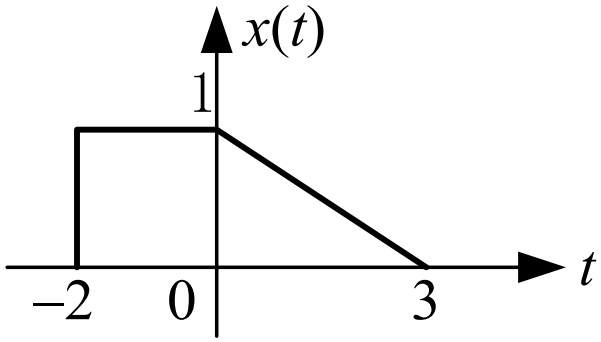


- 信号的相乘：$x(t) = x_1(t) \cdot x_2(t)$

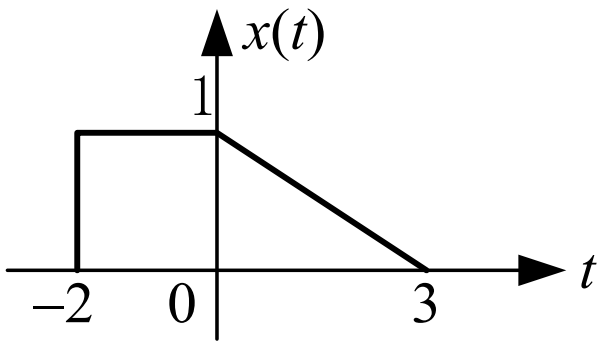


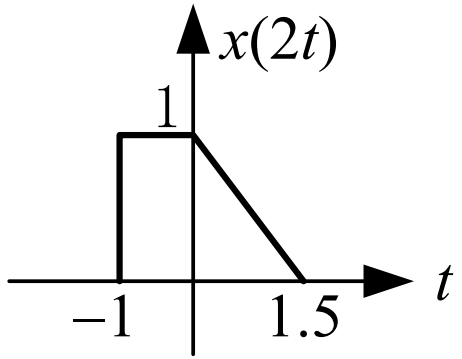
信号的运算

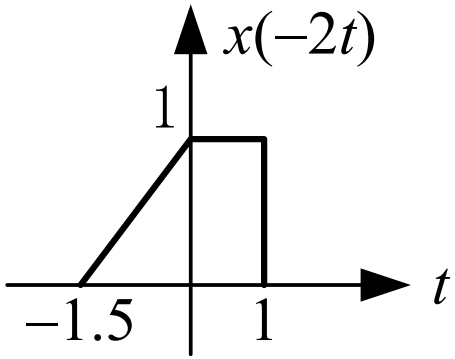
- 已知$x(t)$的波形如图所示，试画出$x(6-2t)$的波形

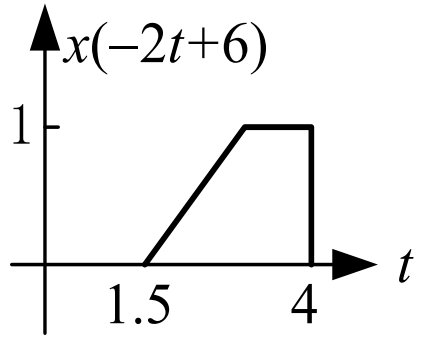


$$x(t) \xrightarrow{\text{尺度变换}} x(2t) \xrightarrow{\text{翻转}} x(-2t) \xrightarrow{\text{右移}} x[-2(t-3)]$$









离散信号的翻转、时移

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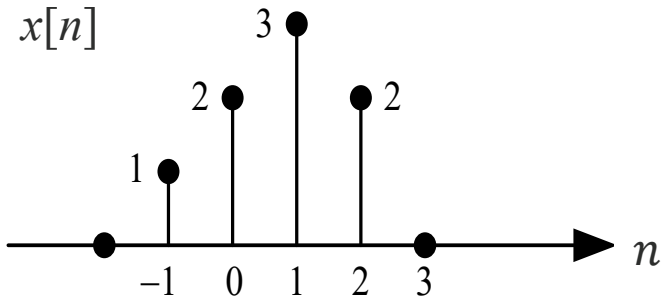
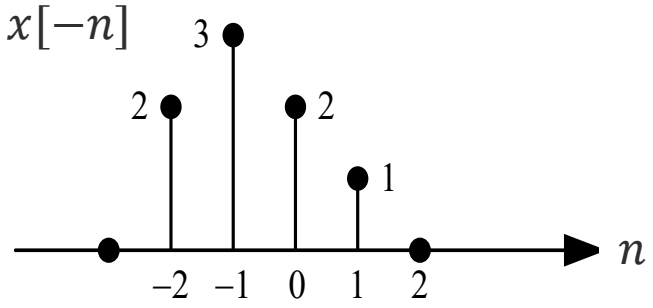
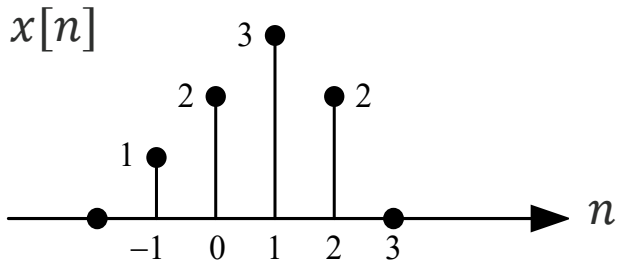
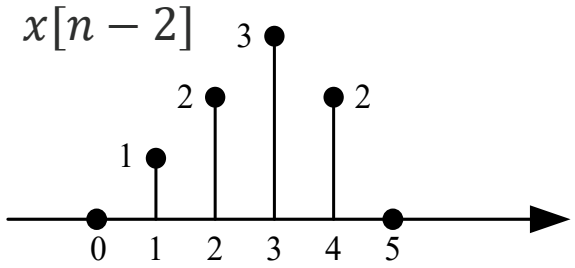
  - 将 $x[n]$ 以纵轴为中心作180度翻转

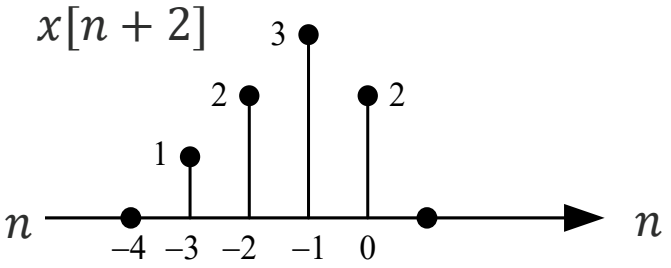




- 时移：$x[n] \rightarrow x[n \pm k],\ k > 0$
  - $x[n-k]$表示将$x[n]$右移k个单位
  - $x[n+k]$表示将$x[n]$左移k个单位

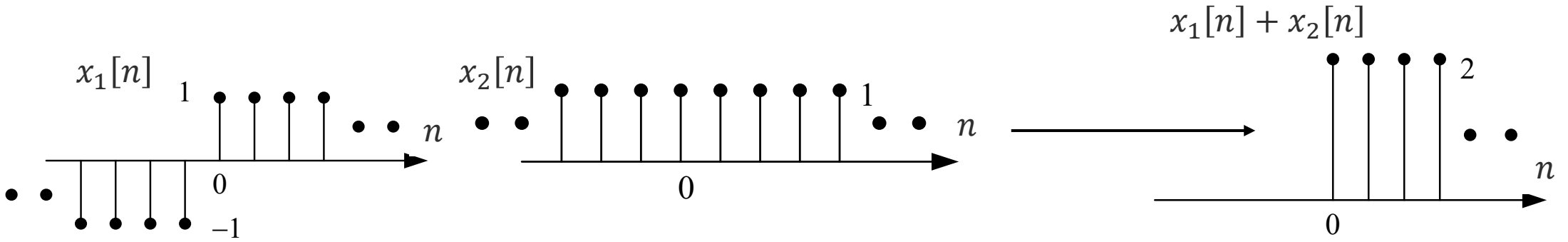






离散信号的加和乘

- 相加：$y[n] = x_1[n] + x_2[n]$



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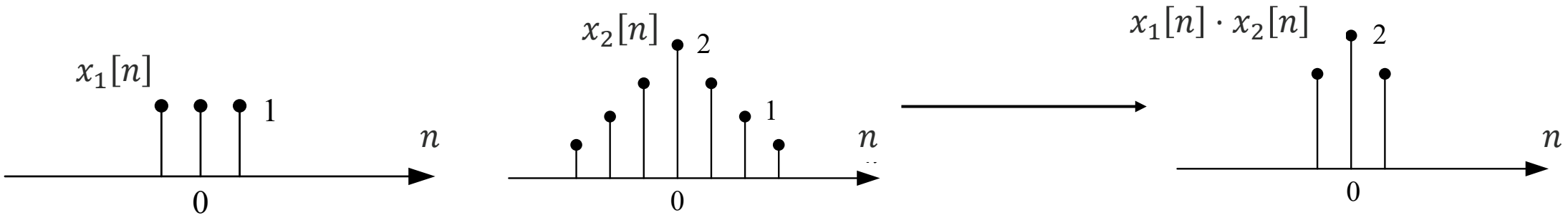

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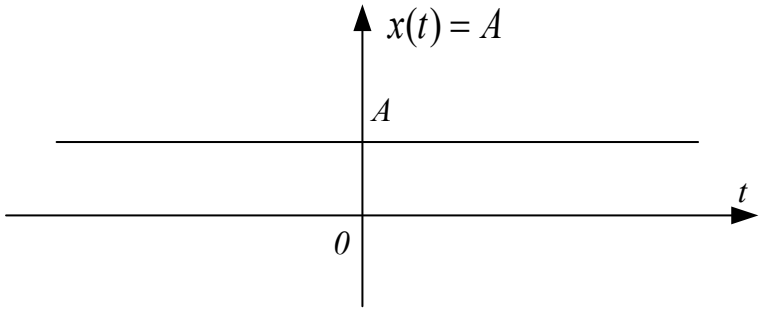
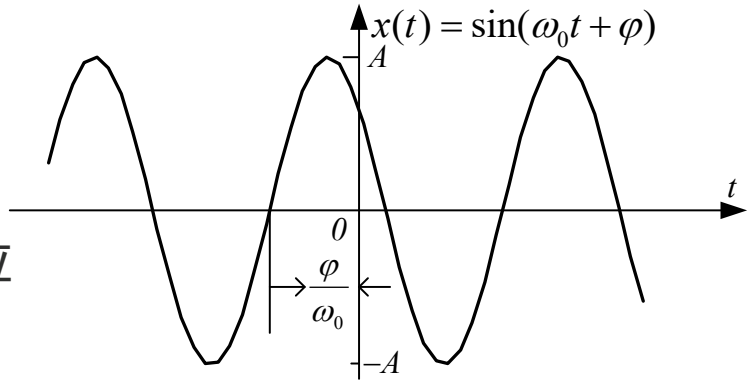
基本连续信号

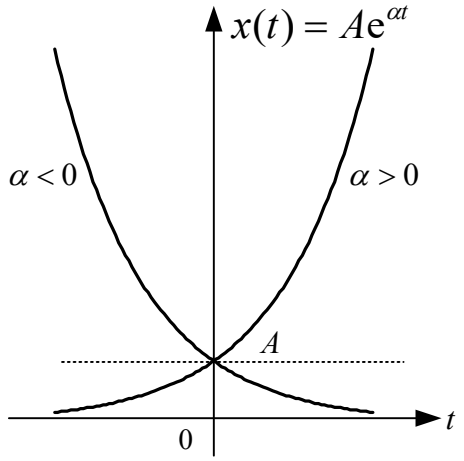
- 直流信号
  - $x(t) = A,\ -\infty < t < \infty$



- 正弦信号
  - $x(t) = A\sin(\omega_0 t + \varphi)$
  - A：振幅；
  - ω_0：角频率；φ：初始相位
  - 周期为$\mathrm{T} = \frac{2\pi}{|\omega_0|} = \frac{1}{f}$



- 实指数信号
  - $x(t) = Ae^{\alpha t}$
  - A和α是实数，取值不同时，信号特征不同

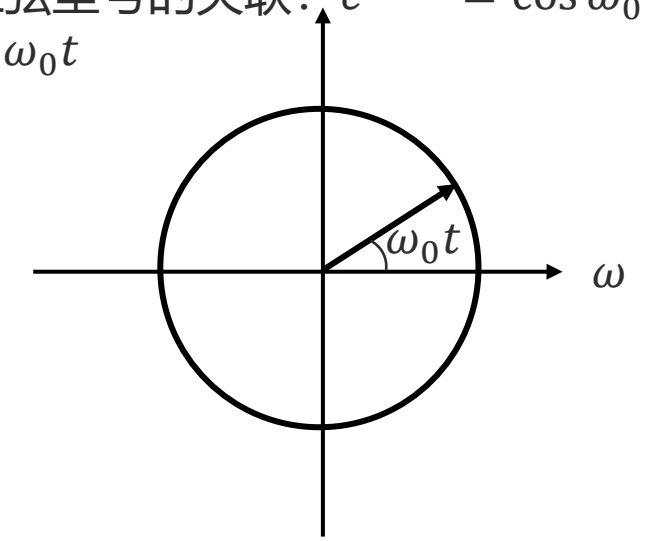


基本连续信号

- 虚指数信号：限制$x(t) = Ae^{\alpha t}$中α为纯虚数
  - 假设A = 1，则$x(t) = e^{j\omega_0 t}$
  - **周期**性：$x(t) = x(t + T_0) = e^{j\omega_0 t}$=$e^{j\omega_0 (t+T_0)}$
  - **若$\boldsymbol{\omega \neq 0}$**，基波周期满足$\omega_0 T_0 = 2\pi k$;

    $k = \pm 1, \pm 2, \cdots$，即

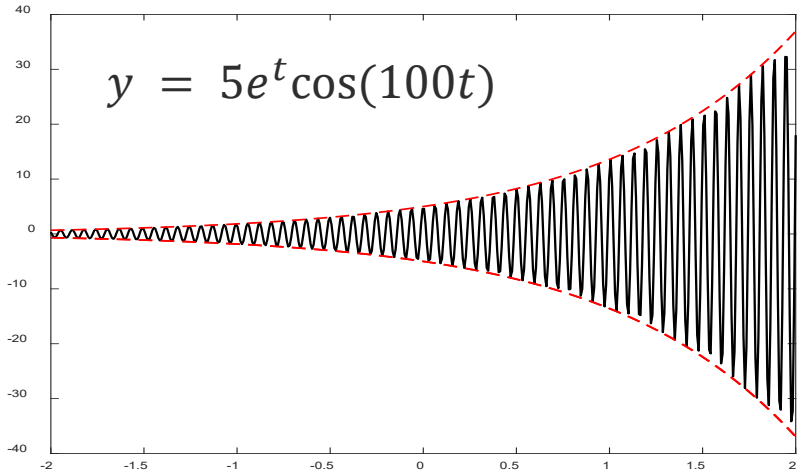
    $$T_0 = \frac{2\pi}{|\omega_0|}$$
  - 与正弦型号的关联：$e^{j\omega_0 t} = \cos \omega_0 t + j \sin \omega_0 t$



- 复指数信号$x(t) = Ae^{st}$
  - A使用极坐标表示：$A = |A|e^{j\theta}$
  - s使用笛卡尔坐标表示：$s = r + j\omega_0$

    $x(t) = Ae^{st} = |A|e^{j\theta}e^{(r+j\omega_0)t} = |A|e^{rt}e^{j(\omega_0 t+\theta)}$
  - 考虑复数和三角函数的关系：

    $x(t) = Ae^{st} = |A|e^{rt}\cos(\omega_0 t + \theta) + j|A|e^{rt}\sin(\omega_0 t + \theta)$
  - 具有指数衰减振幅的正弦信号常称为**阻尼正弦振荡** (*damped sinusoids*)

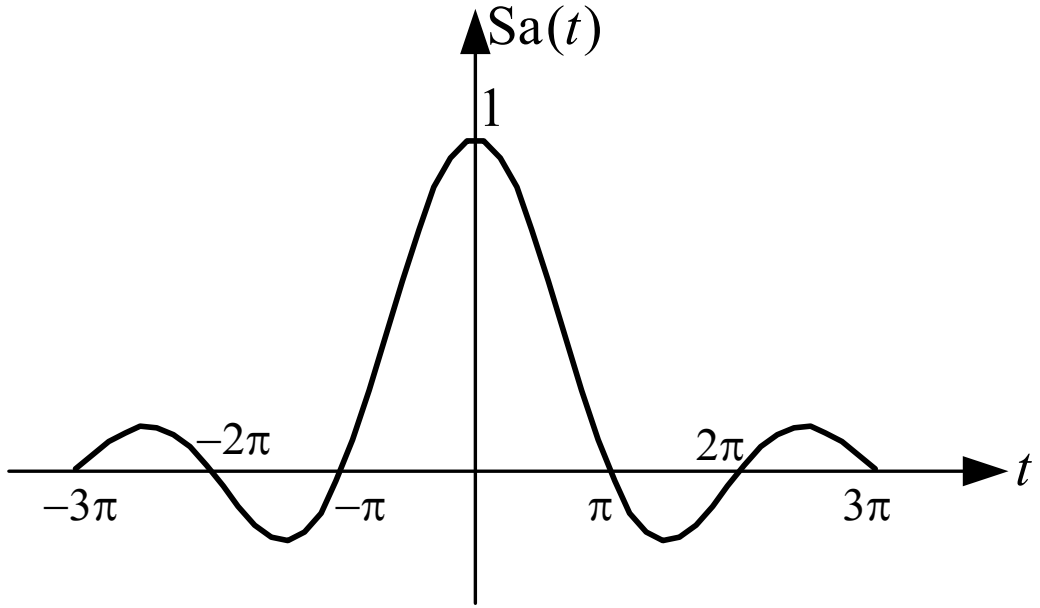


基本连续信号

- 抽样信号

$$\mathrm{Sa}(t)=\frac{\sin t}{t}$$

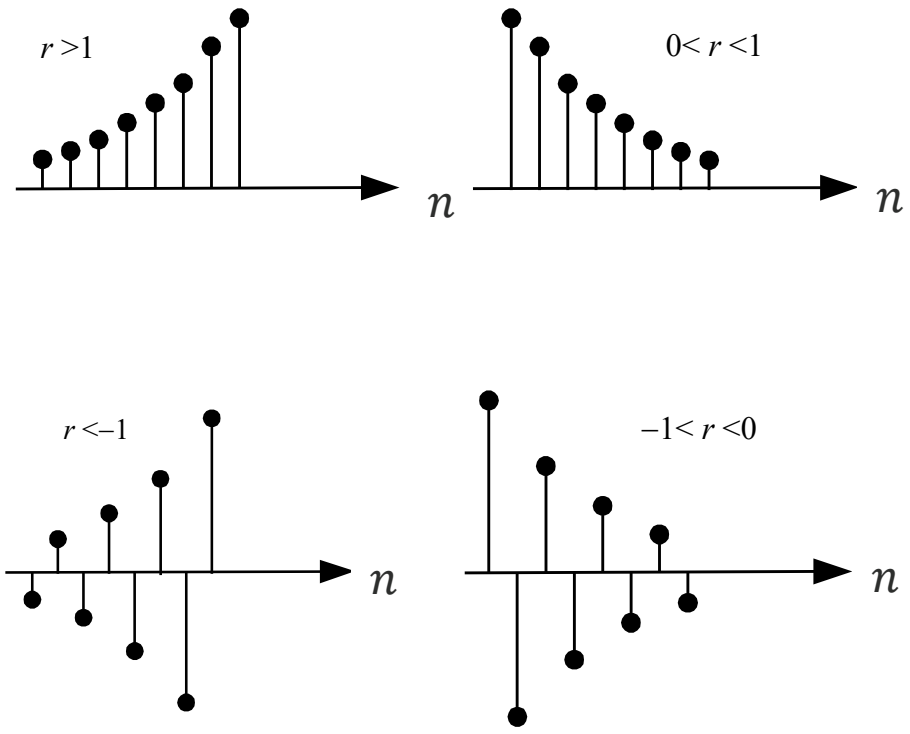
- 主要性质
  - $\mathrm{Sa}(0)=\lim_{t\to 0}\frac{\sin t}{t}=1$
  - $\mathrm{Sa}(k\pi)=0, k=\pm 1,\pm 2,\cdots$
  - $\int_{-\infty}^{\infty}\mathrm{Sa}(t)\,\mathrm{d}t=\pi;\quad \int_{0}^{\infty}\mathrm{Sa}(t)\,\mathrm{d}t=\frac{\pi}{2}$



- $\mathrm{sinc}(t)=\frac{\sin \pi t}{\pi t}$

基本离散时间序列

- 实指数序列：$x[n] = Ar^n, n \in Z$
  - 若令$r = e^{\beta}, x[n] = Ae^{\beta n}, n \in Z$



- 虚指数序列：$x[n] = e^{j\omega_0 n}$
  - 和正弦信号的关系

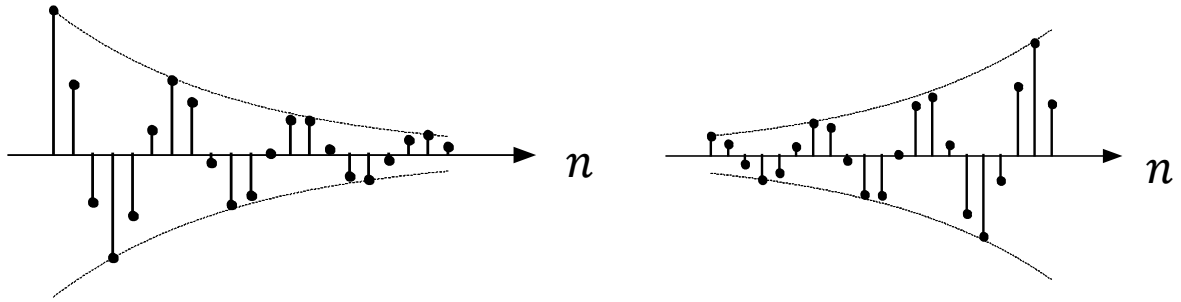
$$e^{j\omega_0 n} = \cos\omega_0 n + j\sin\omega_0 n$$

$$A\cos(\omega_0 n + \phi) = \frac{A}{2}e^{j\phi}e^{j\omega_0 n} + \frac{A}{2}e^{-j\phi}e^{-j\omega_0 n}$$

- 复指数序列：

$$x[n] = Az^n = |A|e^{j\theta}\left(|z|e^{j\omega_0}\right)^n$$

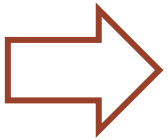
$$= |A||z|^n\cos(\omega_0 n + \theta) + j|A||z|^n\sin(\omega_0 n + \theta)$$



衰减正弦信号　　增幅正弦信号

连续信号性质的推广？

- 虚指数信号：$x(t) = e^{j\omega_0 t}$
  - **性质1**：ω_0越大，信号震荡速率越高

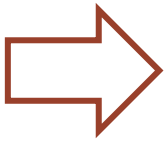


- 虚指数序列：$x[n] = e^{j\omega_0 n}$
  - **性质1**：ω_0和信号震荡速率的关系

- **性质2**：
  - **周期**性：$x(t) = e^{j\omega_0 t} = e^{j\omega_0 (t+T_0)}$

$$T_0 = \frac{2\pi}{|\omega_0|}$$

对任意ω_0都成立



- **性质2**：对任意ω_0都能保持周期性吗？

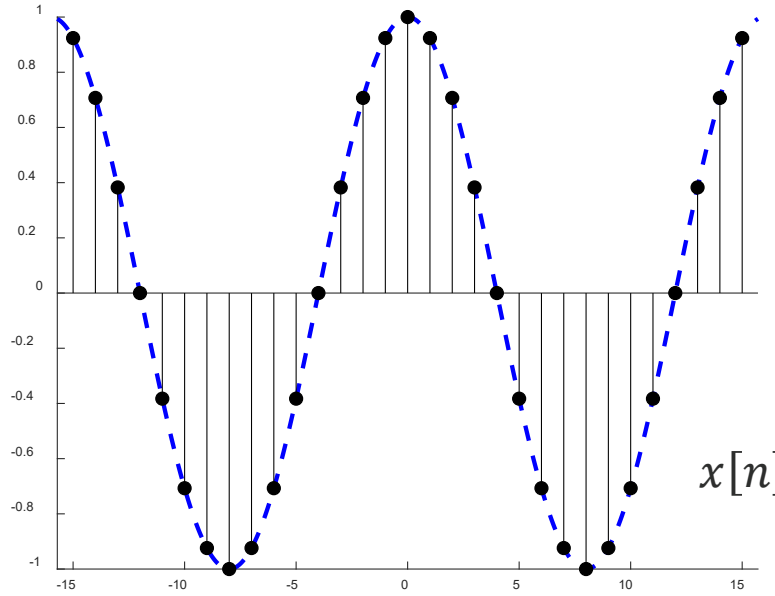
虚指数序列频率震荡的周期性

- 虚指数序列：$x[n] = e^{j\omega_0 n}$
  - （针对ω_0，频率震荡的）周期性

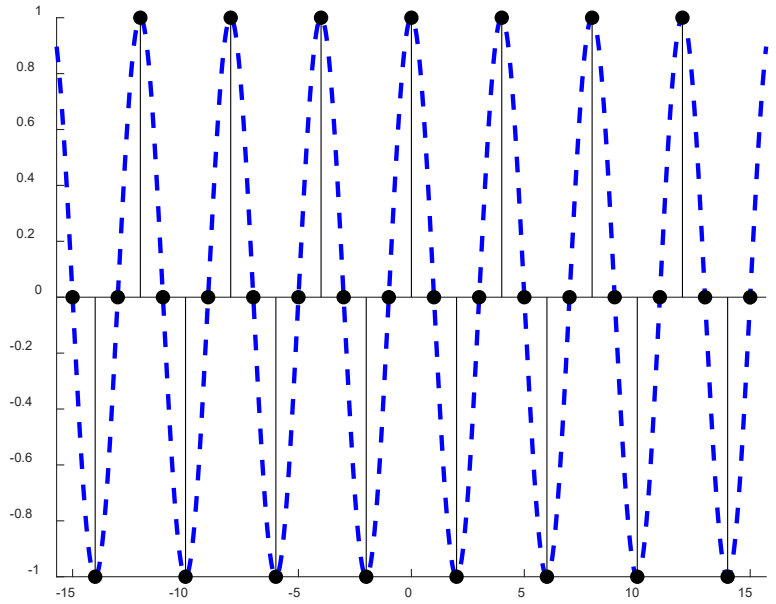
$$e^{j(\omega_0+2\pi)n} = e^{j\omega_0 n}e^{j2\pi n} = e^{j\omega_0 n}$$

信号频率震荡具有周期性

频率为$\omega_0, \omega_0 \pm 2\pi, \omega_0 \pm 4\pi$ …的虚指数信号，只要考虑一个大小为2π的间隔即可



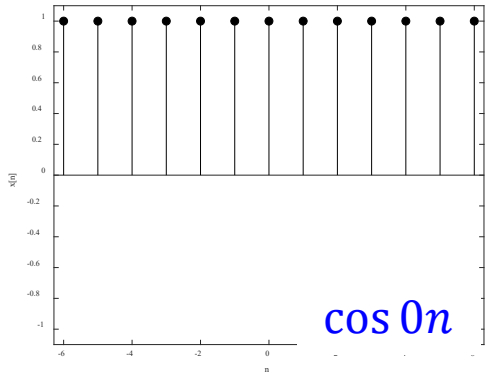
$x[n] = \cos(\pi n/8)$

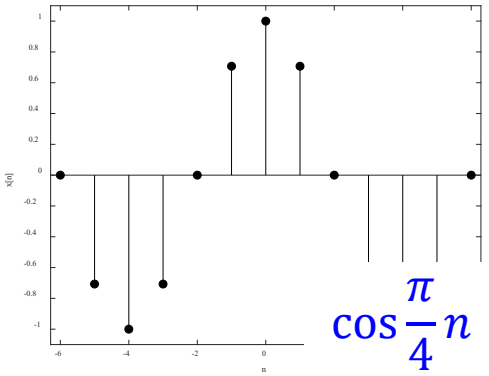


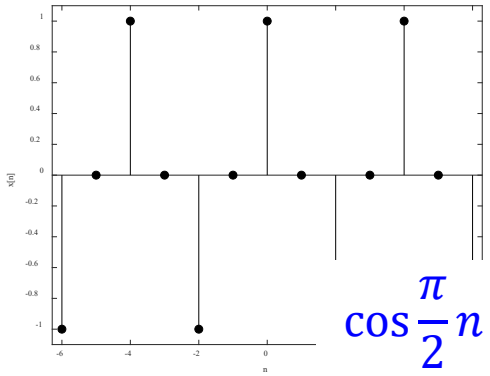
$x[n] = \cos(\pi n/2)$

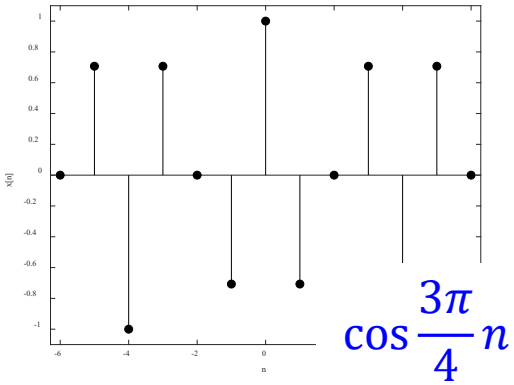
虚指数序列频率震荡的周期性

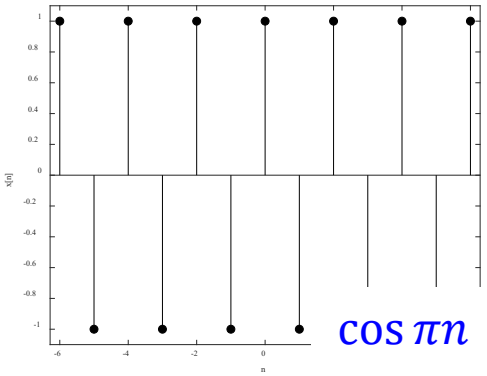
- 虚指数序列：$x[n] = e^{j\omega_0 n}$
  - $e^{j\omega_0 n}$周期为2π，不具有随ω_0增加而增加**震荡速率**的性质
  - $\cos(\omega_0 + 2k\pi)n = \cos\omega_0 n$

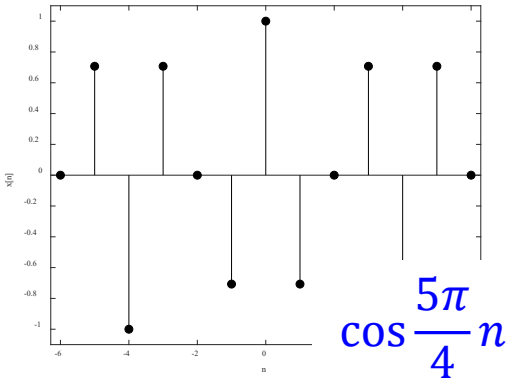


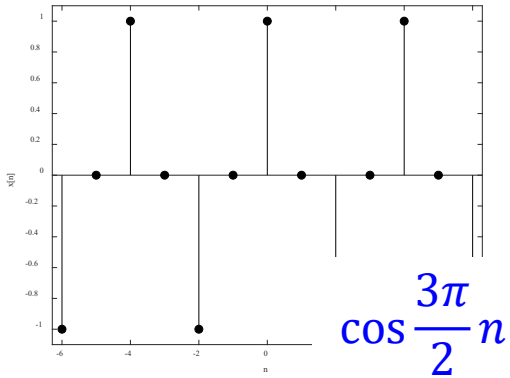


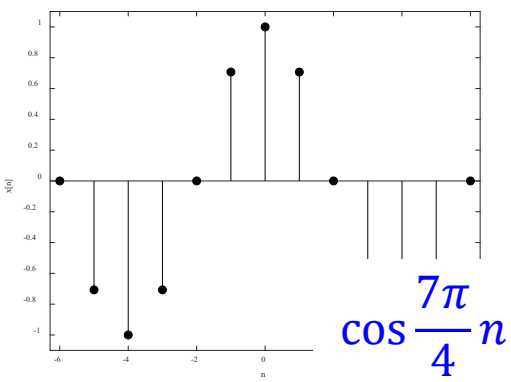






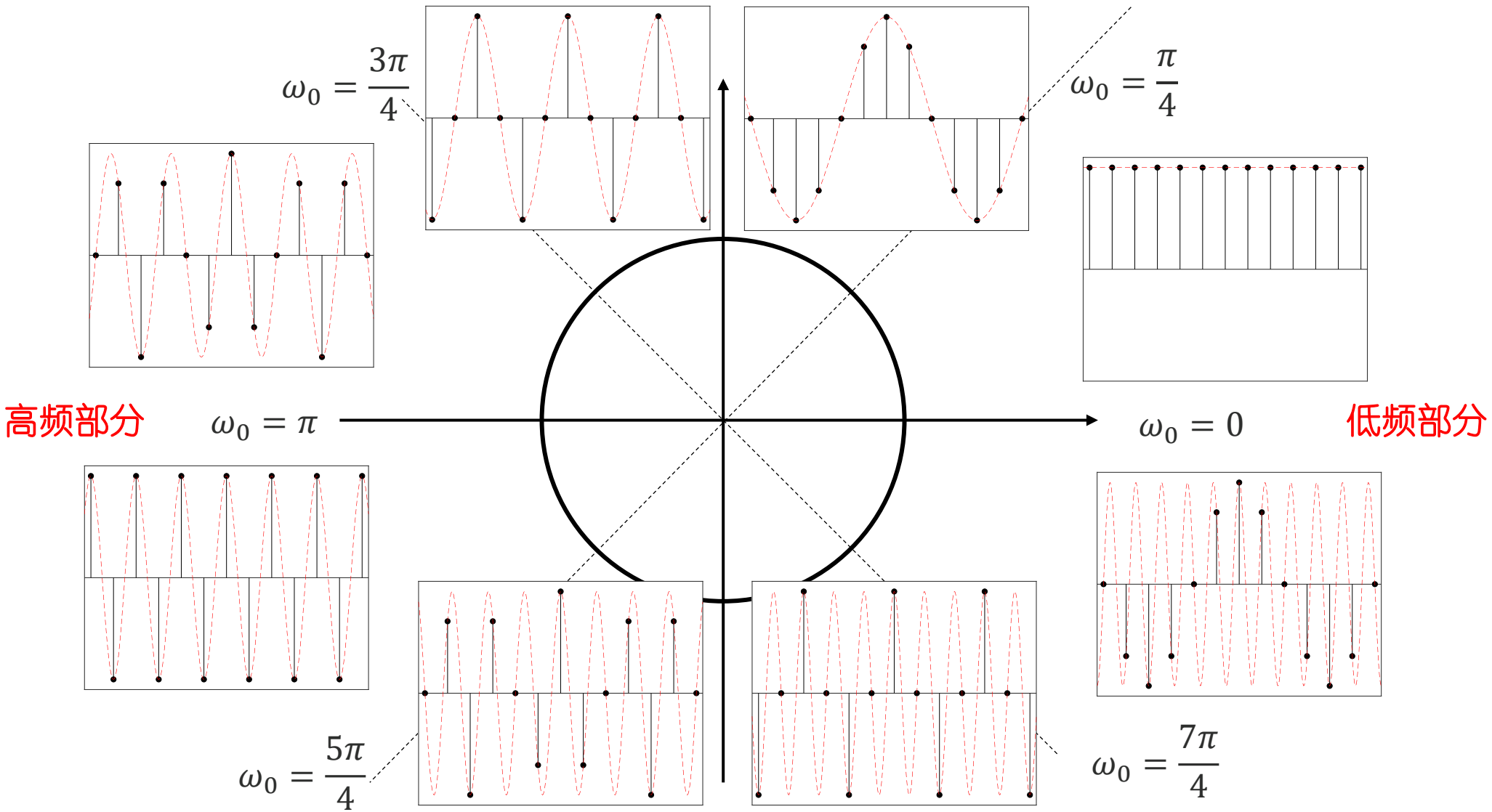






虚指数序列频率震荡的周期性

虚指数序列：$\cos(\omega_0 + 2k\pi)n = \cos\omega_0 n$



虚指数序列关于时间的周期性

- 虚指数序列：$x[n] = e^{j\omega_0 n}$
  - （**时间单位n的**）周期性（要求$x[n+N]=x[n]$），若

$$e^{j(n+N)\omega_0} = e^{j\omega_0 n}e^{jN\omega_0} = e^{j\omega_0 n}$$

则要求

$$e^{j\omega_0 N} = 1$$

即**$\omega_0 N$需为2π的整数倍**：$\omega_0 N = m2\pi$，$m=$正整数时，信号是周期信号

- 关联离散信号的产生过程（周期信号相乘）
  - 若$e^{j(\omega_0+N)n} = e^{j\omega_0 n}e^{jNn} = e^{j\omega_0 n}$则要求$e^{j\omega_0 N} = 1$
  - 即**$\omega_0 N$需为2π**的整数倍：$\omega_0 N = m2\pi$，m为与N互质的正整数时，信号是周期信号

虚指数序列关于时间的周期性

- 虚指数序列：$x[n] = e^{j\omega_0 n}$
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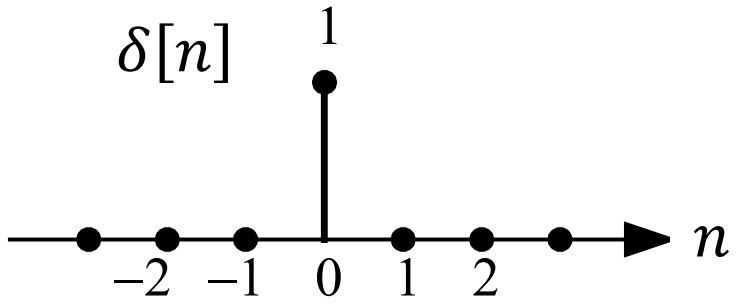
  - 即$\boldsymbol{\omega_0 N}$需为$\boldsymbol{2\pi}$的整数倍：$\omega_0 N = m2\pi$，m为与N互质的正整数时，信号是周期信号

连续信号$e^{j\omega_0 t}$	**离散时间序列信号$e^{j\omega_0 n}$**
ω_0不同，信号不同	ω_0相差2π整倍数，信号相同
任何ω_0信号均为周期信号	满足$\omega_0 N = m2\pi$才是周期信号
基波周期：$\begin{cases} \omega_0 = 0, \text{无定义} \\ \omega_0 \neq 0, T_0 = 2\pi/\omega_0 \end{cases}$	基波周期：$\begin{cases} \omega_0 = 0, \text{无定义} \\ \omega_0 \neq 0, N = 2\pi m/\omega_0 \end{cases}$

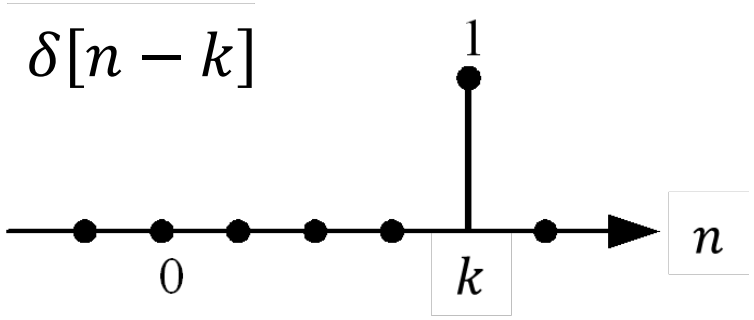
离散时间单位脉冲

- 单位脉冲（unit impulse）序列

$$\delta[n]=\begin{cases}1, n=0\\0, n\neq 0\end{cases}$$

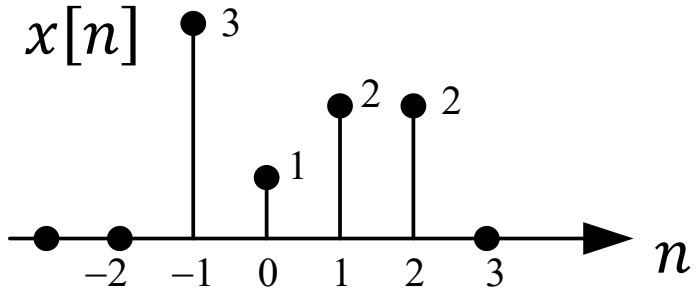


- 单位脉冲序列的基本性质
  - $x[n]\delta[n]=x[0]\delta[n]$;
  - $x[n]\delta[n-n_0]=x[n_0]\delta[n-n_0]$



- 使用单位脉冲表示任意离散时间信号

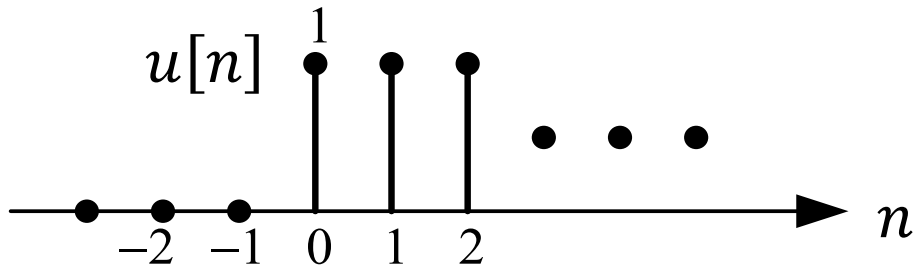
$$x[n]=3\delta[n+1]+\delta[n]+2\delta[n-1]+2\delta[n-2]$$



离散时间单位阶跃

- 单位阶跃（unit step）序列

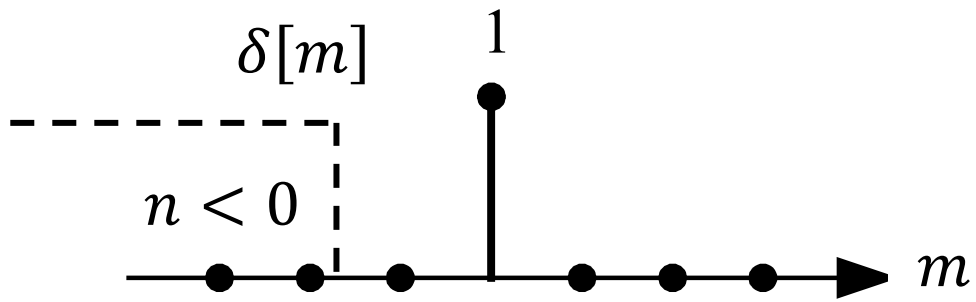
$$u[n]=\begin{cases}1, n\geq 0\\ 0, n<0\end{cases}$$

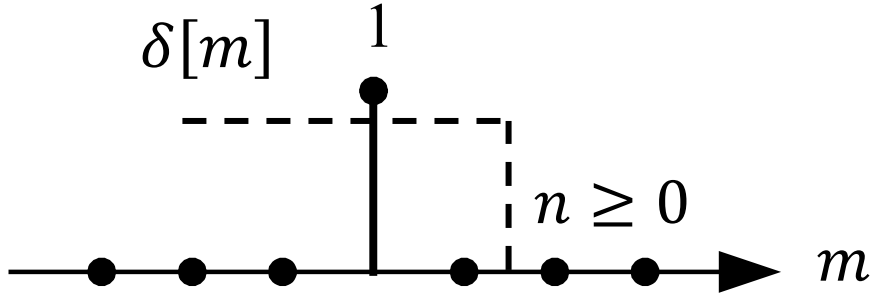


- 单位阶跃序列和单位脉冲序列的相互表示

$$\delta[n]=u[n]-u[n-1]$$

$$u[n]=\sum_{m=-\infty}^{n}\delta[m]$$

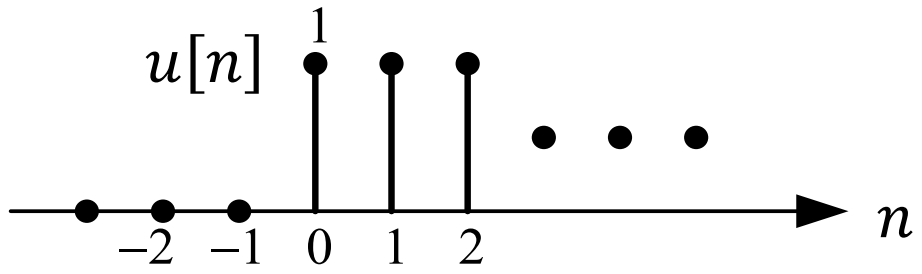




离散时间单位阶跃

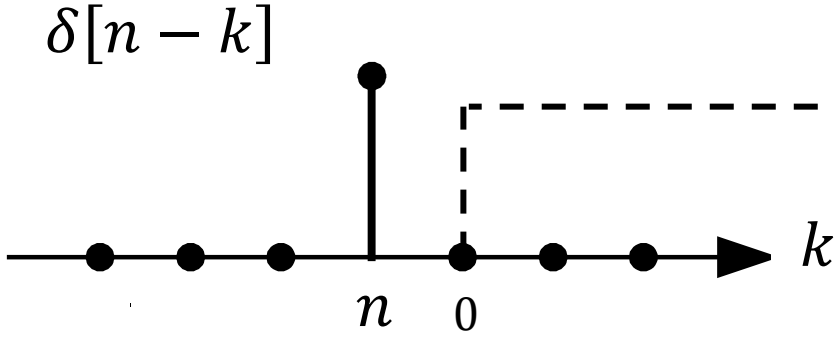
- 单位阶跃（unit step）序列

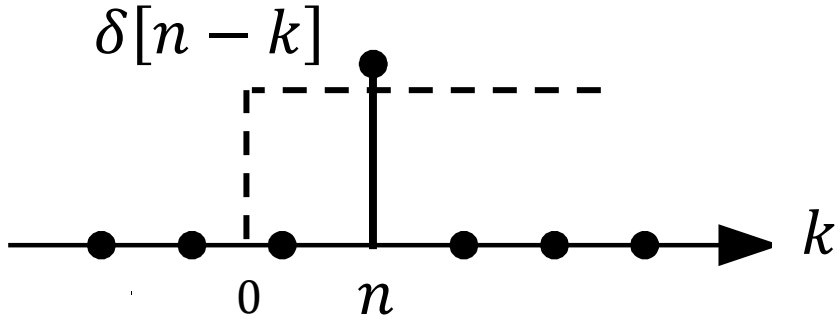
$$u[n]=\begin{cases}1, n\geq 0\\ 0, n<0\end{cases}$$



- 单位阶跃序列和单位脉冲序列的相互表示
  - 将$u[n]$看作延时脉冲的叠加

$$u[n]=\sum_{k=0}^{\infty}\delta[n-k]$$



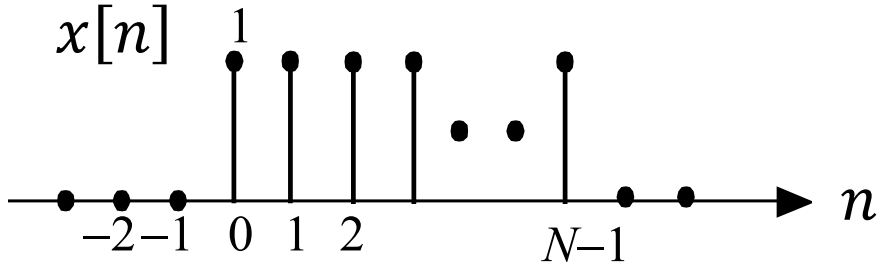


其他基本离散时间序列

- 矩形序列

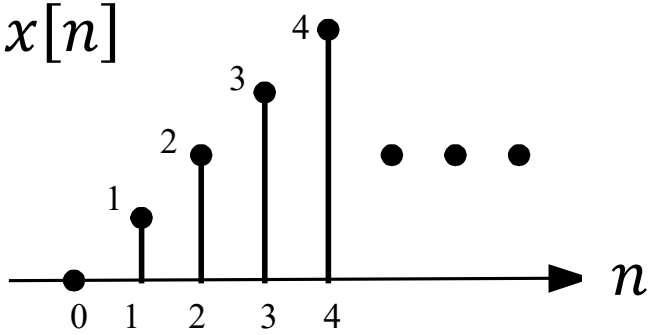
$$r[n]=\begin{cases}1, 0\le n\le N-1\\ 0, \quad o.w.\end{cases}$$

$$r[n]=u[n]-u[n-N]=\sum_{m}^{N-1}\delta[n-m]$$



- 斜变序列

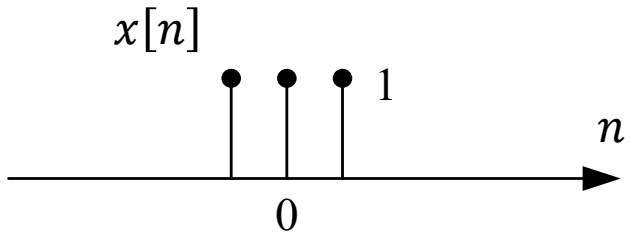
$$x[n]=nu[n]=\sum_{k=0}^{\infty}k\delta[n-k]$$

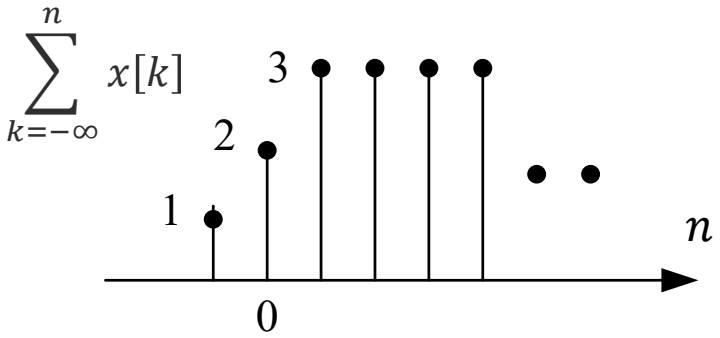


离散信号的差分和求和

- 求和

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$





- 差分

前向差分

- 一阶：$\Delta x[n] = x[n+1] - x[n]$
- 二阶：$\Delta^2 x[n] = \Delta(\Delta x[n])$

$= x[n+2] - 2x[n+1] + x[n]$

后向差分

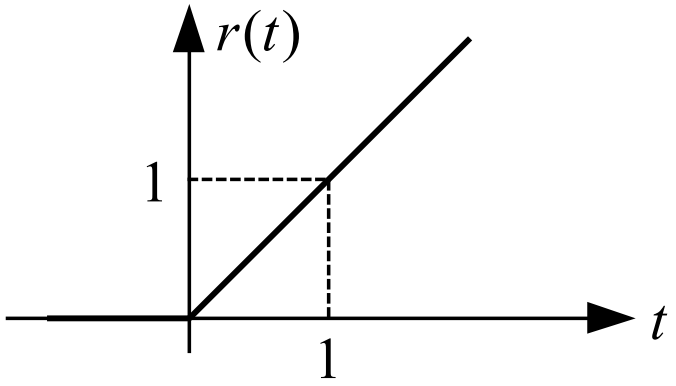
- 一阶：$\nabla x[n] = x[n] - x[n-1]$
- 二阶：$\nabla^2 x[n] = \nabla(\nabla x[n])$

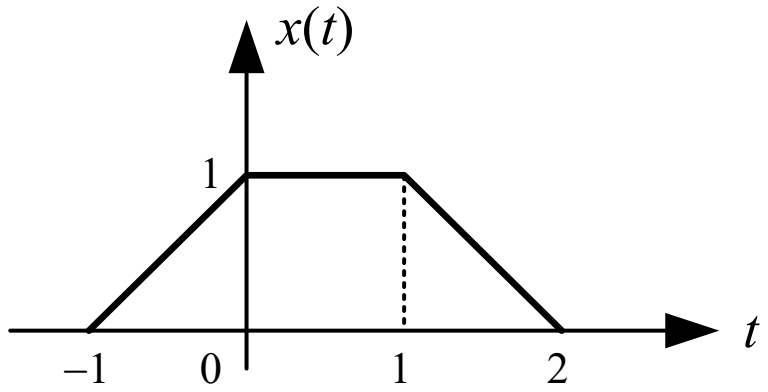
$= x[n] - 2x[n-1] + x[n-2]$

奇异信号

- 奇异信号：信号本身**或其导数**具有**不连续点**（跳变）
- 单位斜变信号

$$r(t)=\begin{cases}t, t\geq 0\\ 0, t<0\end{cases}$$



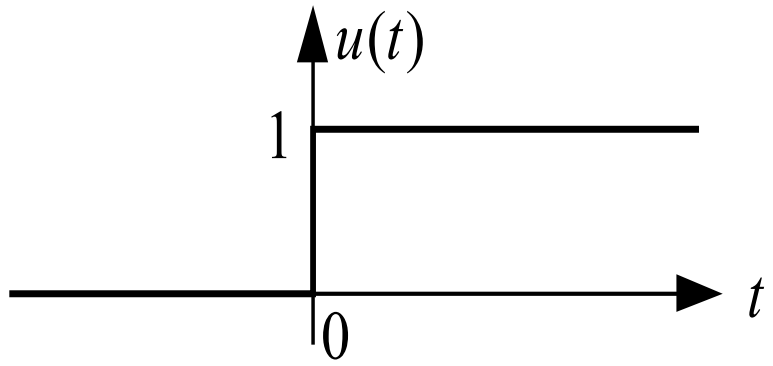


$$x(t)=r(t+1)-r(t)-r(t-1)+r(t-2)$$

单位阶跃信号

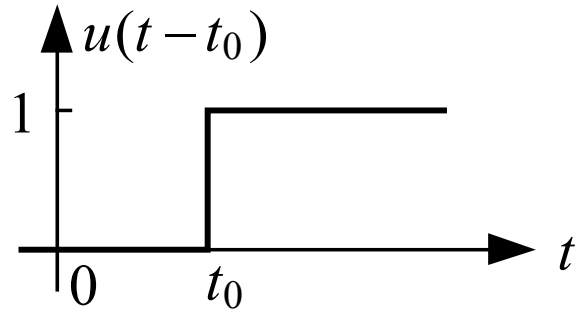
- 物理意义：t=0对某电路接入单位电源，并无线持续

$$u(t)=\begin{cases}1; t>0\\0; t<0\end{cases}$$



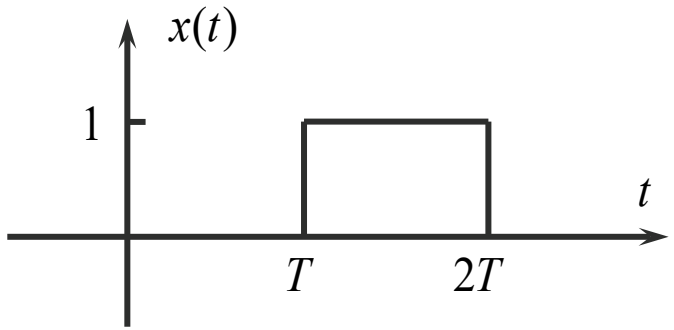
信号移位

$$u(t-t_0)=\begin{cases}1; t>t_0\\0; t<t_0\end{cases}$$



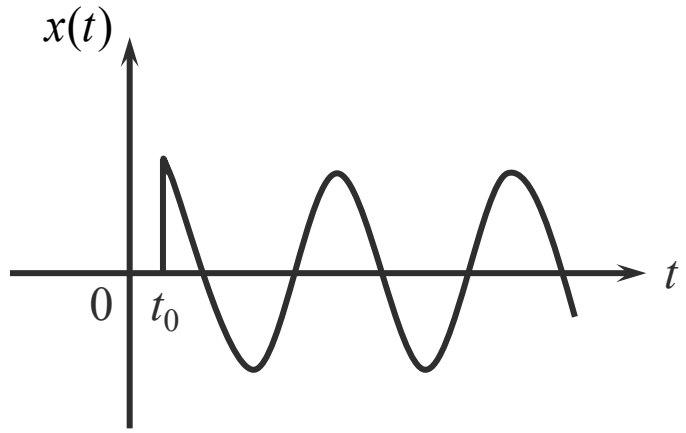
方波表示

$$x(t)=u(t-T)\text{-}u(t-2T)$$



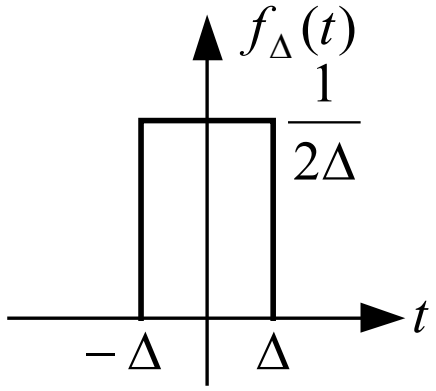
信号截断

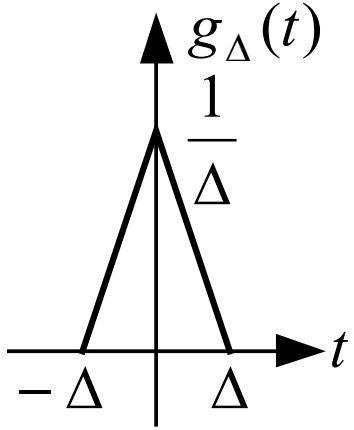
$$x(t)=\sin\omega_0 t\ u(t-t_0)$$

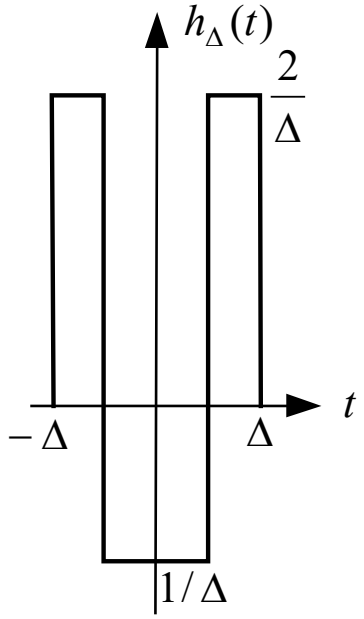


冲激信号

- 冲激信号的极限模型
  - $\delta(t) = \lim_{\Delta\to 0} f_\Delta(t) = \lim_{\Delta\to 0} g_\Delta(t) = \lim_{\Delta\to 0} h_\Delta(t)$







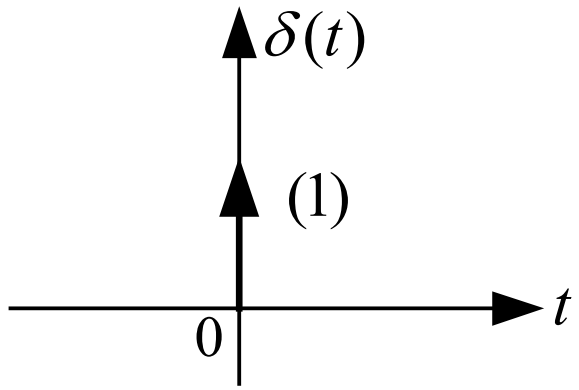
- 冲激信号的狄拉克(Dirac)定义

$$\delta(t) = 0, t \neq 0;$$

$$\int_{\infty}^{\infty} \delta(t)\,\mathrm{d}t = 1$$

冲激信号

- 冲激信号
  - 表征**作用时间极短**，**作用值很大**的物理现象的数学模型
    - 如闪电、瞬间作用冲击力、抽样脉冲等



  - 冲激信号的移位$\delta(t-t_0)$
  - 冲激信号**具有强度**，其强度就是冲激信号对时间的定积分值。在图中用括号注明，以区分信号的幅值。
  - 冲激信号可用于**表示其他任意信号**或**表示信号间断点的导数**

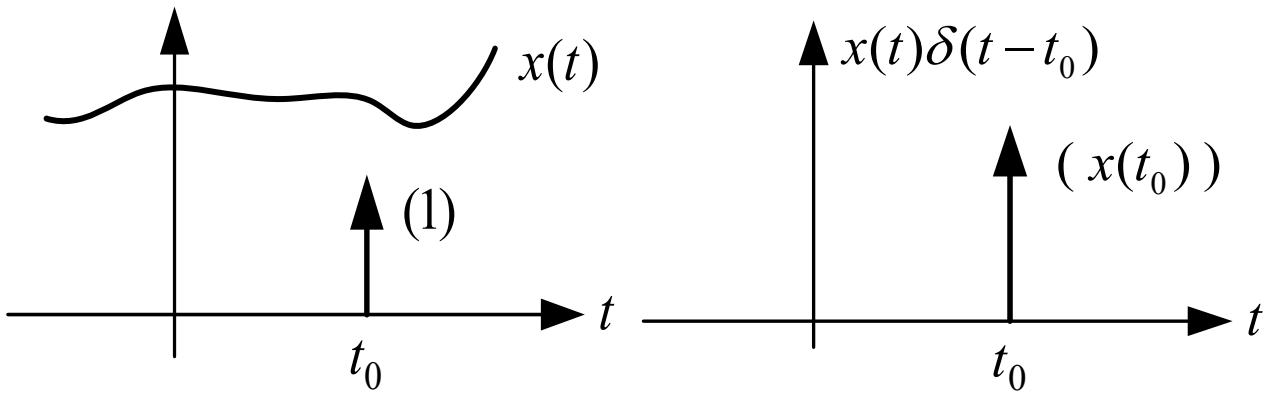
冲激信号的性质

- **抽样（筛选）**特性：

$$x(t)\delta(t-t_0)=x(t_0)\delta(t-t_0)$$

$$\int_{-\infty}^{\infty}x(t)\delta(t-t_0)\mathrm{d}t=\int_{-\infty}^{\infty}x(t_0)\delta(t-t_0)\mathrm{d}t$$

$$=x(t_0)\int_{-\infty}^{\infty}\delta(t-t_0)\mathrm{d}t=x(t_0)$$



- 信号的冲激表示
  - 信号$x(t)$可用冲激信号作为"单位"进行表示

$$\int_{-\infty}^{\infty}x(t)\delta(t-t_0)\mathrm{d}t=x(t_0)$$

令$t=\tau$　　令$t_0=t$

$$\int_{-\infty}^{\infty}x(\tau)\delta(\tau-t)\mathrm{d}\tau=x(t)$$

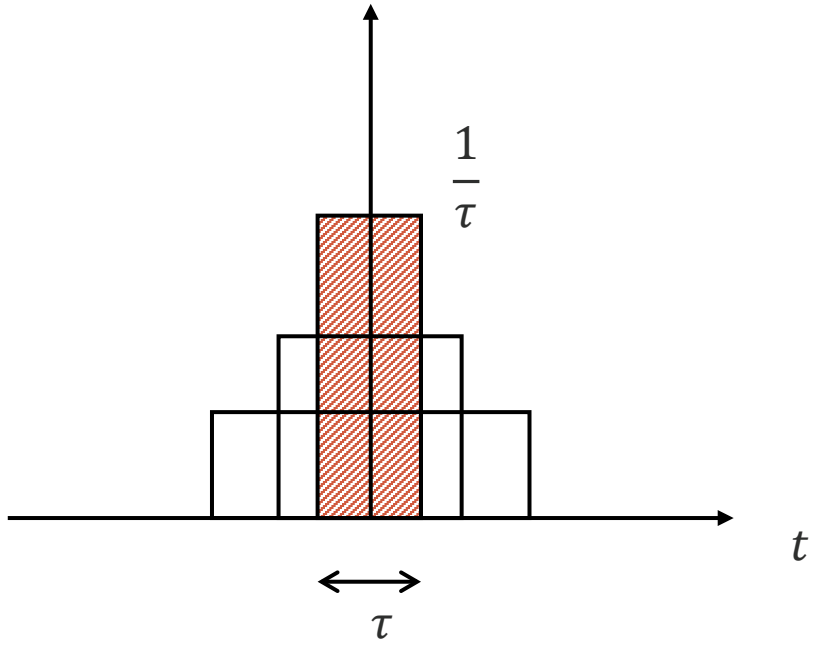
冲激信号的性质

- **展缩**特性：

$$\delta(\alpha t) = \frac{1}{|\alpha|}\delta(t), (\alpha \neq 0)$$

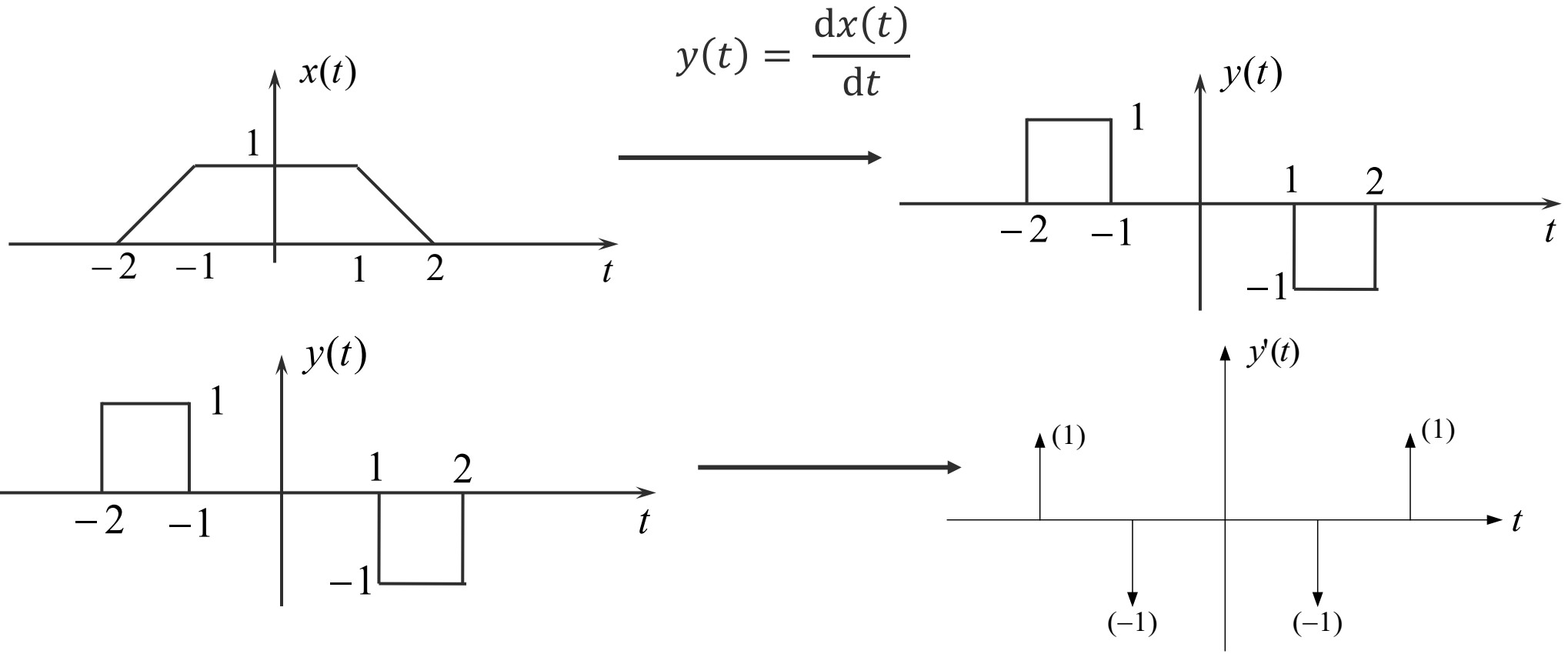
  - $\alpha = -1$有$\delta(-t) = \delta(t)$, 因此$\delta(t)$为**偶函数**

- 对于$\delta(at + b)$形式的冲激信号，要先利用冲激信号的**展缩特性**将其化为$\delta(t + b/a)\ /|a|$形式后，方可利用冲激信号的抽样特性与筛选特性。



信号的微分

- 信号的微分

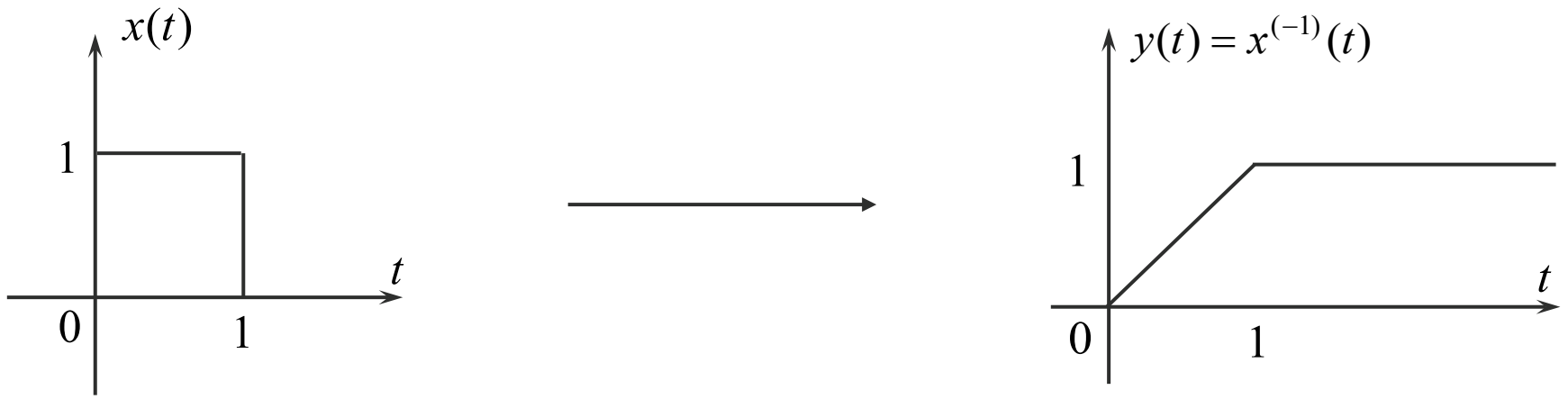


- 信号经过微分运算后突出显示了它的变化部分，起到了**锐化**的作用

信号的积分

- 信号的积分

$$y(t) = \int_{-\infty}^{t} x(\tau)\mathrm{d}\tau = x^{(-1)}(t)$$



- 信号经过积分运算后，使得信号突出变化部分变得平滑了，起到了**模糊**的作用；利用积分可以削弱信号中噪声的影响。

本章概要

1. 信号的分类：重点关注信号的周期性，功率特性

2. 信号的运算：信号的尺度变换、翻转、时移 ...

3. 典型的信号：
一般信号和奇异信号

4. 信号的分解：
如何有效地“表示”信号

基于冲激信号进行信号分解

- 脉冲分量：信号分解为冲激信号之和

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\mathrm{d}\tau$$

  - 当求解信号通过系统产生的响应时，只需求解**冲激信号**通过该系统产生的响应，然后利用线性时不变系统的特性，进行**迭加和延时**即可求得信号$x(t)$产生的响应。

- 离散信号的分解

  - 任意序列可以分解为单位脉冲序列及其位移的和

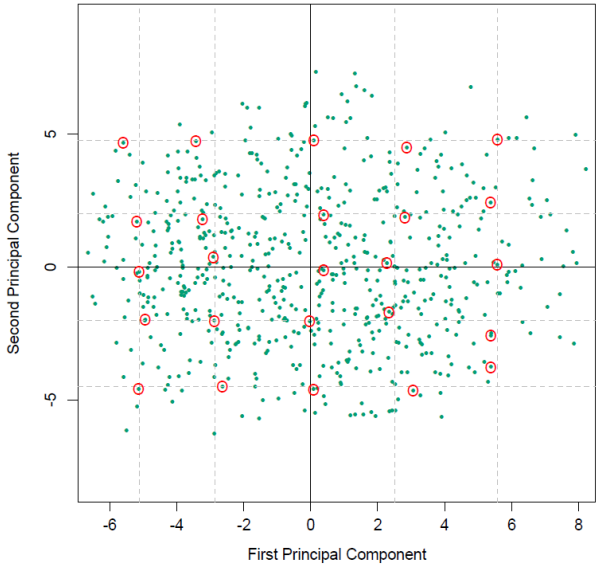
$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

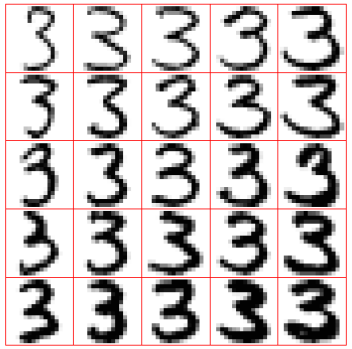
针对一组向量的分解

- 【类比】主成分分析 （PCA）：针对一组向量的分解
  - 原样本点x_i与基于投影重构的样本点$\hat{x}_i$之间的距离为

$$\sum_{i=1}^{m}\left\|\sum_{j=1}^{d'} z_{ij}\boldsymbol{w}_j-\boldsymbol{x}_i\right\|_2^2=\sum_{i=1}^{m}\boldsymbol{z}_i^{\mathrm{T}}\boldsymbol{z}_i-2\sum_{i=1}^{m}\boldsymbol{z}_i^{\mathrm{T}}\mathbf{W}^{\mathrm{T}}\boldsymbol{x}_i+\text{const}$$

$$\propto -\mathrm{tr}\left(\mathbf{W}^{\mathrm{T}}\left(\sum_{i=1}^{m}\boldsymbol{x}_i\boldsymbol{x}_i^{\mathrm{T}}\right)\mathbf{W}\right)$$





$$\hat{f}(\lambda)=\text{[3]}+\lambda_1\cdot\text{[3]}+\lambda_2\cdot\text{[3]}.$$

  - ω_j是正交基，$\sum_i x_i x_i^{\mathrm{T}}$是协方差矩阵

是否能够借鉴向量分解的方法针对信号（函数）进行分解?

信号的“向量”表示

将分析向量的基本概念进行扩展

- 范数
  - 表示信号作用的**强度**

$$\|\boldsymbol{x}\|_1 = \int_{-\infty}^{\infty} |x(t)| \mathrm{d}t$$

  - （平方）表示信号的**能量**

$$\|\boldsymbol{x}\|_2 = \sqrt{\int_{-\infty}^{\infty} |x(t)|^2 \mathrm{d}t}$$

  - 表示信号的**峰值**

$$\|\boldsymbol{x}\|_\infty = \sup|x(t)|$$

- 内积
  - 一定程度上刻画了信号的相似性

$$\langle x_1(t), x_2(t) \rangle = \int_{-\infty}^{\infty} x_1(t) x_2(t)\, \mathrm{d}t$$

  - 范数与内积

$$\langle x(t), x(t) \rangle = \int_{-\infty}^{\infty} x^2(t)\, \mathrm{d}t = \|\boldsymbol{x}\|_2$$

正交函数

- 在区间（$t_1 < t < t_2$）内用函数$x_2(t)$重构$x_1(t)$，即

$$x_1(t) = c_{12}x_2(t)$$

- 优化c_{12}，使**均方误差**最小

$$\overline{\epsilon^2} = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} [x_1(t) - c_{12}x_2(t)]^2 \, \mathrm{d}t$$

- 得

$$c_{12} = \frac{\int_{t_1}^{t_2} x_1(t)x_2(t) \, \mathrm{d}t}{\int_{t_1}^{t_2} x_2^2(t) \, \mathrm{d}t} = \frac{\langle x_1(t), x_2(t) \rangle}{\langle x_2(t), x_2(t) \rangle}$$

- 若$\langle x_1(t), x_2(t) \rangle = 0$则两函数**正交**

正交函数集

- 若n个函数$g_1(t), g_2(t), \dots, g_n(t)$构成函数集，且在区间$(t_1, t_2)$满足

$$\begin{cases} \langle g_i(t), g_j(t) \rangle = 0, i \neq j \\ \langle g_i(t), g_i(t) \rangle = K_i^2 \end{cases}$$

- 则函数集为**正交函数集**。重构函数$x(t) = \sum_{r=1}^{n} c_r g_r(t)$

$$\overline{\epsilon^2} = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} \left[x(t) - \sum_{r=1}^{n} c_r g_r(t) \right]^2 \mathrm{d}t$$

- 有

$$c_r = \frac{\int_{t_1}^{t_2} x(t) g_r(t)\, \mathrm{d}t}{\int_{t_1}^{t_2} g_r^2(t)\, \mathrm{d}t} = \frac{\langle x(t), g_r(t) \rangle}{K_r^2}$$

完备函数集

- 若n个函数$g_1(t), g_2(t), \dots, g_n(t)$构成正交函数集，不存在$g(t)$满足

$$0 < \int_{t_1}^{t_2} g^2(t)\,\mathrm{d}t < \infty$$

且

$$\langle g(t), g_r(t)\rangle = 0, r = 1, \dots, n$$

则$g_1(t), g_2(t), \dots, g_n(t)$构成**完备**正交函数集，使$\overline{\epsilon^2} = 0$

- **帕塞瓦尔定理**

$$\int_{t_1}^{t_2} x^2(t)\,\mathrm{d}t = \sum_{r}^{\infty} c_r^2$$

  - 信号所含有的功率恒等于此信号在完备正交函数集中各分量功率的总和

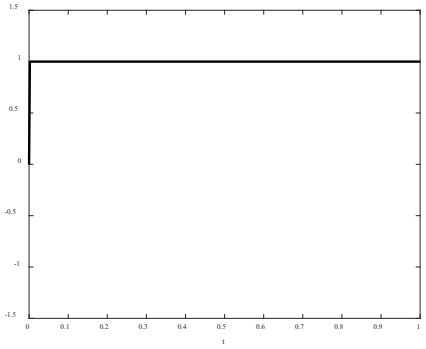
完备函数集

- 三角函数集
- 复指数函数集

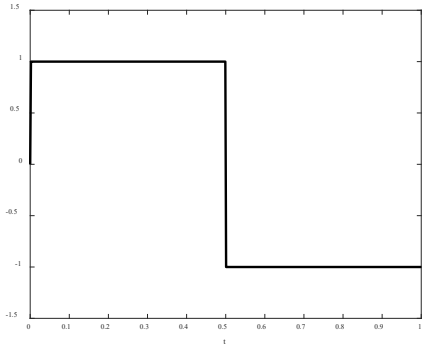
} 傅里叶变换中使用

- Rademacher函数集
  - 使用相互正交的矩形信号表示$f(t)$

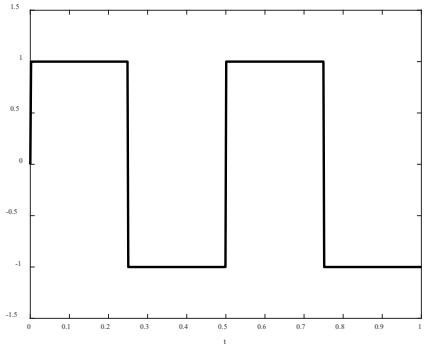
$$r_k(t) = sign \sin 2^k \pi t, \qquad t \in [0,1], \qquad k = 0,1,2, \ldots.$$



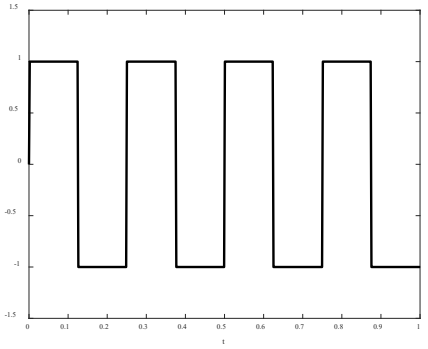
$r_0(t)$



$r_1(t)$



$r_2(t)$

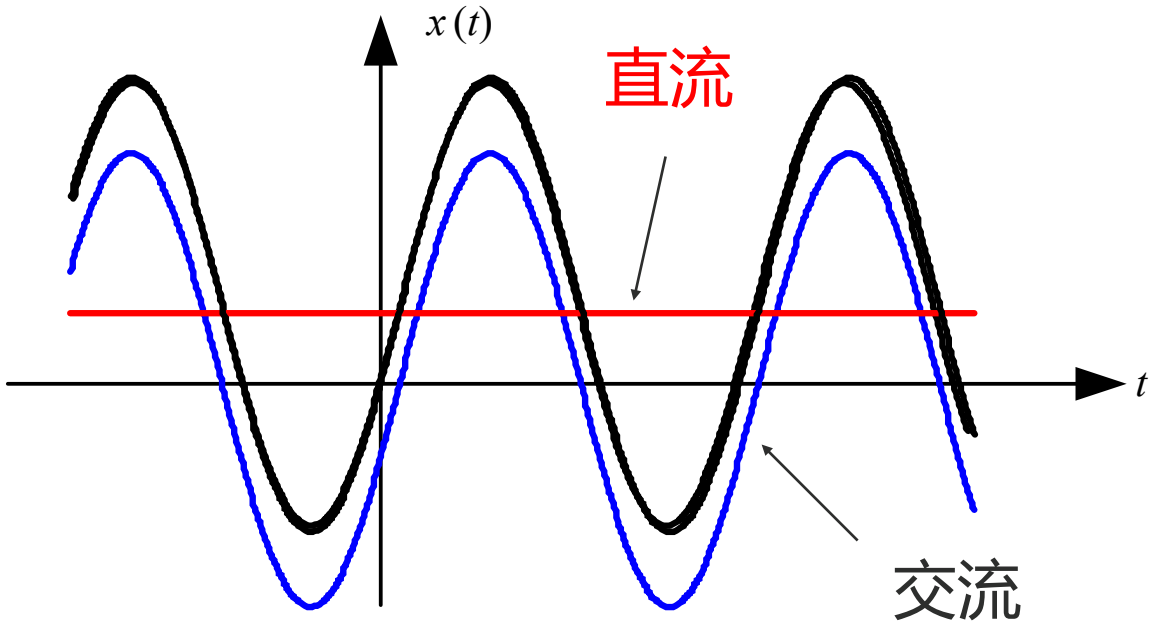


$r_3(t)$

信号的分解

- 直流分量与交流分量

$$x(t) = x_{\mathrm{DC}}(t) + x_{\mathrm{AC}}(t)$$



$$x_{\mathrm{DC}}(t) = \frac{1}{b-a}\int_a^b x(t)\mathrm{d}t$$

信号的分解

- 偶分量与奇分量

$$x(t) = x_e(t) + x_o(t)$$

偶分量　　奇分量

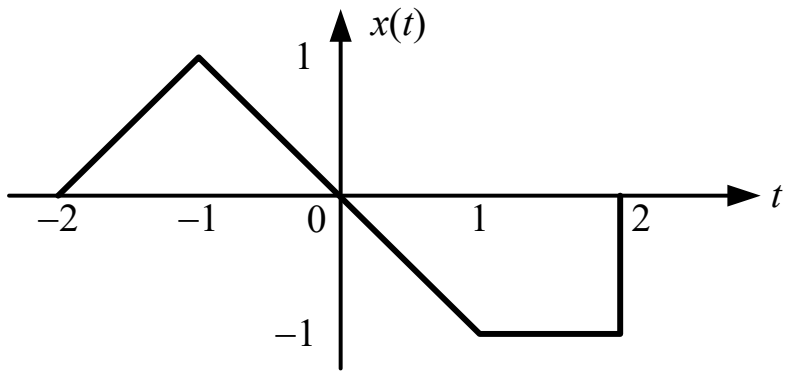
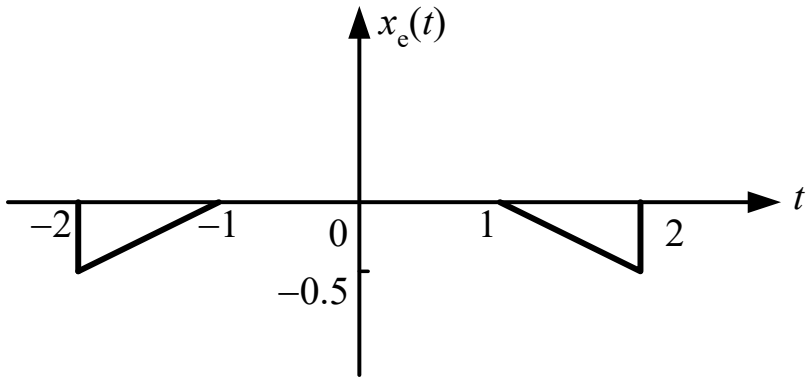
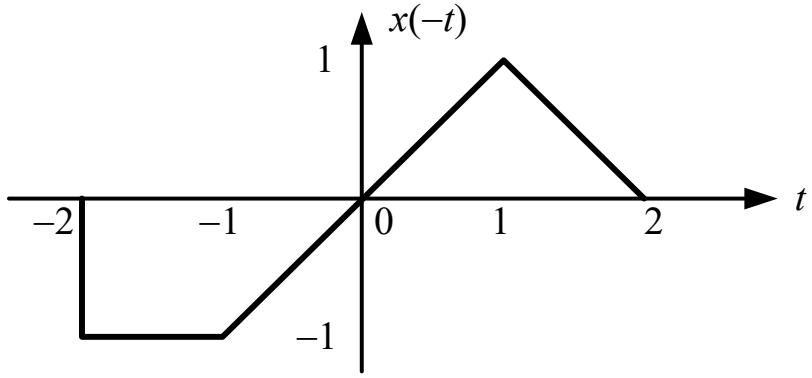
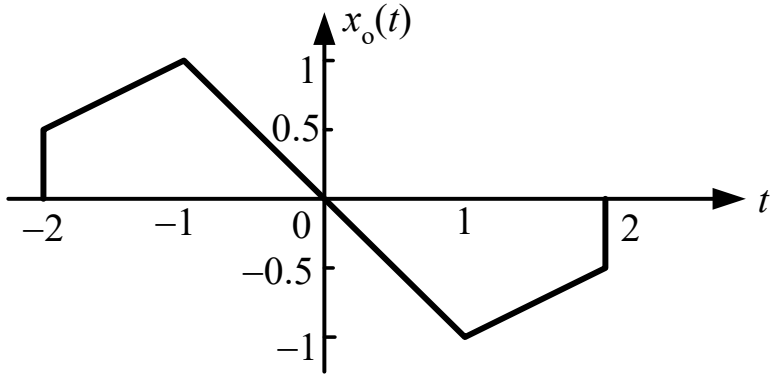
$$x_{\mathrm{e}}(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$x_{\mathrm{e}}(t) = x_{\mathrm{e}}(-t)$$

$$x_{\mathrm{o}}(t) = \frac{1}{2}[x(t) - x(-t)]$$

$$x_{\mathrm{o}}(t) = -x_{\mathrm{o}}(-t)$$

信号的分解

- 偶分量与奇分量
  - 连续时间信号：$x(t) = x_e(t) + x_o(t)$

$$\begin{cases} x_{\mathrm{e}}(t) = \dfrac{1}{2}[x(t) + x(-t)] \\ x_{\mathrm{o}}(t) = \dfrac{1}{2}[x(t) - x(-t)] \end{cases}$$